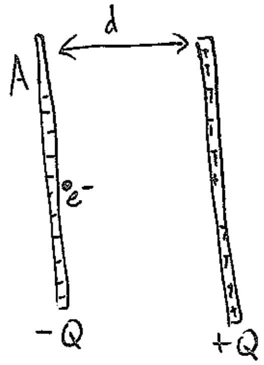
PROBLEM 3

a)



Suppose the charges on the plates are $+Q$ and $-Q$. Then the charge density is $\eta = \frac{Q}{A}$.

We want an electron to have speed v after moving between the plates. So its kinetic energy should increase by

$\frac{1}{2}mv^2$ (or $(\gamma-1)mc^2$ for v close to c)

Thus, the difference in potential energy for an electron at the right plate vs. the left plate should be $-\frac{1}{2}mv^2$.

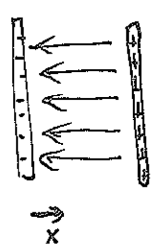
This means that the difference in potential should be

$$(*) \qquad \Delta V = \frac{mv^2}{2e}$$

(the potential energy diff. divided by $-e$, the electron charge)

The electric field between parallel plates is constant, with magnitude $|\vec{E}| = \frac{\eta}{2\varepsilon_0} + \frac{\eta}{2\varepsilon_0} = \frac{\eta}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$. Since the

Contributions from 2 plates.



electric field is related to the derivative of the potential,

$$E_x = -\frac{dV}{dx},$$

the potential between the plates must be

$$V = \frac{Qx}{\varepsilon_0 A}$$

The potential difference is then $\Delta V = \frac{Qd}{\varepsilon_0 A}$ between the plates. To get the desired ΔV in $(*)$, we need to choose Q so

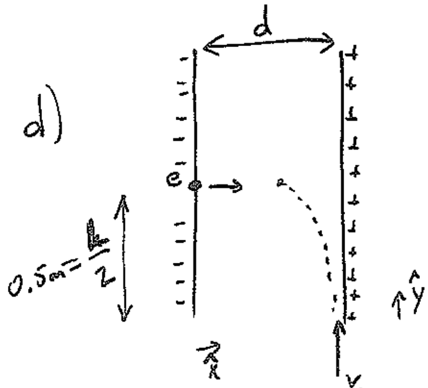
$$\frac{Qd}{\varepsilon_0 A} = \frac{mv^2}{2e} \implies \boxed{Q = \frac{\varepsilon_0 A m v^2}{2ed}}$$

b) We have: $\Delta V = \dfrac{Qd}{\varepsilon_0 A}$, so $\dfrac{Q}{\Delta V}$ is $\dfrac{\varepsilon_0 A}{d}$. This "capacitance" depends on ε_0 and the geometrical properties of the plates. We can increase it by increasing A, decreasing d, or putting a material inside so ε_0 is replaced with a bigger quantity.

c) For relativistic speeds, we need to replace $\frac{1}{2}mv^2$ with $(\gamma-1)mc^2$ in part a), so our result changes to

$$Q = \frac{\varepsilon_0 A}{e\, d} \cdot (\gamma - 1) mc^2$$
$$= 7\times 10^{-6}\,C$$

d)



The field between the plates is $\vec{E} = -\frac{\eta}{\varepsilon_0}\hat{x}$. The electron's trajectory is determined from

$$\vec{a} = \frac{1}{m}\vec{F} = \frac{1}{m}\vec{E}\cdot q = \frac{(-e)}{m}\cdot\left(-\frac{\eta}{\varepsilon_0}\hat{x}\right) = \frac{eQ}{mA\varepsilon_0}\hat{x}$$

This is constant acceleration, so we get:

$$\vec{v}(t) = \frac{eQ}{mA\varepsilon_0}t \cdot \hat{x}$$

$$x(t) = \frac{1}{2}\frac{eQ}{mA\varepsilon_0}t^2$$

We want to choose Q so $v = 10^6\,\mathrm{m/s}$ at $x = \frac{d}{2}$. We find (from these equations or from part a),

$$Q = \frac{\varepsilon_0 A m v^2}{e\cdot d}$$

The electron reaches the middle at t s.t. $\frac{d}{2} = \frac{1}{2}\frac{eQ}{mA\varepsilon_0}t^2 \Rightarrow t = \sqrt{\frac{mA\varepsilon_0 d}{eQ}}$

$$= \frac{d}{v} = \frac{0.1\,\mathrm{m}}{10^6\,\mathrm{m/s}}$$
$$= 10^{-7}\,\mathrm{s}$$

The proton's trajectory will be:

$$y(t) = v_y t$$
$$x(t) = d - \tfrac{1}{2}a_x t^2 \quad \text{where } a_x = \frac{F}{m_p} = \frac{eQ}{m_p A\varepsilon_0} = \frac{m_e v^2}{m_p d}$$

The proton reaches $x = \frac{d}{2}$ when

$$d - \frac{1}{2} a_x t_p^2 = \frac{d}{2}$$

$$\Rightarrow \quad t_p = \sqrt{\frac{d}{a_x}} = \sqrt{\frac{m_p}{m_e}}\,\frac{d}{V} = 4.3\times10^{-6}\,s$$ (relative to when the proton is released)

At this time, the proton should be at $y = \frac{L}{2} = 0.5\,m$, so

$$V_y t_p = 0.5\,m$$

$$\Rightarrow V_y = \frac{0.5\,m}{t_p} = 1.17\times10^{5}\,m/s$$

Thus, the proton should be inserted with speed $V_y =$ at a time $t = \sqrt{\frac{m_p}{m_e}}\,\frac{d}{V} - \frac{d}{V} = 4.2\times10^{-7}\,s$ before the electron is released.