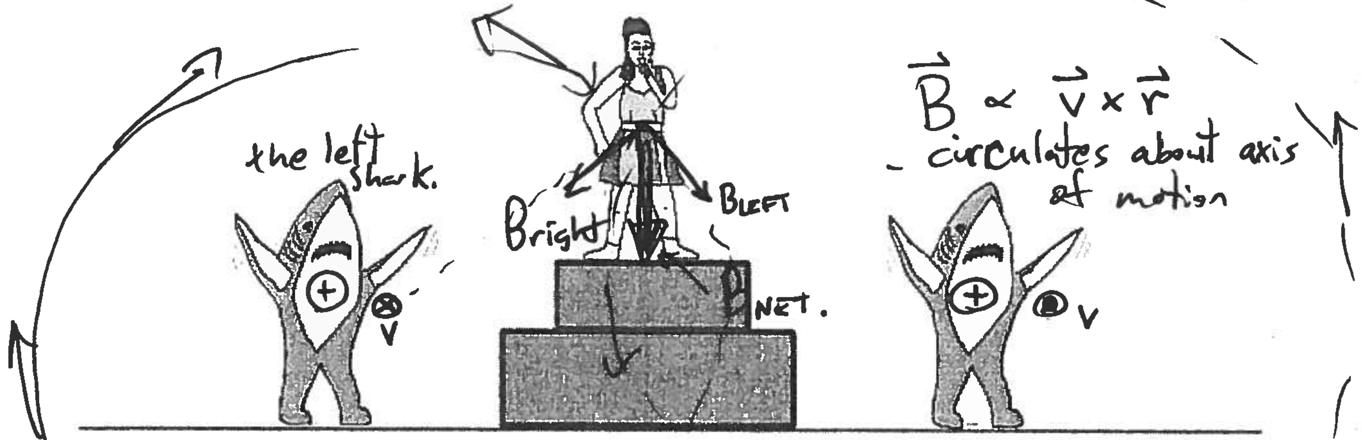
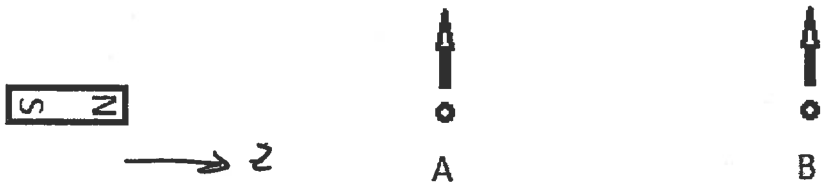
SOLUTIONS.



Question 1: The two dancing sharks are positively charged. If the left shark starts moving backward (into the page) when the right shark starts moving forward (out of the page), the magnetic field at the location of Katy Perry (directly between but above the sharks) will be:

A) To the right B) To the left C) Upwards D) Downwards
E) Up and to the left F) Down and to the right G) The field is zero



acceleration $= \frac{F}{m} = \frac{q}{m}\vec{v}\times\vec{B}$

$= \frac{q}{m} v B$

q, m, v same

Question 2: The two particles above have equal positive charges and equal upward velocities. The acceleration of particle A (half as far from the magnet) is roughly

B from magnet like E from dipole falls off like $\frac{1}{z^3}$ $z_A = \frac{1}{2} z_B$

A) half as large as the acceleration of particle B
B) twice as large as the acceleration of particle B
C) four times as large as the acceleration of particle B
D) eight times as large as the acceleration of particle B
E) sixteen times as large as the acceleration of particle B
F) the same as the acceleration of particle B, since they are both zero.

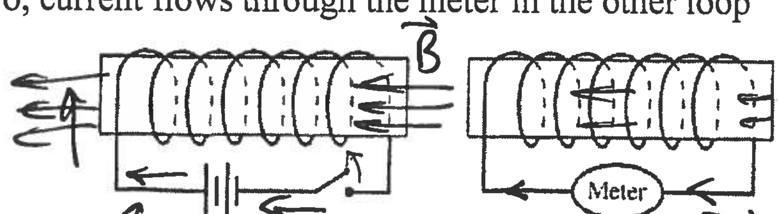
$\therefore B_A = 8 B_B$

$\therefore a_A = 8 a_B$

Question 3: The switch has been closed for a long time, and then is opened. As the current in the left loop drops to zero, current flows through the meter in the other loop

LENZ'S LAW.

A) from left to right.
B) from right to left.
C) There is no current.

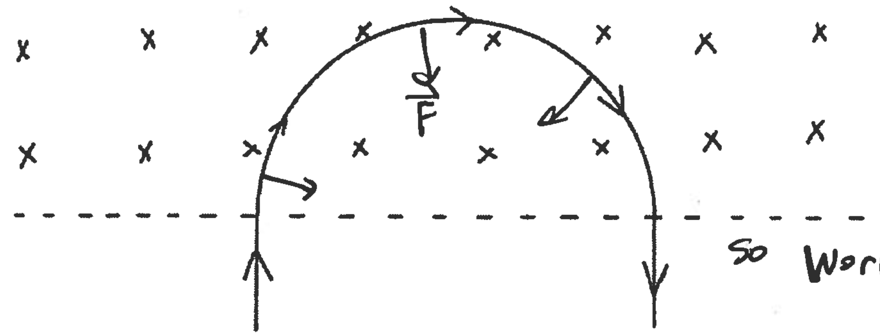


When switch opened, B decreases ∴ flux through right coil decreases

current when switch closed

induced I

Current induced s.t. induced B replaces lost B
∴ want current in same direction as original



$\vec{F}$ always perp. to $\vec{v}$:

$\vec{F} = q\vec{v} \times \vec{B}$

So Work $\vec{F} \cdot d\vec{x}$ is zero always

$\therefore$ no change in kinetic energy speed. same

Question 4: A charged particle enters a magnetic field, follows a circular path, and exits the magnetic field again. Assuming there are only magnetic forces on the particle, we can say that the speed of the particle when it exits the field will be

A) Greater than when it entered
B) The same as when it entered
C) Less than when it entered
D) Any of the above are possible depending on the initial velocity of the particle.

Question 5: If another particle with the same mass and charge but larger velocity enters the field as in question 4,

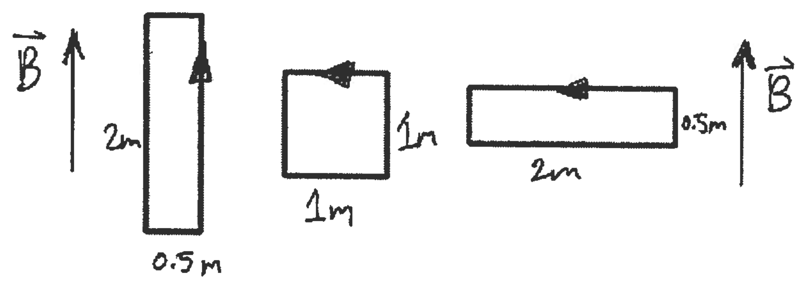
Have: $F = ma$

$\Rightarrow qvB = m\frac{v^2}{R}$

$\Rightarrow R = \frac{mv}{qB}$

$\therefore$ larger $v \Rightarrow$ larger R

A) The radius of the circular path will be larger.
B) The radius of the circular path will be smaller.
C) The radius of the circular path will be the same.

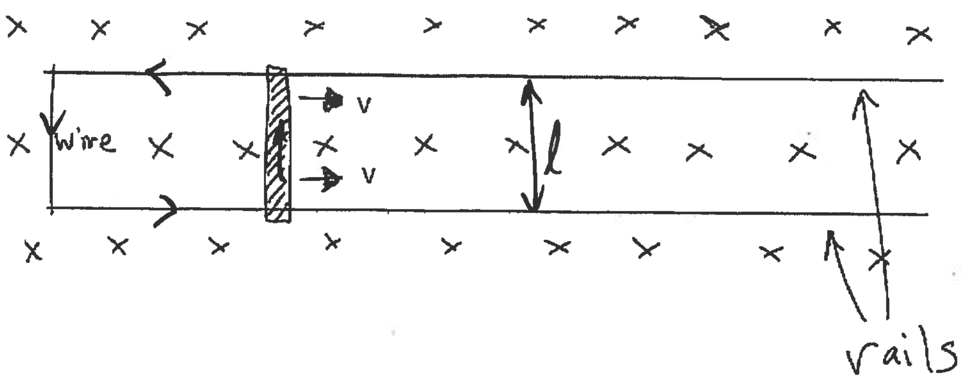


Question 6: The three rectangular current loops shown have equal current and are in equal magnetic fields. Which experiences the greatest torque?

Magnetic moment $= IA \rightarrow$ Same for all.

$\vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow$ same for all

A) The first one
B) The second one
C) The third one
D) The first and third have the same torque, larger than the second
E) All the torques are equal



Question 7:

In the picture above, a metal rod slides to the right on conducting rails which are connected by a wire. We can say that:

A) There will be an upward current in the rod.
B) There will be a downward current in the rod.
C) There will be no current in the rod.

Flux into page is increasing
Lenz's Law ⇒ current flows to counter this i.e. induced current causes flux out of page
∴ I counterclockwise in loop, upward in rod

OR: charges in rod moving to right
∴ upward force on + charges $F = q\vec{v} \times \vec{B}$

Question 8:

In the question above, we can say that the rod will experience

A) a force to the left that increases with time
B) a force to the left that stays constant in time
C) a force to the left that decreases with time
D) a force to the right that increases with time
E) a force to the right that stays constant in time
F) a force to the right that decreases with time
G) no net force

Force on rod $= I\ell B$ to the left.
B constant, ℓ constant
I is $\frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d\Phi}{dt}$

Note: assume that any changes in the rod's speed are due to electromagnetic forces only (i.e. ignore friction).

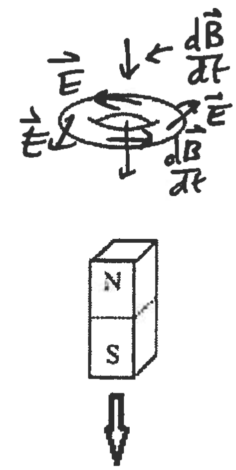
this decreases as the rod slows down due to the force.
∴ I decreases w time
∴ F decr. with time.

Question 9: Give a brief explanation (2-3 sentences) for each of the following:

a) How can we determine the direction and strength of the magnetic field at some point in space? **(2 points)** ONE POSSIBLE ANSWER:

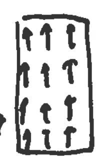
A small test magnet will experience torques that align it with the field. Direction of $\vec{B}$ = direction that N end of test magnet points. Magnitude of B can be deduced from magnitude of torque on test magnet via $\vec{\tau} = \vec{\mu} \times \vec{B}$.

b) In the situation shown, a current is induced in the stationary loop of wire (mmm…metal donut). Explain how/why the current arises. **(2 points)**



As the magnet moves downward, the magnetic field at the location of the loop is changing w/ time. This induces an electric field which circulates around the vertical axis (via Faraday's Law). The electric field gives a net EMF in the loop which causes a current.

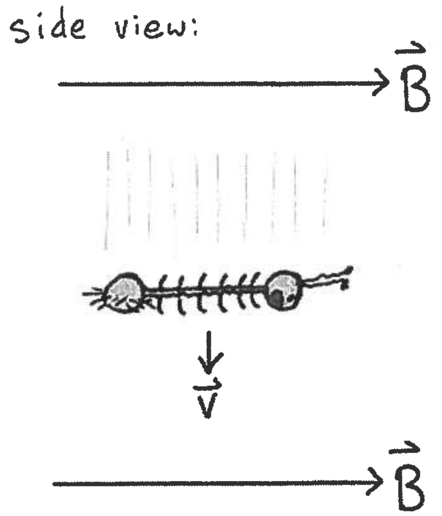
c) Why does a permanent magnet stop acting like a magnet if we make it hot enough? **(2 points)**

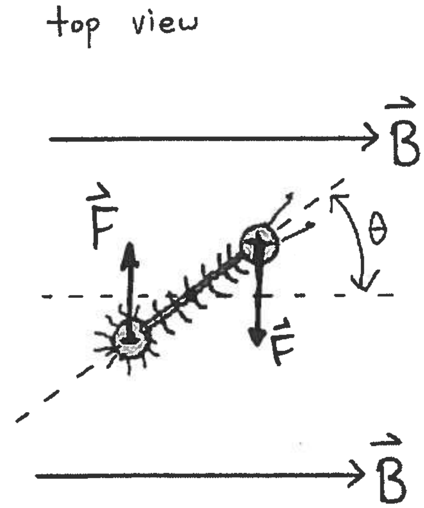


each electron like little magnet.

add up to big magnet when spins aligned.

A permanent magnet has its electron spins aligned (on average) in some specific direction. This is a lower energy but also lower entropy situation. At high temperatures, free energy will be minimized in a configuration where the spins are not aligned, so there will no longer be a net magnetic field.



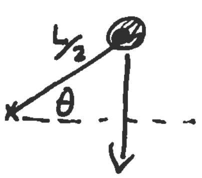


Question 10: A space plankton is falling to Earth at a constant terminal velocity v. The plankton has a narrow body of length L, with a "head" carrying positive charge Q and a tail carrying negative charge –Q. If the plankton is horizontal and θ describes the angle between the plankton's body and the Earth's (horizontal) magnetic field B, determine an equation/equations that describe how θ changes with time. The plankton has mass M and moment of inertia I. You do not need to solve your equation(s).
(6 points)

The charged head and tail moving through the $\vec{B}$ field experience magnetic forces:

$$\vec{F} = q\vec{v} \times \vec{B}$$

These have magnitude qvB. There is no net force, but we have a torque



$$\tau = r_\perp^+ F_+ + r_\perp^- F_-$$

$$= \frac{L}{2}\cos\theta \cdot qvB + \frac{L}{2}\cos\theta\, qvB$$

$$= qvBL\cos\theta$$

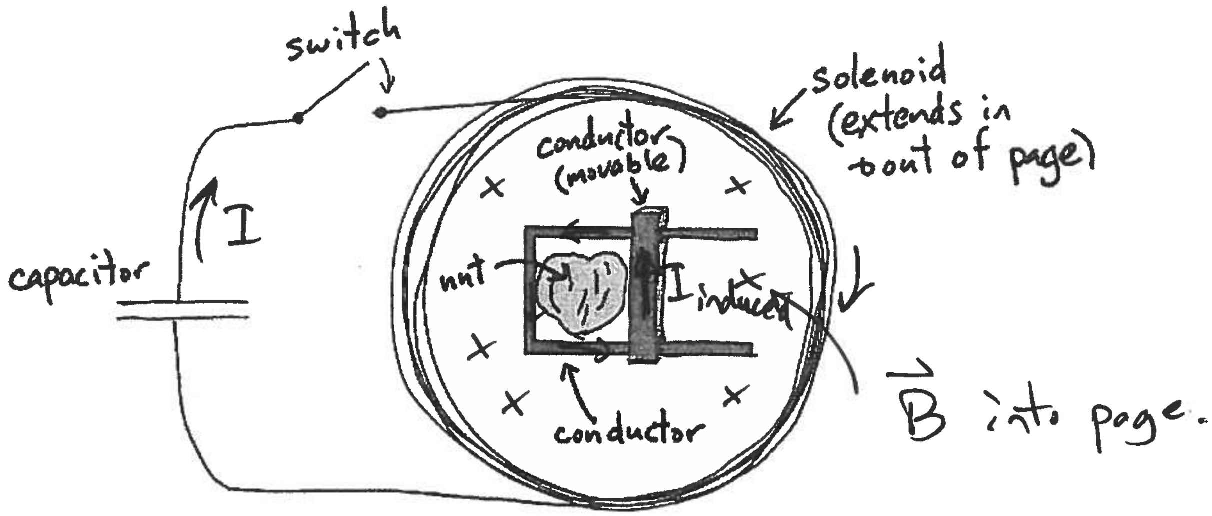
The torque causes an angular acceleration

$$\alpha = \frac{\tau}{I} = \frac{qvBL}{I}\cos\theta$$

We can determine how the angle changes using:

$$\frac{d\theta}{dt} = \omega \qquad \frac{d\omega}{dt} = \frac{qvBL}{I}\cos\theta$$

e.g. using Euler's method.

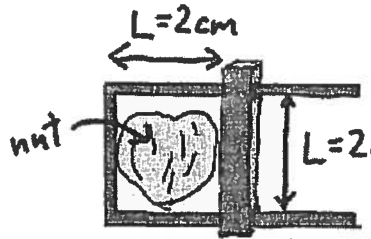


Question 11: Back at Sally's So-So Inventions, Sally has just come up with an idea for a not-entirely-practical electromagnetic nut cracker, depicted above. To crack a nut, Sally closes the switch, which discharges the capacitor through a solenoid. When the capacitor is discharged, a movable conductor inside the coil is observed to slide forcefully along another conductor, cracking the nut.

a) Explain why this nut cracker works **(3 points)**

When the switch is closed, the current in the coil rapidly increases, causing a rapid increase in the magnetic field inside the solenoid. This gives rise to a rapidly changing flux through the square nutcracker loop, so we have an induced EMF. The induced current flows counter clockwise to cause an induced ~~field~~ B that opposes the change in flux. There is a magnetic force on the movable conductor $\vec{F} = I\vec{l} \times \vec{B}$ which points to the left, and this pushes the movable piece against the nut to crack it.

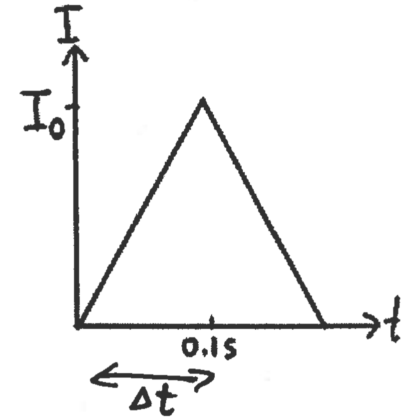
b) Suppose that the coil has 1000 turns in 10cm ℓ, and that the dimensions and resistance of the cracker circuit are as shown at the right. The current in the *solenoid* increases linearly to a maximum value I_0 ~~and then decreases linearly to a maximum value I~~ in time t=0.1s and then decreases linearly to zero, also in the time 0.1s. What value of I_0 is necessary to crack the nut (which requires 1000N of force)? **(3 points)**



resistance of square: $R = 10^{-4}\,\Omega$

We have the magnetic field in the solenoid equal to

$$B = \mu_0 \frac{N}{\ell} \cdot I$$



So the flux through the square loop is $\Phi = B \cdot A = \mu_0 \frac{N}{\ell} \cdot L^2 \cdot I$

The induced EMF is

$$\mathcal{E} = \frac{d\Phi}{dt} = \mu_0 \frac{N}{\ell} \cdot L^2 \cdot \frac{dI}{dt}$$

The induced current (by Ohm's Law) is:

$$I_{ind} = \frac{\mathcal{E}}{R} = \frac{\mu_0 N L^2}{\ell \cdot R} \cdot \frac{dI}{dt}$$

The magnetic force on the movable conductor is:

$$F = I L B$$

$$= \frac{\mu_0 N L^3}{\ell \cdot R} \cdot \frac{dI}{dt} \cdot \mu_0 \frac{N}{\ell} \cdot I = \left(\frac{\mu_0 N}{\ell}\right)^2 \cdot \frac{L^3}{R} \cdot I \cdot \frac{dI}{dt}$$

$\frac{dI}{dt}$ is $\frac{I_0}{\Delta t}$, and the max current is I_0, so the max force is

$$F_{MAX} = \left(\frac{\mu_0 N}{\ell}\right)^2 \frac{L^3}{R} \cdot \frac{I_0^2}{\Delta t} \quad \therefore \text{ We need: } I_0 = \left[F_{BREAK} \cdot \left(\frac{R \Delta t}{L^3}\right)\right]^{\frac{1}{2}} \cdot \left(\frac{\ell}{\mu_0 N}\right)$$

$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{Tm/A}$ $\quad = 2.8 \times 10^3\,\mathrm{A}$

not-entirely-practical

$F = 1000\,\mathrm{N}$ $\quad R = 10^{-4}\,\Omega$ $\quad \Delta t = 0.1\,\mathrm{s}$ $\quad L = 0.02\,\mathrm{m}$ $\quad \ell = 0.1\,\mathrm{m}$ $\quad N = 1000$