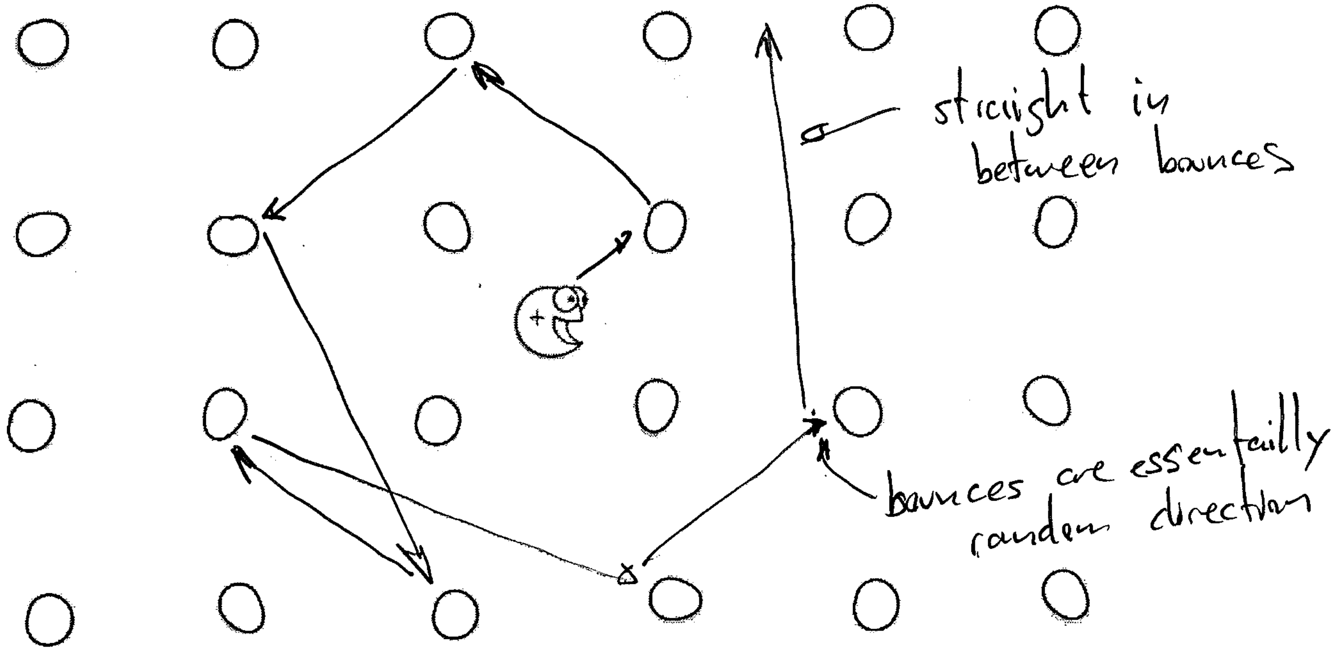
Ion drift through membrane channels
(or how I learned to stop worrying and love current)

Question 1

Imagine an ion bumping around in water, but the water is effectively stationary and represented by the grid of dots below. If the ion starts in the middle going at velocity $\mathbf{v}_0$ (choose initial direction), draw a possible path for the ion.



Now imagine a bunch of ions, all with initial velocities in random directions. What is the average velocity of all the ions at some time?

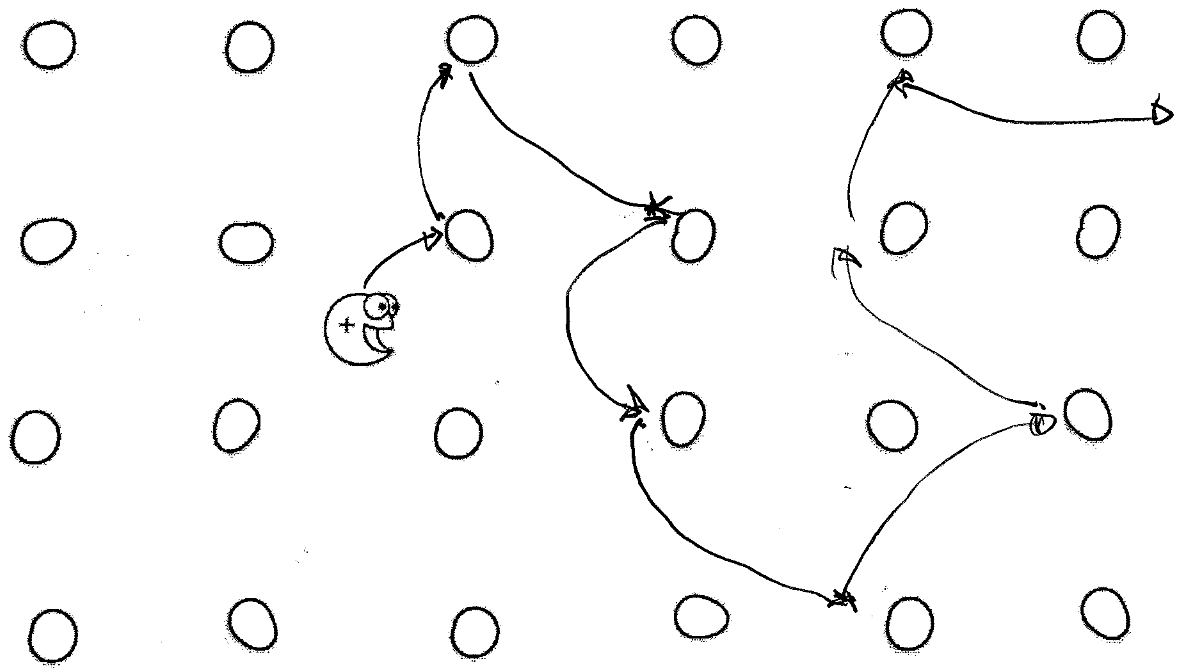
$\langle \mathbf{v} \rangle = 0$

The brackets indicate average for a bunch of ions.

Explain: Since ~~the ions are all~~ each ion is bouncing in a random direction at any given time the net motion of them is zero. For this to be true you must have many ions.

Question 2

Now imagine an ion with charge q and mass m moving in the presence of an electric field E pointing from left to right. Draw a possible path for the ion below.



Now think of the motion between two bounces. Right after a bounce, the ion has some velocity v_0, but it also gets accelerated by an electric field. Write the expression for the velocity between bounces (answer in terms of v_0, q, E, and m, and t, the time since the last bounce).

$$v = \vec{V}_0 + \vec{a}t = \vec{V}_0 + \frac{q\vec{E}}{m}t$$

Now think of a bunch of ions bouncing like this. Suppose that on average, the time between bounces is t_{avg} and that the velocity v_0 after a bounce is essentially random.

What is the average velocity of all these ions at some time? (*Hint: use your previous result for one ion and take the average of this expression for all the ions.*)

from before.

$$\langle v \rangle = \cancel{\langle V_0 \rangle}^{\,0} + \left\langle \frac{q\vec{E}}{m} t \right\rangle$$

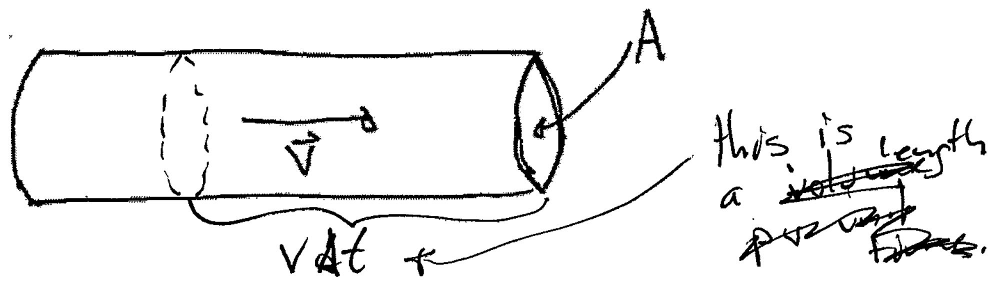
Explain:

$$= \frac{qE}{m} t_{avg}$$

Between each bounce the electron gets accelerated a little bit to the right.

Question 3

The average velocity that the ions move is called the drift velocity. Imagine the channel was a cylinder (wire) with cross-sectional area ***A*** and that a bunch of ions were moving down it with some average velocity <***v***>.



The net flow of these ions (or any other charge) is called current. Assume that the number density of particles carrying charge in the wire is ***n***. Write an expression for the total number of particles that go through ***A*** in a time **Δt**.

$$N = nV = n\langle v\rangle\Delta t\, A$$

Using the above, what is the number of charge carrying particles per unit time that passes through ***A***?

$$\frac{N}{\Delta t} = n\langle v\rangle A$$

If each ion has charge ***q***, what is the total charge per unit time that passes through ***A***?

$$I = q\frac{N}{\Delta t} = q\,n\langle v\rangle A$$

The amount of charge per unit time that passes through ***A*** is called the current. It has unit amperes (amps), and we denote it with a capital ***I***.

Question 4

If you haven't already done so, substitute the equation you derived for the average speed of electrons in question 2 into the expression you derived for current above.

$$I = q n_q \left(q \frac{E}{m} \Delta t \right) A$$

The quantities in the expression for I can be labelled as either macroscopic or microscopic quantities.

I = (microscopic)(macroscopic)

σ

$$I = \left(\frac{q^2 n \Delta t}{m} \right)\left(A \quad E \right)$$

The microscopic quantities are grouped together into one variable called the conductivity σ. How does the current relate to the electric field?

The current is proportional to the electric field. A current only exists if E does.

If you're so inclined, you can use the extra space below to write the current density $J = I/A$ in terms of the electric field and the conductivity.

$$J = \frac{I}{A} = \sigma E$$ how pretty.

Ta Dah!

Question 5 (extra)

~~The drift~~ speed vs. ~~electron speed.~~

a) A 1.0A current passes through a 1.0 mm wire, which is a typical size for circuit wires. What is the drift speed of the electrons? Assume $n_e = 6 \times 10^{28}\ m^{-3}$.

b) Assume that the electrons in a conductor act like gas near room temperature, calculate the average speed of an electron bouncing around in a conductor.

c) Compare the two numbers.