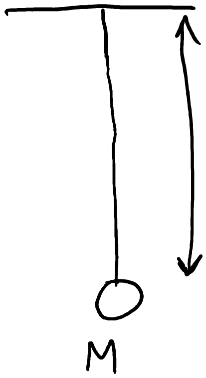
1



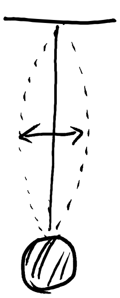
The tension in the string will be Mg, so the wave speed on the string is

$$V = \sqrt{(\text{mass density})^{-1} \times (\text{tension})}$$

$$= \sqrt{\left[(0.0004132\ lb/in) \times \frac{1}{2.2}\frac{kg}{lb} \times \frac{1}{0.0254}\frac{in}{m}\right]^{-1} \times 9.8\,m/s^2 \cdot M}$$

$$= \sqrt{M \cdot 1325\,\frac{m^2}{s^2 \cdot kg}}$$

To play middle C, we want a frequency of $261.6\,s^{-1}$. The longest wavelength standing wave on the string will have $\lambda = 2L = 1.26m$. So we need a wave speed of



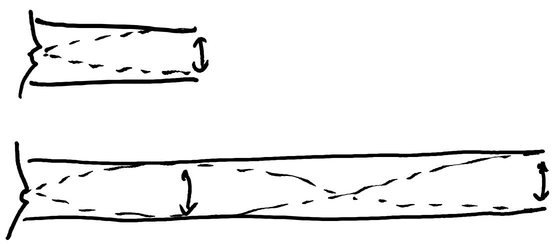
$$V = \lambda \cdot f$$

$$= 1.26m \cdot 261.6\,s^{-1}$$

$$= 330\ m/s$$

Thus, we need $\sqrt{M \cdot 1325 \frac{m^2}{s^2 kg}} = 330\,m/s$

$$\Rightarrow \quad M = 82\,kg$$



For the kelp horn, we want a frequency

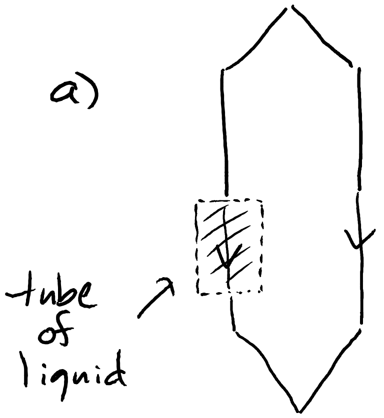
$f = 392$ Hz. Since the speed of sound in air is about 340m/s, we need a wavelength of $\lambda = \frac{v}{f} = 0.867$m. From the pictures above, we can see that the two shortest tubes for which standing waves of this wavelength can live are

$$L = \frac{\lambda}{4} = 0.217\text{m}$$

and $$L = \frac{3\lambda}{4} = 0.651\text{ m}$$

Problem 2

a)

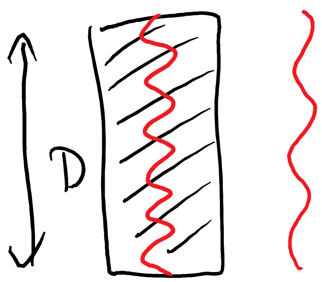


For this setup, the key point is that the larger index of refraction in the drink means that v is smaller, and since frequency will be the same everywhere, the wavelength $\lambda = \frac{v}{f}$ will be shorter in the liquid. This means that the number of wavelengths along the two paths will differ, and the difference will depend on how much liquid is in the tube. When we have a difference of a whole # of wavelengths, there will be constructive interference, while if the difference is an integer $+ \frac{1}{2}$ wavelength, we'll have destructive interference. As liquid is added, we go between these possibilities, so the image gets brighter & darker.

b) Suppose the depth of liquid is D. The wavelength in the liquid is

$$\lambda = \frac{v}{f} = \frac{c}{nf} = \frac{\lambda}{n}$$

$$\lambda = \frac{v}{f} = \frac{c}{nf} = \frac{\lambda}{n}$$



The number of wavelengths in the liquid is then $N_1 = \dfrac{D}{\lambda/n}$. The number of wavelengths in the same distance on the other path is:

$N_2 = \dfrac{D}{\lambda}$. So the difference is:

$$N_1 - N_2 = \frac{D}{\lambda}(n-1)$$

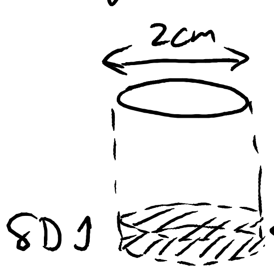
When this increases by $\frac{1}{2}$, we'll go from constructive to destructive interference.

So the change in liquid height will be:

$$\frac{\delta D}{\lambda}(n-1) = \frac{1}{2}$$

We are given that $\lambda = 500\text{nm}$, and can calculate that $\delta D = \dfrac{0.5 \times 10^{-9}\,\text{m}^3}{\pi \cdot (0.01\,\text{m})^2}$

$= 1.59 \times 10^{-6}\,\text{m}$



$\delta V = \text{Area} \times \delta D$

$\Rightarrow 0.5 \times 10^{-9}\,\text{m}^3 = \pi \cdot (0.01\,\text{m})^2 \times \delta D$

So: $n = 1 + \dfrac{\lambda}{2 \cdot \delta D} = 1.157$