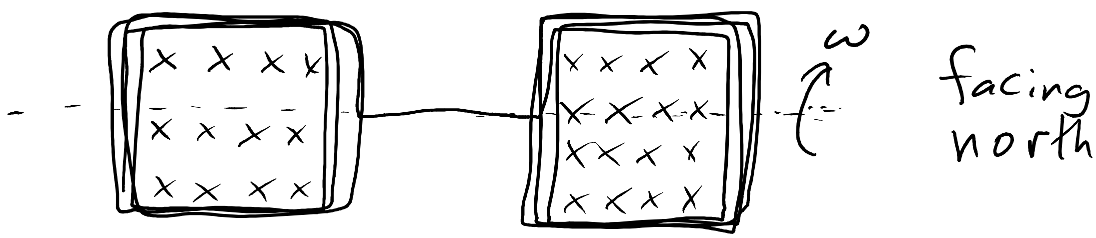
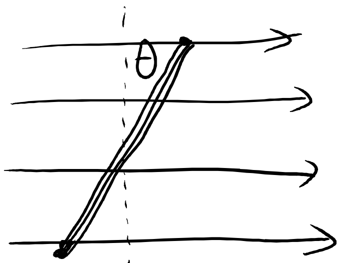
PS7 SOLUTIONS

(1)



side view



The EMF is determined by Faraday's Law:

$$\mathcal{E} = -\frac{d\Phi_M}{dt}.$$ Here, the flux is determined by

$$\Phi_M = N\int \vec{B}\cdot d\vec{A} = N\vec{B}\cdot\vec{A} = N\cdot B\cdot A\cdot\cos\theta$$

We have: $A = 2\cdot(0.5m)^2 = 0.5m^2$ $\quad N = 200$

$B = 0.5\times10^{-4}\,T$

used chain rule

So: $$\mathcal{E} = -\frac{d\Phi_M}{dt} = NB\cdot A\cdot\sin\theta\cdot\frac{d\theta}{dt}$$

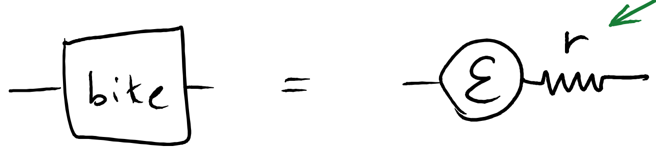
$$= NBA\omega\cdot\sin(\omega t)$$

$\omega = 2\pi\cdot f$
$= 2\pi\cdot 4s^{-1}$

The max EMF occurs when $\sin\theta = 1$, so:

$$\mathcal{E}_{MAX} = N\cdot B\cdot A\cdot\omega = 0.1257\,V$$

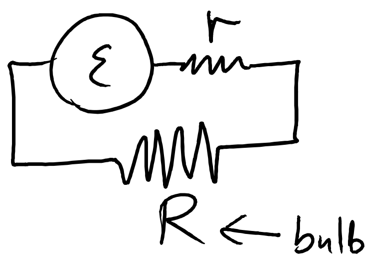
(2) The bike coils produce an EMF, but also contribute to the net resistance of the circuit. So we can think of:



0 gauge copper wire: $0.3235\,\Omega/\text{km}$

$\therefore$ for 800 m get $r = 0.2584$

For a single bike attached to the bulb, the circuit is:



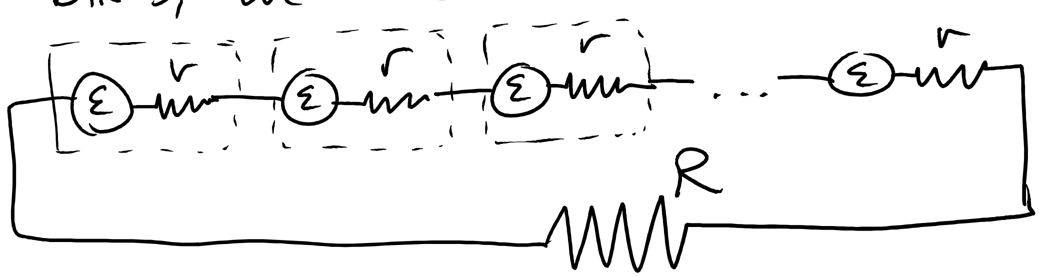
Here, the current is $I = \dfrac{\varepsilon}{r+R}$ and the power used by the bulb is

$$P = I^2 R = \frac{\varepsilon^2 R}{(r+R)^2}$$

$$= 1.261 \times 10^{-3}\ \text{W}$$

Not big enough!

For n bikes, we have:



$\varepsilon_{TOT} = n\,\varepsilon$ $\qquad R_{TOT} = R + n\cdot r$

So the current is

$$I = \frac{\mathcal{E}_{TOT}}{R_{TOT}} = \frac{n\,\mathcal{E}}{(R + n r)}$$

The power through the bulb is now:

$$P = I^2 R = \frac{n^2 \mathcal{E}^2 \cdot R}{(R + n r)^2} = \frac{\mathcal{E}^2 \cdot R}{\left(r + \frac{R}{n}\right)^2}$$

As n gets larger, the denominator gets smaller, so P increases. The power P of the bulb increases as n increases. By trial and error, the smallest n to get $P > 1\,W$ is $n = 68$, so including the coordinator, it takes all 69 Science One students to light the bulb.

(3) Starting from rest, constant torque gives a constant angular acceleration

$$\alpha = \frac{\tau}{I} = \frac{100\,\mathrm{Nm}}{0.3\,\mathrm{kg\,m^2}} = 333\ \mathrm{s}^{-2}$$

Since $\alpha = \frac{d\omega}{dt}$, and $\omega(0) = 0$, we get:

$$\omega(t) = \alpha t$$

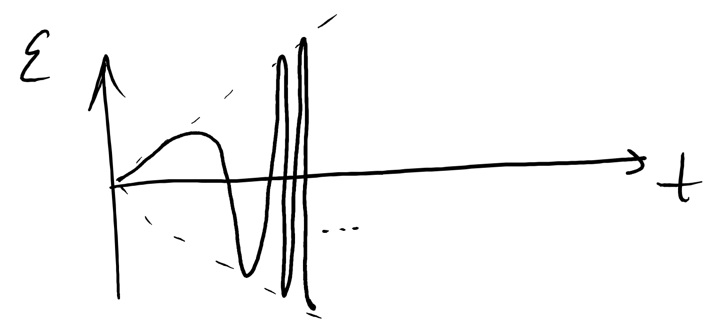
Also: $\omega = \frac{d\theta}{dt}$. Assuming $\theta(0) = 0$, we get

$$\theta(t) = \frac{1}{2}\alpha t^2$$

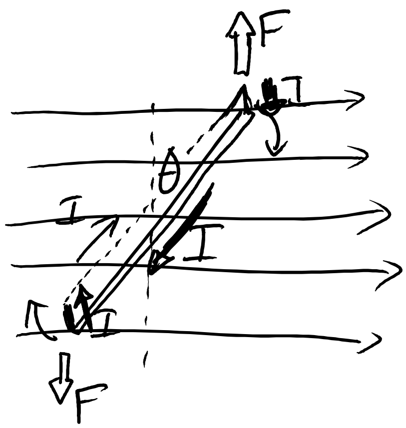
From Q1,

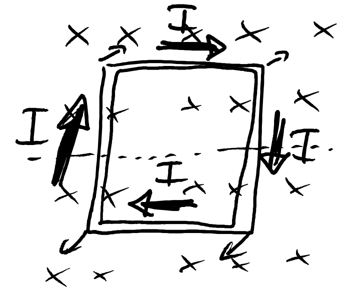
$$\mathcal{E} = -\frac{d\Phi_M}{dt} = NBA \cdot \sin\theta\ \omega$$

$$= NBA \cdot \sin\left(\tfrac{1}{2}\alpha t^2\right) \cdot (\alpha t)$$



(4)





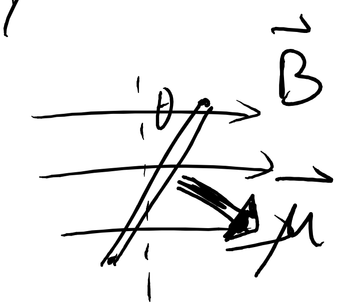
With the circuit connected, we get $I = \frac{\mathcal{E}}{r}$ so

$$I = \frac{NBA\sin\theta\,\omega}{r}$$

The various parts of the wire then experience magnetic forces that result in a net torque on the loop (opposing the applied torque)

We can calculate this by

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$



The magnetic moment for a loop with N turns is:

ωt

$$\mu = N \cdot I \cdot A = \frac{N^2 A^2 B \sin\theta\omega}{r}$$

$$\text{So: } |\tau| = \mu B \sin\theta = \frac{N^2 A^2 B^2 \sin^2\theta \cdot \omega}{r}$$

$$= 2.43 \times 10^{-3}\ \text{Nm} \cdot \sin^2(\omega t)$$

The average of $\sin^2(\omega t)$ is $\frac{1}{2}$ over time, so the average torque from the induced current is 1.21×10^{-3} Nm. The time to get up to 4 revs/s with a torque τ is

$$t = \frac{\omega_f}{\alpha} = \frac{\omega_f}{\tau/I} = \frac{\omega_f \cdot I}{\tau}$$

since $\omega(t) = \alpha t$

With the circuit closed we have a torque that is less by a factor

$$\frac{100 - 1.2 \times 10^{-3}}{100}$$

so the time will be greater by a factor of approximately

$$\frac{100}{100 - 1.2 \times 10^{-3}} \approx 1.0012$$

i.e. 0.000028 % greater.

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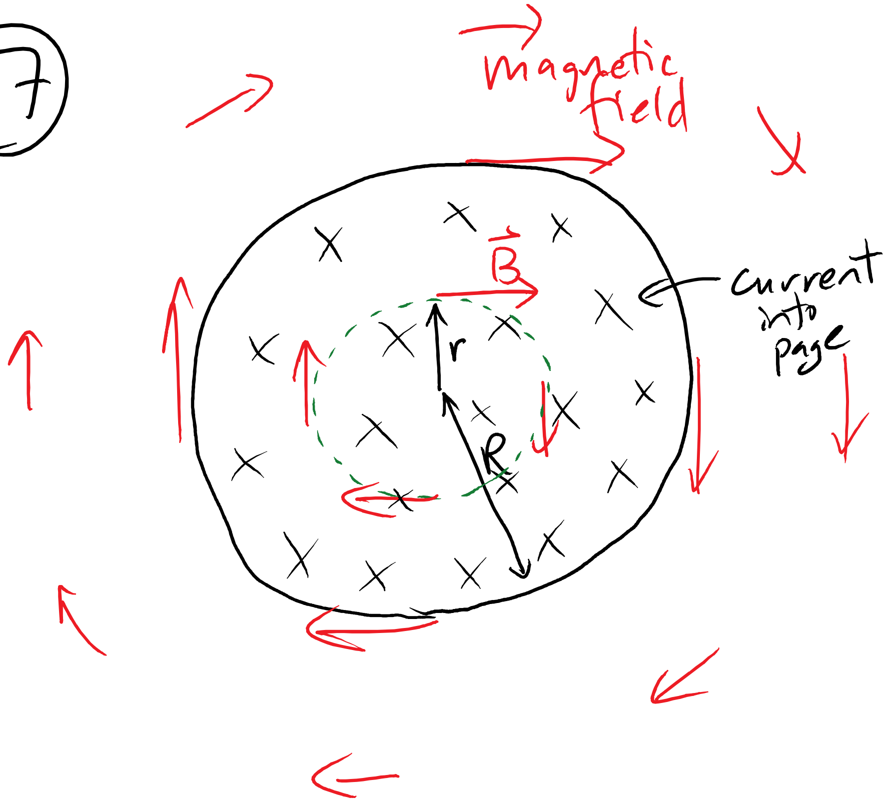

The magnetic field from the current points in the direction that circulates around the wire according to the RHR.

To find the magnitude at radius r, we can use Ampere's Law:

$$\int \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$$

We evaluate this for a closed circular path, shown by the dotted green line in the figure. The $\vec{B}$ is always in the direction $d\vec{s}$ along the path and has the same magnitude everywhere, so the left side is just $|\vec{B}| \times \int ds = |\vec{B}| \times 2\pi r$.

sum of length around path

The enclosed current on the right side is the total current times the fraction of wire enclosed by the path:

$$I_{encl} = \frac{\pi r^2}{\pi R^2} \times I = \frac{r^2}{R^2} I .$$

Thus, Ampere's Law gives:

$$B \cdot 2\pi r = \frac{r^2}{R^2} I \cdot \mu_0$$

$$\Rightarrow \quad B = \frac{r}{2\pi R^2} \cdot I \cdot \mu_0$$