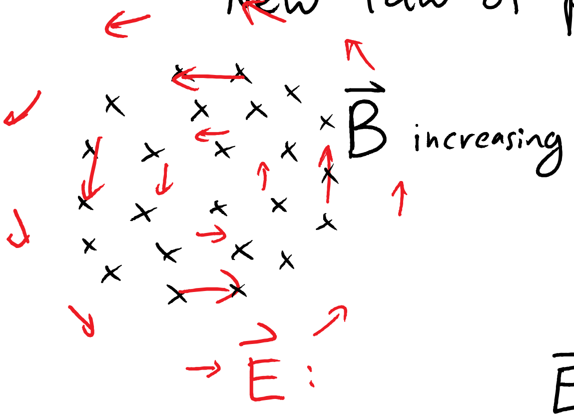
LAST TIME:

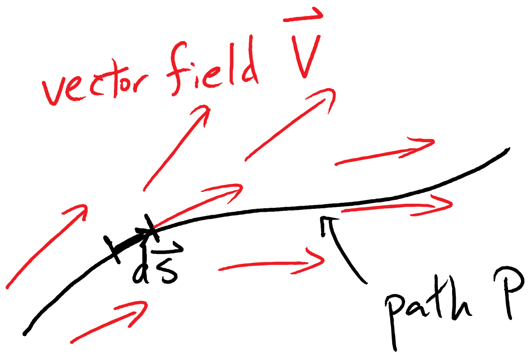
New law of physics: Changing $\vec{B} \longrightarrow \vec{E}$



$\vec{E}$ induced in direction that *would* produce a current satisfying Lenz's Law

$\vec{E}$ from $-\frac{d\vec{B}}{dt}$ looks like $\vec{B}$ from $\vec{J}$ (current density)

Rules for both of these laws use LINE INTEGRALS.



$\int_P \vec{V} \cdot d\vec{s}$: break up path into little segments $\vec{\ }$
- add up $\vec{V} \cdot d\vec{s}$ for each segment

e.g. Work done on charge by electric field:

$= \text{sum of } \vec{F} \cdot d\vec{x} \text{ along path } = \int \vec{F} \cdot d\vec{x}$

FARADAY'S LAW:

$$\text{EMF } \mathcal{E} = \left| \frac{d\Phi_M}{dt} \right|$$

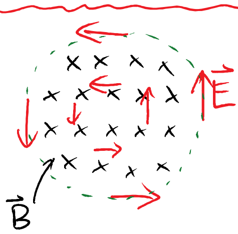
recall: for fixed loop, $\mathcal{E}q$ = work done on charge by induced $\vec{E}$ going around loop

$$\mathcal{E}q = \int \vec{F} \cdot d\vec{x} = q \int \vec{E} \cdot d\vec{s}$$

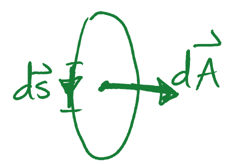
CONCLUSION:

$$\int \vec{E} \cdot d\vec{s} = -\frac{d\Phi_M}{dt}$$

FARADAY'S LAW



chosen so Lenz's Law works

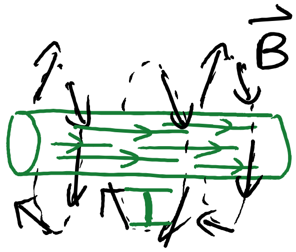


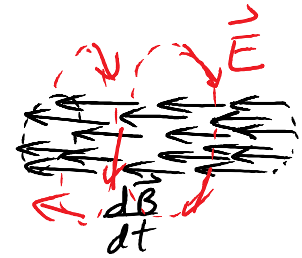
conventions for relative direction of $d\vec{s}$ and $d\vec{A}$

example: At radius R inside solenoid with changing $\vec{B}$, Faraday's Law gives:

$$E \cdot (2\pi R) = (\pi R^2) \cdot \frac{dB}{dt}$$

$$\Rightarrow E = \frac{R}{2} \cdot \frac{dB}{dt}$$





Rule for finding $\vec{B}$ from I same as rule for finding $\vec{E}$ from $-\frac{d\Phi_B}{dt}$

ANALOGY:

$$\vec{E} \longrightarrow \vec{B}$$

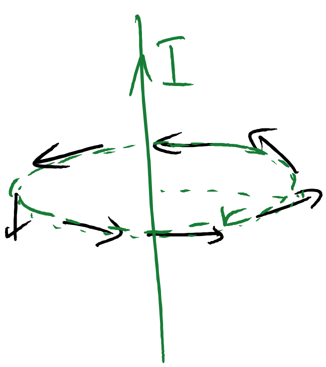
$$-\frac{d\Phi_B}{dt} \longrightarrow \mu_0 I$$

$$-\frac{d\vec{B}}{dt} \longrightarrow \mu_0 \vec{J} \quad \text{(current density)}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

AMPERE'S LAW

example: what is magnitude of $\vec{B}$ at distance R from wire?



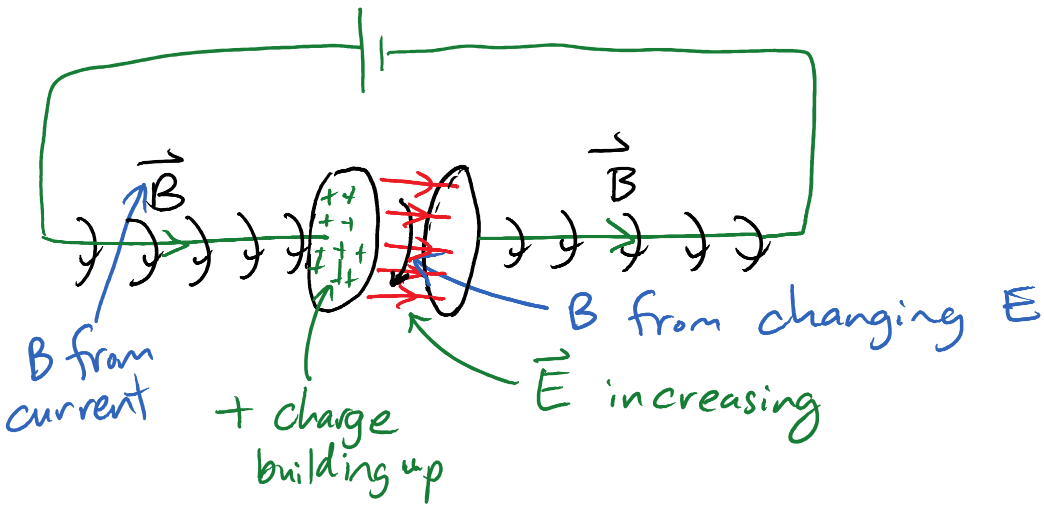
Ampere's Law gives

$$B \cdot (2\pi R) = \mu_0 I$$

so $$B = \frac{\mu_0 I}{2\pi r}$$

→ same result as from Biot-Savart Law

One last law: $\vec{B}$ from $\frac{d\vec{E}}{dt}$



Maxwell: realized that increasing $\vec{E}$ inside capacitor must act like a current to produce a magnetic field

$$\int \vec{B} \cdot d\vec{s} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

AMPERE'S LAW PLUS MAXWELL "DISPLACEMENT CURRENT"