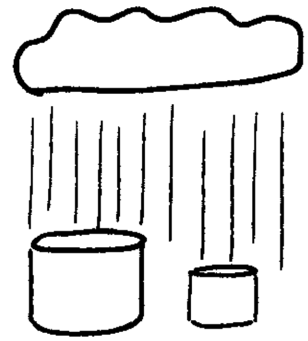
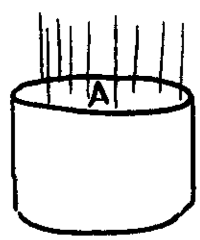
Suppose it's raining really hard and we want to measure just how hard it's raining. We might put out a bucket and see what volume of water goes into the bucket per unit time. This is the **FLUX** of water through the opening of the bucket.



But this will depend on the size of the bucket: if we used a bucket with twice the opening area, we would get twice as much water per unit time. So to measure how hard it's raining, we really want to take the volume of rain going into the bucket per unit time and divide by the area of the bucket. This number gives us a direct measurement of how hard it's raining. Let's call it the **INTENSITY** of the rainfall.

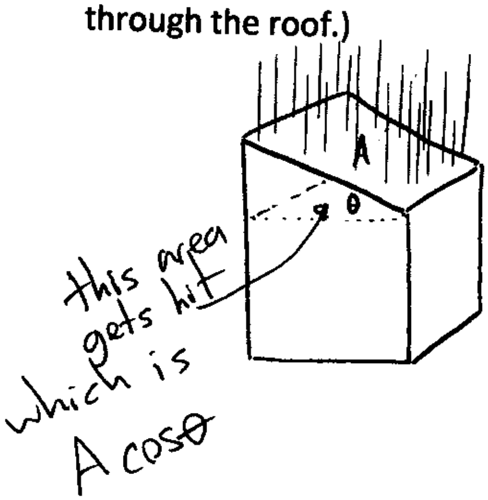
Question 1

a) Suppose we have rain with intEnsity **E** (measured in liters per second per m^2) falling directly down into a bucket with opening area A. Using our definitions, what is the flux of water into the bucket?



Flux = EA

b) The same rain with intensity **E**, falls directly down onto a slanty roof with area A at an angle θ to the horizontal as shown. What is the flux of rain hitting the surface? (If you want, you can imagine the rain going straight through the roof.)

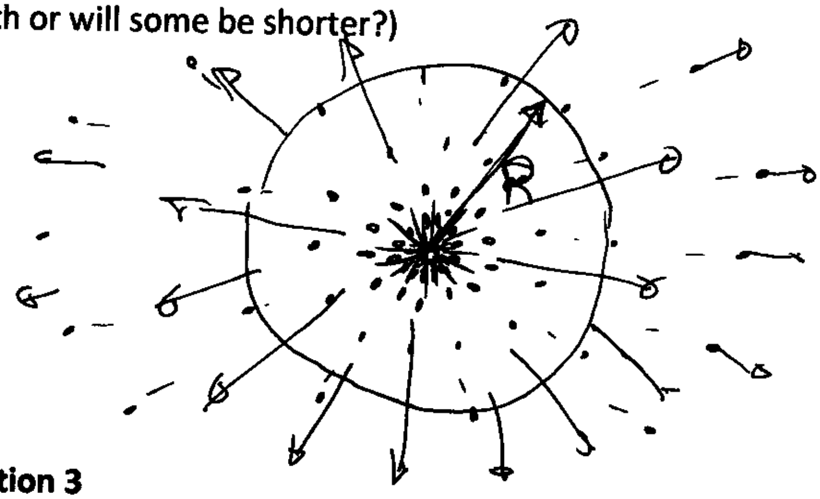


$$\Phi = EA\cos\theta$$
$$= \vec{E}\cdot\vec{A}$$

Question 2

We can use the same definition (amount of water per time per area) to describe the intensity of water flow in any other situation (e.g. coming from a sprinkler). In these cases, it might be that the water is flowing more intensely in one place than another (e.g. narrow vs wide parts of a river). It may also be flowing in different directions in different places. So it's useful to draw vectors to show the direction and intensity of flow at various places. The vector points in the flow direction and the length is the intensity.

On the picture below, water is shooting out in all directions from a source (some droplets are shown). Draw some vectors to show the flow direction and intensity at various places, ignoring effects of gravity. (Should all your vectors be the same length or will some be shorter?)



theres less water further away, (think of getting sprayed) So the arrows are shorter.

Question 3

Let's define **Q** to be the Quantity of water per unit time coming from the source.

a) Consider a spherical surface of radius R (area $4\pi R^2$) around the sprinkler head (you can draw it on the picture above). What is the flux of water through this surface?

Flux = $Q = EA = E 4\pi R^2$

The flux is the amount of water coming out of the spout. All the water must go through the surface. It's also equal to EA.

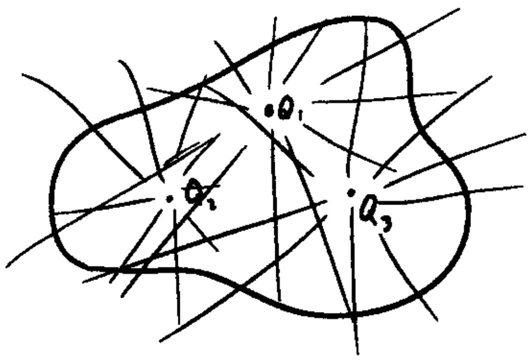
b) What is the intensity E of water flow at the surface?

E = $\frac{Q}{4\pi R^2}$

Hopefully, you have found that ***the flow intensity E of water is related to the flow rate Q from a source in exactly the same way that the electric field from a point charge is related to the charge*** *(if we divide the right side by ε_0). This means that anything that is true of water flow from a collection of sprinklers will be true about the electric field from a collection of charges.*

Question 4

Suppose we have a bunch of water sources, all shooting water out uniformly in all directions. If the flow rates are given by Q_1, Q_2, Q_3, etc.... What is the total flux of water through the surface shown?

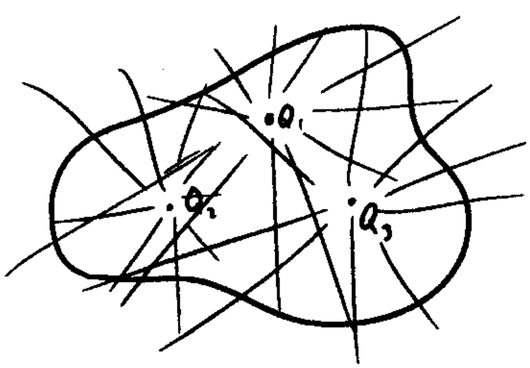


$\Phi = Q_1 + Q_2 + Q_3 + \ldots$

All the water from each sprinkler must go through the surface.

Question 5

We can define ELECTRIC FLUX in terms of the electric field E in the same way that flux of water was related to the flow intensity vectors. Using our analogy and your answer to the previous question, what can we say about the electric flux through the surface below?



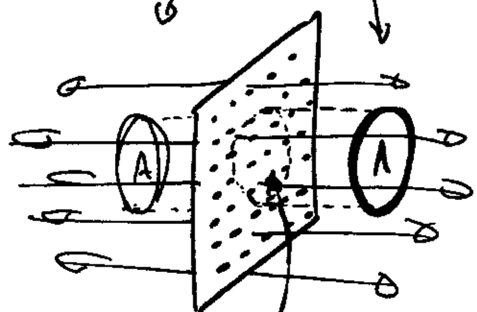
$\Phi = \frac{Q_1}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0} + \frac{Q_3}{\varepsilon_0} + \ldots$

Question 6

Suppose we have an infinite plane of water sources, putting out a total of η liters per second for each unit of area on the plane. What is the flux of water through the disk of area A shown? What is the flow intensity at this surface? *Hint: start by drawing the flow of water, remembering that the plane is infinite (and that the water can go in either direction)*

the water must flow straight out of the plane.

same amount goes each way

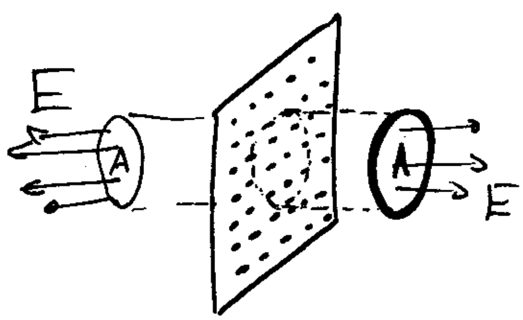


$$\Phi = Q_{inside} = \eta A = (2) A E$$

$$\Rightarrow E = \frac{\eta}{2}$$

Question 7

Suppose we have an infinite plane of charge, with η Coulombs of charge for each unit of area on the plane. What is the electric flux through the surface of area A shown? What is the electric field at this surface? *Hint: it's the same question. Just use our analogy.*



$$\Phi = \frac{Q_{encl}}{\varepsilon_0} = \eta A$$

$$= EA + EA = 2E$$

$$\Rightarrow \boxed{E = \frac{\eta}{2\varepsilon_0}}$$

constant field!