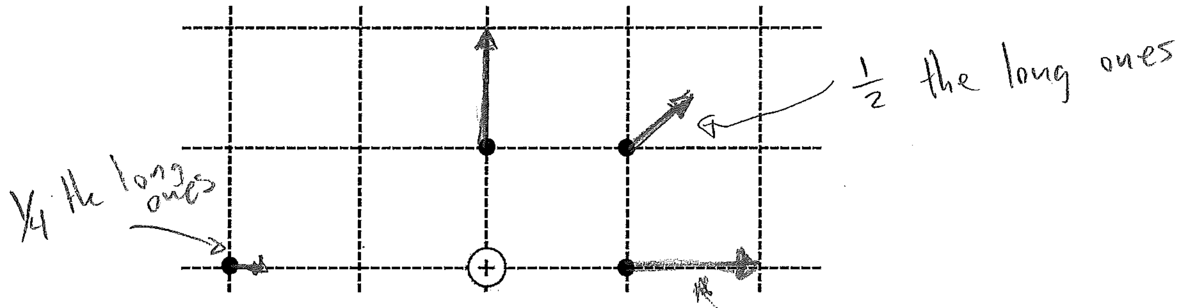
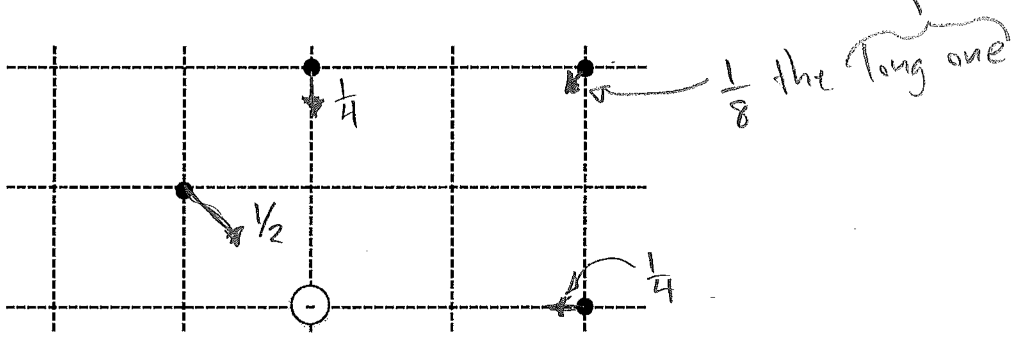
Physics – Electric Field Worksheet

Question 1

a) At each of the four points draw an electric field vector with the proper direction and whose length is proportional to the electric field strength at that point.

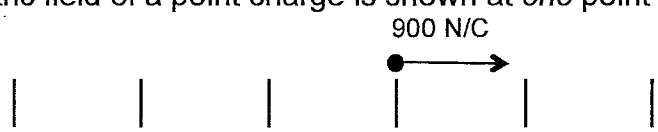


b) Do the same for the field of this negative charge.



Question 2

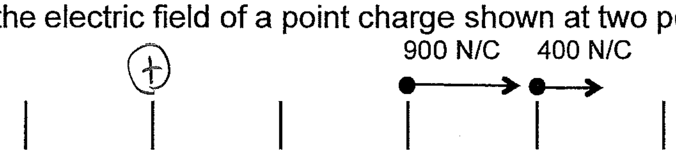
a) The electric field of a point charge is shown at *one* point in space.



Can you tell if the charge is + or -? Explain?

No. The charge could be ⊕ and to the left of the point or ⊖ and to the right.

b) Here is the electric field of a point charge shown at two positions in space.



Can you tell if the charge is + or -? Explain?

Yes. The field decreases as we go further right, so we know we're moving away from the charge, so it's to the left. The field lines point away, so the charge must be positive.

c) Can you determine the location of the charge? If so, draw it on the figure. If not, why not?

We have 2 equations

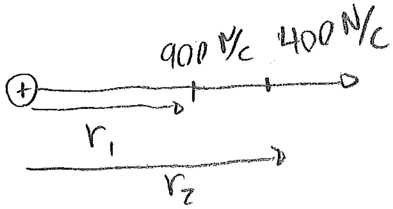
$$E = 900\,N/C = \frac{kQ}{r_1^2}$$

$$E = 400\,N/C = \frac{kQ}{r_2^2}$$

$$\frac{900}{400} = \frac{r_2^2}{r_1^2}$$

$$\frac{r_2}{r_1} = \frac{3}{2}$$

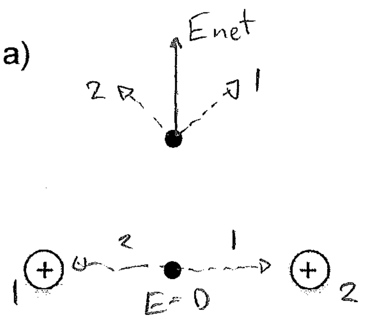
r_2 is $\frac{3}{2}$ times as long as r_1.



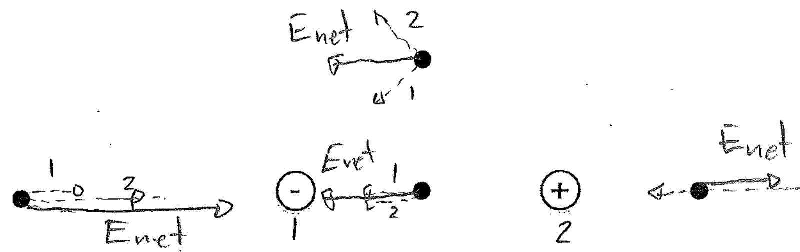
Question 3

At each of the dots, using dashed lines, draw and label the electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ due to the two point charges. Make sure the relative lengths of your vectors indicate the strength of each electric field. Using a solid line, draw the net electric field $\mathbf{E}_{net}$.

a)



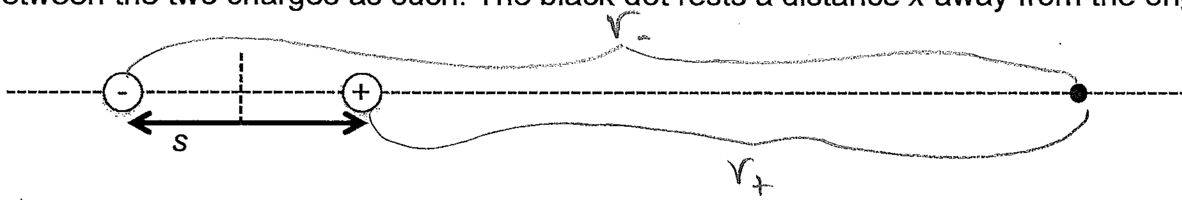
b)



Question 4

You might recognise the charge configuration in question 3 b) as an electric dipole. In chemistry class Pierre mentioned that the electric field from the dipole falls off as $1/r^3$, rather than the $1/r^2$ we would expect from an electric field. Our goal is to derive that dependence on r.

Consider two charges $+q$ and $-q$ separated by a distance s. Place the zero of the x-axis between the two charges as such. The black dot rests a distance x away from the origin.



The distance from negative charge to the point is $r_- = x + s/2$ and the distance from the positive charge to the point is $r_+ = x - s/2$.

a) Using this write an expression for the electric field at the black dot. You should have a difference of two terms. Simplify it by pulling out k and q.

$$E = \frac{kq}{r_+^2} + \frac{k(-q)}{r_-^2} = \frac{kq}{(x-\frac{s}{2})^2} - \frac{kq}{(x+\frac{s}{2})^2}$$

$$= kq\left[\frac{1}{(x-\frac{s}{2})^2} - \frac{1}{(x+\frac{s}{2})^2}\right]$$

b) Find a common denominator and bring the two terms together into one.

$$E = kq\left[\frac{(x+\frac{s}{2})^2}{(x-\frac{s}{2})^2(x+\frac{s}{2})^2} - \frac{(x-\frac{s}{2})^2}{(x-\frac{s}{2})^2(x+\frac{s}{2})^2}\right]$$

$$= kq\left[\frac{x^2+xs+\frac{s^2}{4}-x^2+xs-\frac{s^2}{4}}{(x-\frac{s}{2})^2(x+\frac{s}{2})^2}\right] = \frac{kq\,2xs}{(x-\frac{s}{2})^2(x+\frac{s}{2})^2}$$

c) We're interested in points much further away from the dipole than the charge separation, that is $x >> s$. Using this idea, can we simplify the term $(s/2 + x)^2$, and thus simplify our expression from b).

$$x >> s \Rightarrow 1 >> \frac{s}{x}$$

lets write $(x+\frac{s}{2})^2 = x^2(1+\frac{s}{2x})^2$

So we can use $1 >> \frac{s}{x}$.

So

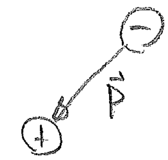
$$E = \frac{kq\,2xs}{x^2(1+\frac{s}{2x})^2 x^2(1-\frac{s}{2x})^2} \sim \frac{k\,2qxs}{x^2x^2}$$

$$= \frac{k\,2qs}{x^3}\ !$$

It goes as x^3!

d) The combination qs is called the dipole moment, and is denoted by the letter **p**. The dipole moment is a vector that points from the negative charge to the positive charge. Write the electric field in terms of **p**. At a fixed distance r, what can one change about the dipole to increase the electric field strength?

We can write $\vec{E} = \frac{k2\vec{p}}{x^3}$, $p = qs$



One can increase the strength of E by increasing the charge separation s, or by increasing the charge q.

Question 5 (extra)

We'll do the same thing again, except this time we'll abuse the definition of the derivative.

This time consider the coordinate system where the origin of the x-axis is on the negative charge.

Write an expression for the electric field as before, but instead of finding a common denominator, see if you can use an abomination of the definition of the derivative to rewrite the difference as a derivative.

The definition of the derivative tells us.

$$\frac{d f(x)}{dx} \sim \frac{f(x+s) - f(s)}{s}$$

$$E = kq\left[\frac{1}{(x+s)^2} - \frac{1}{x^2}\right]$$

let $f(x) = \frac{1}{x^2}$ then

$$= kq\left(f(x+s) - f(s)\right)$$

$$= kqs\frac{d}{dx}f(x) = kqs\frac{d}{dx}\frac{1}{x^2} = -\frac{k2sq}{x^3}$$

negative makes sense because ⊖ is closer to •.