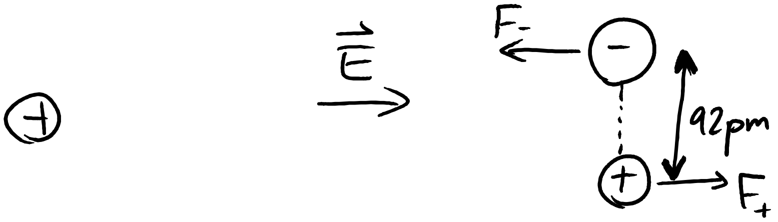
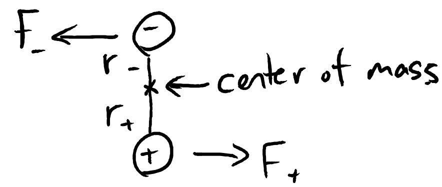
PROBLEM SET 3 SOLUTIONS



To estimate how long it takes for the molecule to orient itself with the field, we first determine the torque due to electrostatic forces. We have

$$F_- = -\frac{kQq}{R^2} \quad \text{and} \quad F_+ = \frac{kQq}{R^2} \qquad \begin{matrix}(R = 1m)\\(Q = 100nC)\end{matrix}$$

on the fluorine & hydrogen respectively (modeling HF as charges $\pm q = \pm 0.41e$ separated by $r = 92pm$).



The torque is

$$\tau = |F_-|r_- + |F_+|r_+$$

$$= \frac{kQq}{R^2}(r_+ + r_-)$$

$$= \frac{kQq}{R^2} \cdot r$$

The angular acceleration is initally:

$$\alpha = \frac{1}{I}\tau$$

where:

$$I = r_-^2 M_F + r_+^2 M_H$$

To be precise, we have $r_- = r \cdot \frac{M_H}{M_F + M_H}$ $\quad r_+ = r \cdot \frac{M_F}{M_F + M_H}$

so $I = r^2 \frac{M_H M_F}{M_H + M_F} \approx$

To estimate the time it takes to orient, we can just assume const. angular acceleration.

This gives:

$$\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2$$

So $$t_{\pi/2} = \sqrt{\frac{\pi}{\alpha}} = \sqrt{\frac{\pi I}{\tau}} = \sqrt{\frac{\pi \cdot I \cdot R^2}{k \cdot q \cdot Q \cdot r}}$$

$$\approx$$

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b)

The force on the HF is:

$$F = -\frac{kQq}{R^2} + \frac{kQq}{(R+r)^2}$$

this is $\approx r \cdot F'(R)$ for $F = +\frac{kqQ}{R^2}$, since $\varepsilon F'(R) \underset{\varepsilon \to 0}{=} F(R+\varepsilon) - F(R)$

Now: $$\frac{1}{(R+r)^2} - \frac{1}{R^2} = \frac{R^2 - (R+r)^2}{R^2(R+r)^2} = \frac{-2Rr - r^2}{R^2(R+r)^2}$$

$$\approx -\frac{2r}{R^3}$$

So $$F \approx -\frac{2kQ\overbrace{q \cdot r}^{\text{this is the dipole moment.}}}{R^3}$$

Now, we have initial acceleration

$$a = \frac{F}{m} = -\frac{2kQq \cdot r}{R^3 m} \approx 0.32 m/s^2$$

If this acceleration stayed constant, we would have $V = 10m/s$ when

$$\Delta t = \frac{10m/s}{a} \approx 30s$$

However in this time, the HF would have travelled over 100m in our approximation, so our assumption of const. acceleration isn't very good. As another approach, we can note that the speed will be 10m/s when the kinetic energy $\frac{1}{2} \cdot m \cdot (10m/s)^2$ equals the change in potential energy:

$$\frac{1}{2} m (10m/s)^2 = kqQr\left\{\frac{1}{R_f^2} - \frac{1}{R^2}\right\}$$

$$\Rightarrow R_f \approx 0.06m$$

Thus, the HF needs to make it almost all the way to Q. Most of the time will be in the first 0.5m. During this time, the accel. changes from $0.32m/s^2$ to $2.4\,m/s^2$ so we can say the time will be somewhere between

$$t = \frac{1}{\sqrt{2.4s^{-2}}} \text{ and } \frac{1}{\sqrt{0.32s^{-2}}} \quad \text{i.e. } 0.65s \leq t \leq 1.8s$$

This gives a more reliable estimate.