Setlist L7 (90 minutes)

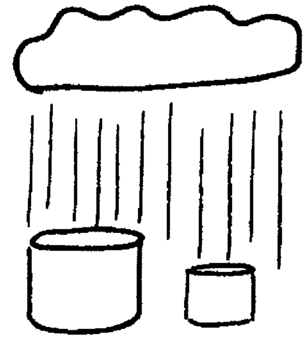
Finish Potential. Do Gauss's Law.
Prep: Gauss's Law worksheet. Electrostatics Cheatsheet. Flux transparencies.

1. Last Class: Talk about how all the concepts are related
2. The relationship between potentials and fields.
3. Clicker Question:
4. Alessandro Volta, lighting, and the Jacob's Ladder. It's Alive!
5. Activity - tell me what's behind the doors. The idea is that you can identify charge distributions simply by field configurations.
6. Gauss bio - He did everything

30 min

7. Worksheet Q1
8. Clicker Question - Flux through a surface - D
9. Worksheet Q2 and Q3
10. Clicker Question - Flux through a closed surface - C
11. Where must the charge be to make a non-zero flux?
12. Clicker Question - Two charges - C
13. Worksheet Q4 - Q7
14. Review Charge distributions. When they drew the surfaces, these were called gaussian surfaces. Choosing the right surface made it easy to calculate the electric field. What would the surface for the wire look like?
15. State Gauss's law just to be clear.
16. Clicker Question - Funny surfaces, charge, and flux - B
17. Clicker Question - Find charge on the inside surface of a conductor - D
18. Clicker Question - Find charge on the outside surface of a conductor - B

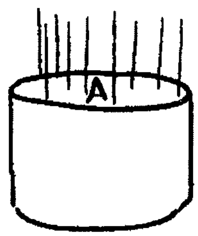
Suppose it's raining really hard and we want to measure just how hard it's raining. We might put out a bucket and see what volume of water goes into the bucket per unit time. This is the **FLUX** of water through the opening of the bucket.



But this will depend on the size of the bucket: if we used a bucket with twice the opening area, we would get twice as much water per unit time. So to measure how hard it's raining, we really want to take the volume of rain going into the bucket per unit time and divide by the area of the bucket. This number gives us a direct measurement of how hard it's raining. Let's call it the **INTENSITY** of the rainfall.

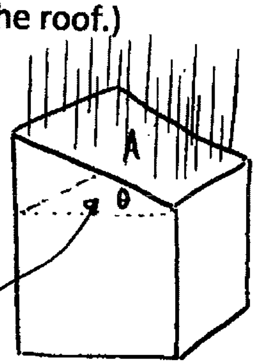
Question 1

a) Suppose we have rain with intEnsity **E** (measured in liters per second per m^2) falling directly down into a bucket with opening area A. Using our definitions, what is the flux of water into the bucket?



Flux = EA

b) The same rain with intensity **E,** falls directly down onto a slanty roof with area A at an angle θ to the horizontal as shown. What is the flux of rain hitting the surface? (If you want, you can imagine the rain going straight through the roof.)



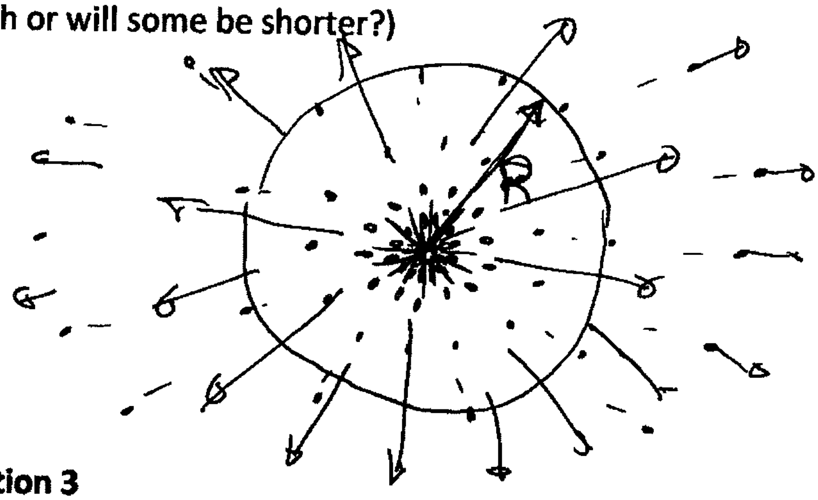
this area gets hit which is $A\cos\theta$

$$\Phi = EA\cos\theta$$
$$= \vec{E}\cdot\vec{A}$$

Question 2

We can use the same definition (amount of water per time per area) to describe the intensity of water flow in any other situation (e.g. coming from a sprinkler). In these cases, it might be that the water is flowing more intensely in one place than another (e.g. narrow vs wide parts of a river). It may also be flowing in different directions in different places. So it's useful to draw vectors to show the direction and intensity of flow at various places. The vector points in the flow direction and the length is the intensity.

On the picture below, water is shooting out in all directions from a source (some droplets are shown). Draw some vectors to show the flow direction and intensity at various places, ignoring effects of gravity. (Should all your vectors be the same length or will some be shorter?)



theres less water further away, (think of getting sprayed) So the arrows are shorter.

Question 3

Let's define **Q** to be the Quantity of water per unit time coming from the source.

a) Consider a spherical surface of radius R (area $4\pi R^2$) around the sprinkler head (you can draw it on the picture above). What is the flux of water through this surface?

Flux = $Q = EA = E\,4\pi R^2$

The flux is the amount of water coming out of the spout. All the water must go through the surface. It's also equal to EA.

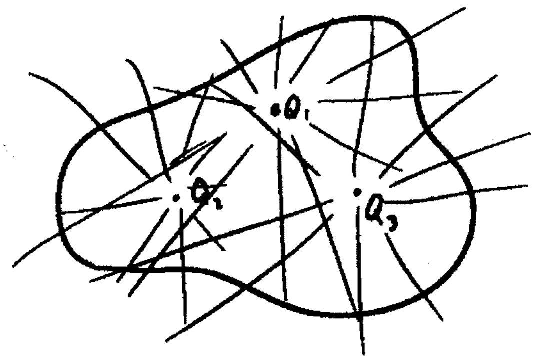
b) What is the intensity E of water flow at the surface?

E = $\frac{Q}{4\pi R^2}$

Hopefully, you have found that ***the flow intensity E of water is related to the flow rate Q from a source in exactly the same way that the electric field from a point charge is related to the charge*** *(if we divide the right side by ε_0). This means that anything that is true of water flow from a collection of sprinklers will be true about the electric field from a collection of charges.*

Question 4

Suppose we have a bunch of water sources, all shooting water out uniformly in all directions. If the flow rates are given by Q_1, Q_2, Q_3, etc.... What is the total flux of water through the surface shown?

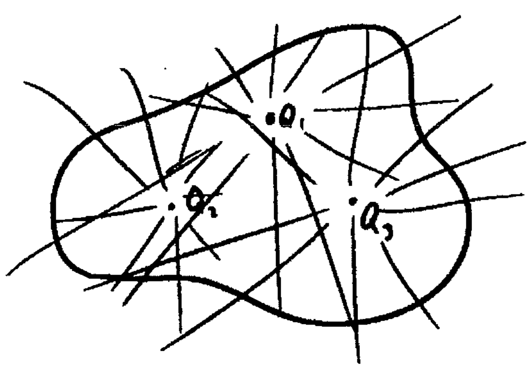


$\Phi = Q_1 + Q_2 + Q_3 + \ldots$

All the water from each sprinkler must go through the surface.

Question 5

We can define ELECTRIC FLUX in terms of the electric field E in the same way that flux of water was related to the flow intensity vectors. Using our analogy and your answer to the previous question, what can we say about the electric flux through the surface below?



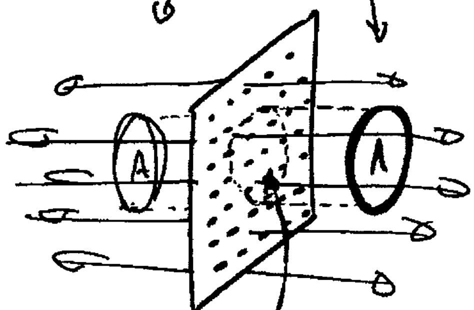
$$\Phi = \frac{Q_1}{\varepsilon_0} + \frac{Q_2}{\varepsilon_0} + \frac{Q_3}{\varepsilon_0} + \ldots$$

Question 6

Suppose we have an infinite plane of water sources, putting out a total of η liters per second for each unit of area on the plane. What is the flux of water through the disk of area A shown? What is the flow intensity at this surface? *Hint: start by drawing the flow of water, remembering that the plane is infinite (and that the water can go in either direction)*

the water must flow straight out of the plane.

same amount goes each way

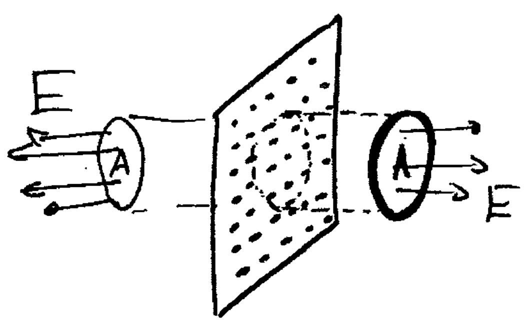


$$\Phi = Q_{inside} = \eta A = 2AE$$

$$\Rightarrow E = \frac{\eta}{2}$$

Question 7

Suppose we have an infinite plane of charge, with η Coulombs of charge for each unit of area on the plane. What is the electric flux through the surface of area A shown? What is the electric field at this surface? *Hint: it's the same question. Just use our analogy.*



$$\Phi = \frac{Q_{encl}}{\varepsilon_0} = \eta A$$

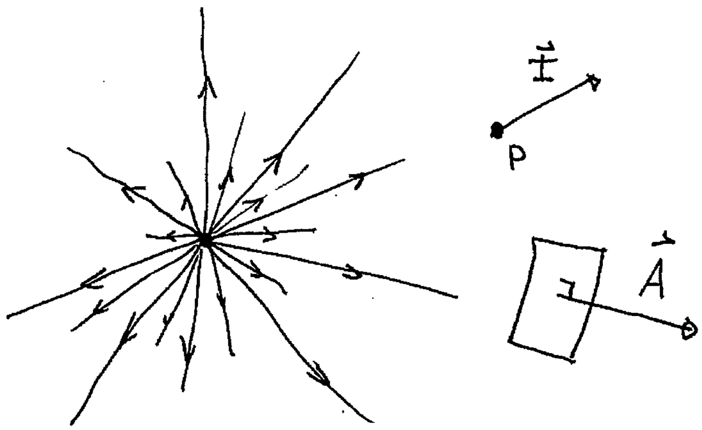
$$= EA + EA = 2E$$

$$\Rightarrow \boxed{E = \frac{\eta}{2\varepsilon_0}}$$

constant field!

Gauss's Law

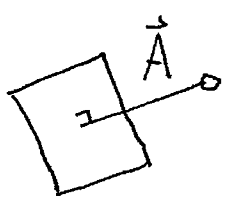
Imagine water spraying in all directions from a point.



Amount (mass) of water per unit time through a unit area at a point (say p) is intesity

$$\vec{I} = \frac{\text{amount}}{S\,m^2}, \text{ in direction of flow}$$

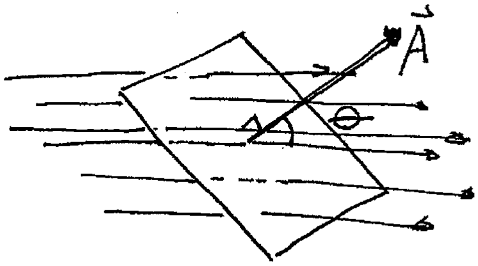
The total water per second is found by multiplying by an area $\vec{A}$,



direction is $\perp$ to surface
Magnitude = area.

The amount of water through this surface depends on orientation. This is the flux

$$\Phi = \vec{I} \cdot \vec{A} = IA\cos\theta$$

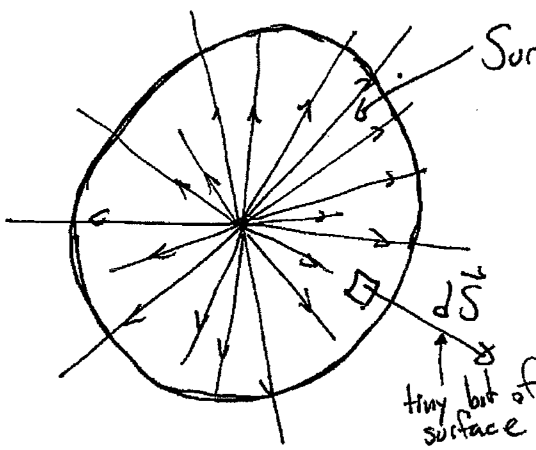


This shows a uniform $\vec{I}$ through a surface $\vec{A}$ at an angle θ.

clicker

Back to the spray of water

Close the spray with a surface. (area vector always points out of a closed surface)



[a sphere of radius r surface area $= 4\pi r^2$]

$$\boxed{\begin{aligned}&\text{Mass of water out of source}\\ &= \text{mass water through } S_1{}^*\end{aligned}}$$

*This is a conservation law

We can write this conservation law as:

$$\text{flow} = \Phi_1 = \oint \vec{I} \cdot d\vec{A}$$

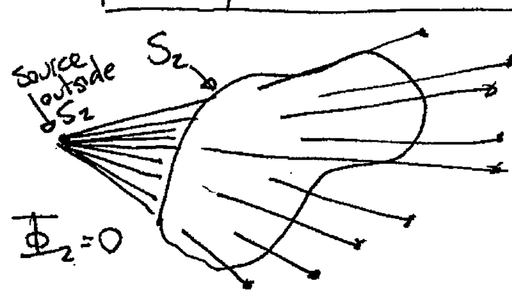
to make the curved surface we add up a buch of tiny surfaces.

$$\Rightarrow I \oint dA = IA = \text{flow}$$

$$\Rightarrow \boxed{I = \frac{\text{flow}}{4\pi r^2}}$$

Important: for all $\vec{I} \cdot d\vec{A}$ we have $I\,dA$, $\theta = 0$. Also I is the same everywhere on the sphere

Intensity decreases as r^{-2} away from the source.

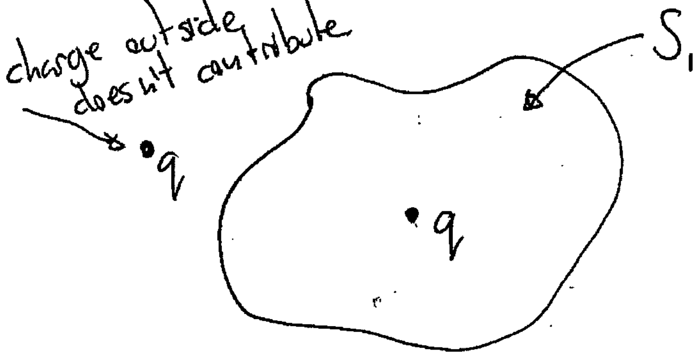


clicker

<u>The analogy</u>: flow (amount of water from spigot) $= \dfrac{q}{\epsilon_0}$

Intensity of water $= \vec{E}$

The electric flux through a surface enclosing charge is



$$\boxed{\Phi_E = \frac{q_{encl}}{\epsilon_0}}$$

clicker

We aslo have:

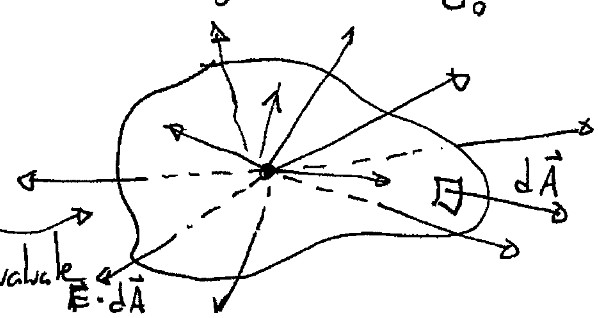
$$I = \frac{\text{flow}}{4\pi r^2} \longrightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = k\frac{q}{r^2}}$$

The electric field of a point charge.

How did we find the electric field? The first step was to guess the symmetry of the system and set up an imaginary surface (Gaussian surface). Such that $\vec{I}\cdot\vec{A} = IA$.

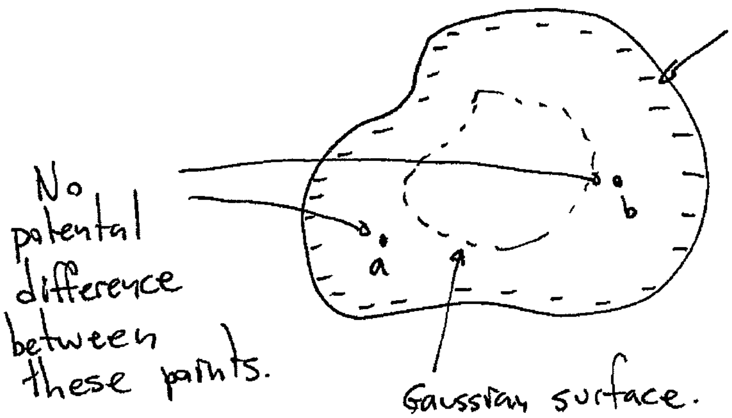
$$\Phi = \oint \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$$

Knowing the symmetry is critical. Without that the problem would be unsolvable.



for this surface you can't evaluate $\vec{E}\cdot d\vec{A}$

Gauss's law tells us about the field inside a charged conductor.



No charge inside Gaussian surface means

$$\Phi = \frac{q_{encl}}{\epsilon_0} = 0$$

$$\Longrightarrow \vec{E} = 0!$$

If $\vec{E} = 0$ inside a conductor it means that $\Delta V = 0$ inside the conductor. The surface is an equipotential!

Also, Gauss's Law is the first step towards formulating Maxwell's equations.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$\Downarrow$ Green's theorem (which you'll learn in later math)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$ ← charge density.

Gauss's Law, Continued

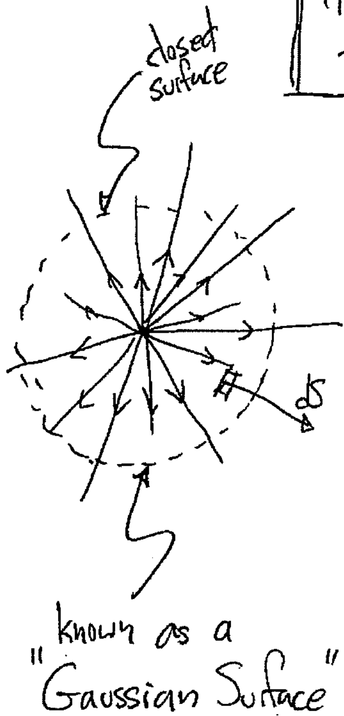
Last class: What happened?

We argued that this was true

$$\boxed{\text{flow of water from point} = \text{mass of water through closed surface}}$$

showed that these are the same. (for point source).

$$I = \frac{\text{flow}}{4\pi r^2}$$



Which is mathematically equivalent to

$$E = \frac{kq}{r^2}$$

Then these must be the same.

$$\boxed{\text{Electric flux through surface} = \text{enclosed charge}/\epsilon_0}$$

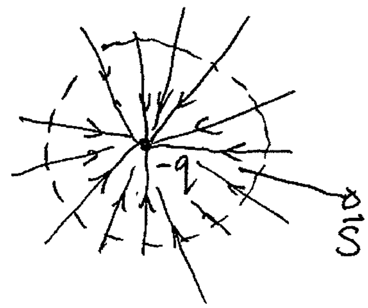
This last box contains Gauss's Law for electric fields. Mathematically, it looks like

$$\Phi_E = \frac{q_{enclosed}}{\epsilon_0}$$

where only the charge enclosed in the Gaussian surface contributes to the flux.

clicker questions

<u>note:</u> for a closed surface the area vector $\vec{A}$ always points <u>outward</u>. Flux from inside to out is positive. and flux from outside in is negative.



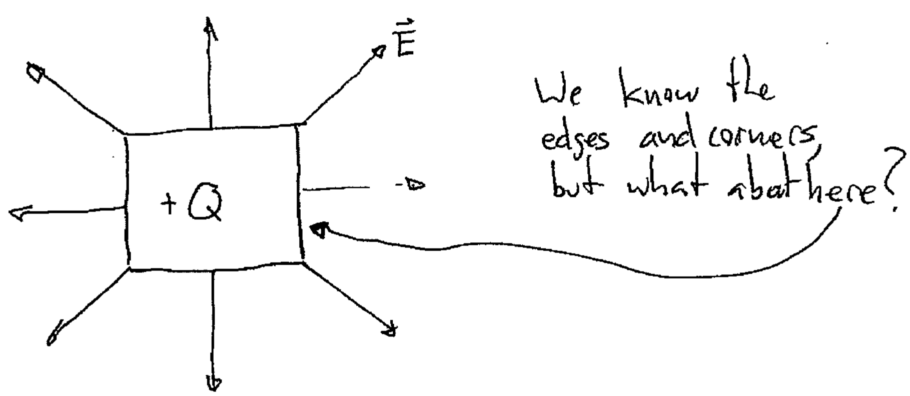
For a negative charge the flux is negative.

$$\Phi_E = \frac{-q}{\epsilon_0}$$

Why does Gauss's Law work?

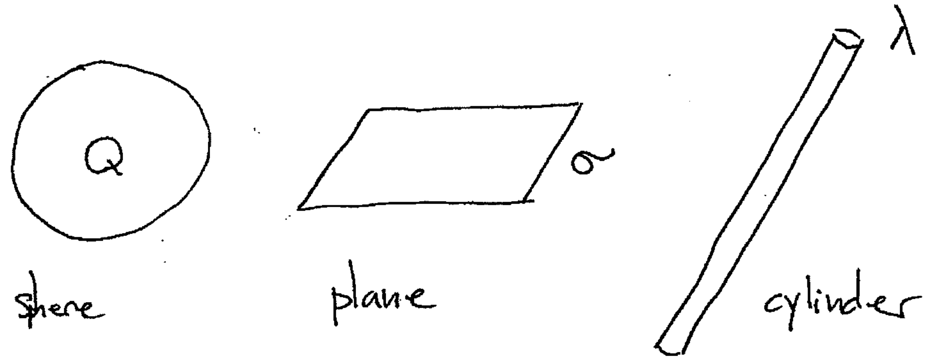
We were only able to find the electric field because we used our intuition to find the symmetry of the electric field, Also, there was a surface that matched the symmetry.

- Does it work for a cube of charge?



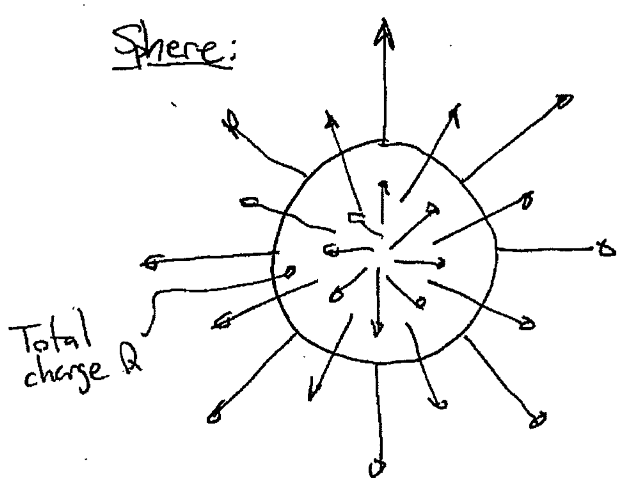
There's no shape that describes this well. (that itsef is easy to describe).

What shapes do work?



Imagine the surface of these were covered in sprinklers. Which way would the spray go?

Sphere:

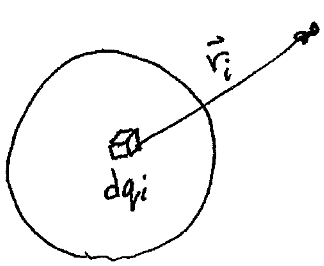


The spray is outwards, just like a point charge.

$$\vec{E} = \frac{kQ}{r^2}\hat{r} \quad , \quad r > R_{sphere}$$

For $r < R_{sphere}$ we need to know how the charge is distributed.

note: Though trivial with Gauss's Law, this is difficult to do with Coulomb's Law.

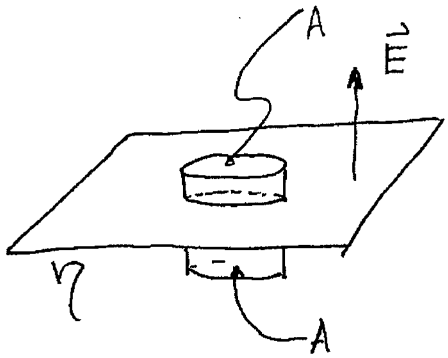


$$\vec{E} = \sum_i \frac{k\,dq_i}{r_i^2}\hat{r}_i$$

Not an impossible integral, but tough enough.

OK, now do a plane of charge.

$$\boxed{\text{Electric Flux} = \text{enclosed charge}/\epsilon_0}$$

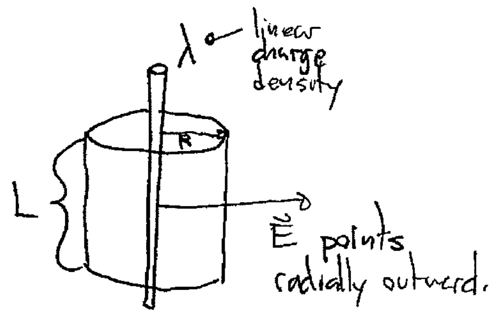


$$\text{enclosed charge}/\epsilon_0 = \frac{\eta A}{\epsilon_0}$$

$$\text{electric flux} = EA + EA = 2EA$$

$$\Longrightarrow \boxed{E = \frac{\eta}{2\epsilon_0}}\,!$$

What about a wire?



$$\left.\begin{aligned} \frac{q}{\epsilon_0} &= L\lambda \\ \Phi_E &= E\,2\pi r L \end{aligned}\right\} \Rightarrow \boxed{E = \frac{\lambda}{2\pi r}}\,!$$

<u>Final note:</u>

Aside from being cute and somewhat useful, Gauss's Law is the first of "Maxwell's Equations", which describe all of E&M (almost).

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} \Longleftrightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

charge density