Name:
Student Number: SOLUTIONS.

Science One Physics Midterm #3

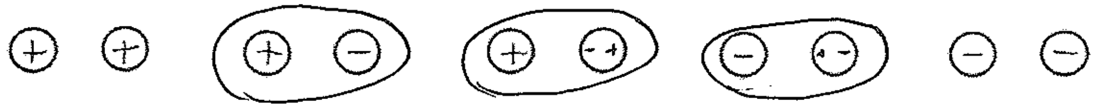
February 11, 2013

Questions 1-9: Multiple Choice: 2 points each
Questions: Explain your work: 22 points total

Multiple choice answers:

#1	C
#2	F
#3	A
#4	D
#5	A
#6	D
#7	B
#8	C
#9	B

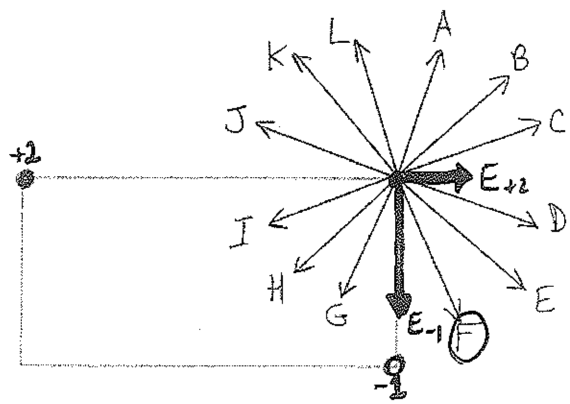
Formula sheet at the back (you can remove it)



Question 1: The diagram above shows pairs of water droplets, which may be positively charged, negatively charged, or neutral. In how many of the pairs above will the two droplets attract each other?

A) 1 B) 2 C) 3 D) 4 E) 5

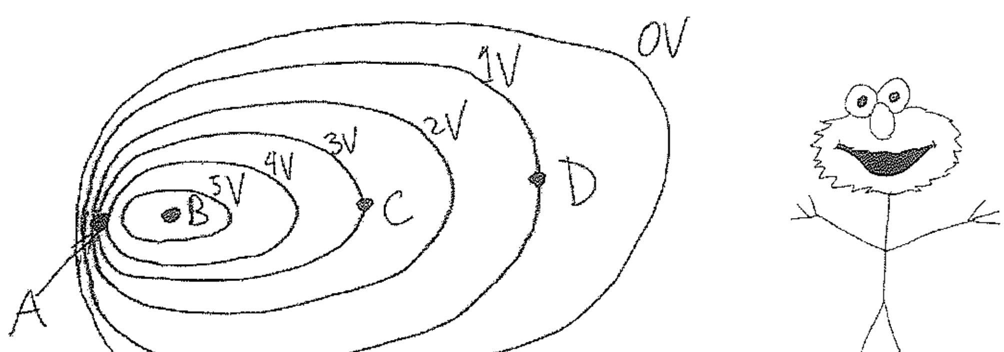
Neutral droplets become polarized and are attracted to a charged droplet.



$E = k\frac{Q}{r^2}$

$\therefore$ E from +2 has magnitude $\frac{2}{(2)^2} = \frac{1}{2}$ compared w. E from -1

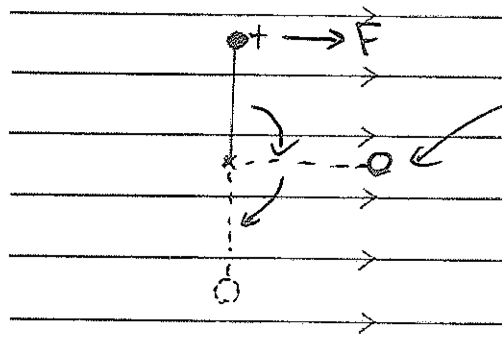
Question 2: If the rectangle above is twice as wide as it is high, what is the direction of the electric field at the top-right corner due to the two charges shown? Note: vectors B, E, H, and K point at 45 degrees to the horizontal.



Question 3: Elmo *loves* electric fields. To which point in the picture above should Elmo go to find the largest electric field, if the curves shown are lines of constant potential?

Answer A, B, C, D, or E) not enough information

E largest where V changes fastest

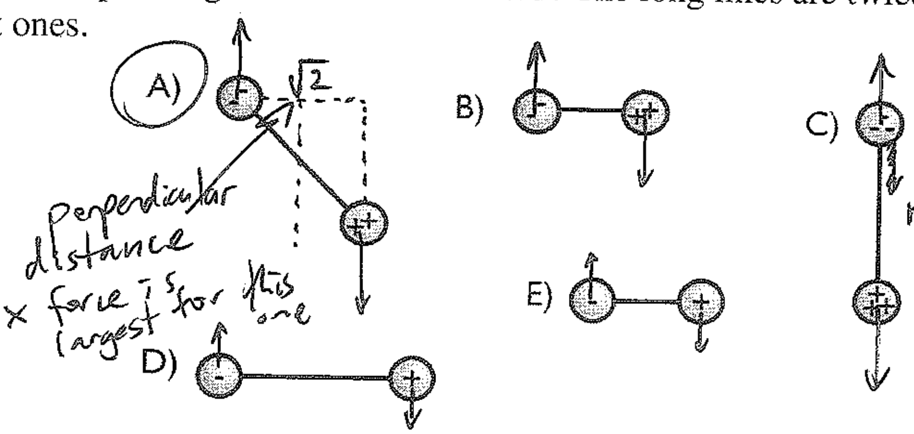


doesn't stop here due to energy conservation
∴ goes back & forth.

Question 4: In the picture above, a uniform electric field points to the right. A positively charged object is attached by a rigid rod to a fixed pivot point (marked with an x). If the charged object is initially held stationary and then released, we can say that *(ignoring gravity)* it will:

A) Rotate clockwise around the pivot until the rod points to the right and remain there.
B) Rotate clockwise around the pivot until the rod points down and remain there.
C) Continue to rotate clockwise around and around the pivot.
D) Oscillate back and forth between the initial configuration and a configuration where the rod points down. Just like pendulum in gravity
E) Stay still unless pushed a little to the right.

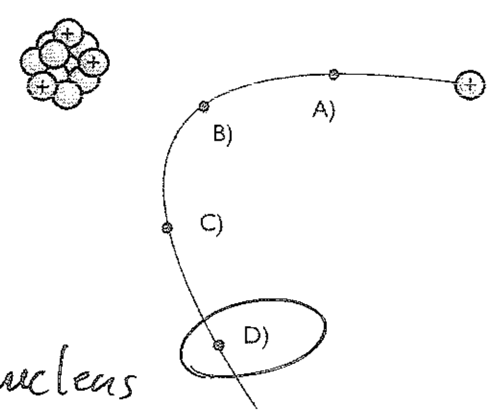
Question 5: All of the balls shown below have the same mass, with charges as indicated. On which of the dipoles will there be the largest torque if they are all in a downward-pointing constant electric field? The long lines are twice the length of the short ones.



torque = sum of $F \cdot r_\perp$

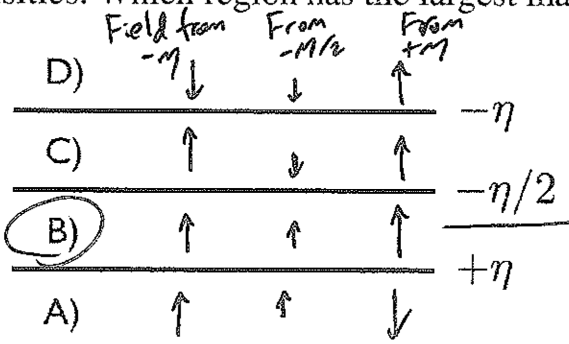
Have: $E < B = D < A$

Question 6: The figure to the right shows the path a proton takes near the nucleus of an atom. At which of the points indicated does the proton have the highest kinetic energy?



$E_{potential} + E_{kinetic}$ constant.
∴ Highest E_k where smallest E_p = furthest from nucleus

Question 7: The diagram below shows three infinite planes of charge with different surface charge densities. Which region has the largest magnitude of electric field?



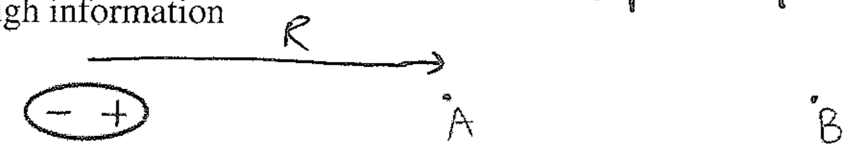
largest here since all in same direction

Question 8: In the picture below, which would have a greater acceleration, an object with charge +1 and mass 4M placed at point A, or an object with charge +2 and mass M placed at point B (twice as far from the dipole)?

$a = \frac{F}{m} = \frac{Eq}{m}$

A) The +1 charge at A
B) The +2 charge at B
C) The accelerations would be the same
D) Not enough information

$E_{dipole} \sim \frac{1}{r^3}$ so $a \sim \frac{q}{mr^3}$

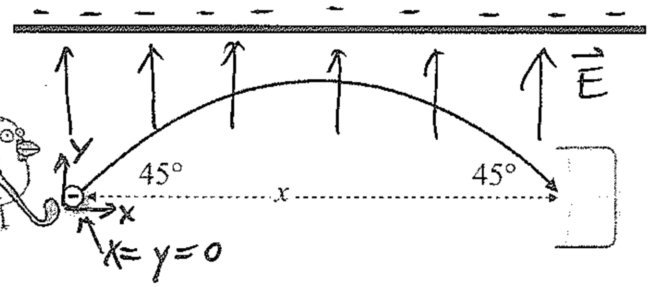


A: $\frac{1}{(4M)\cdot R^3} \sim \frac{1}{4MR^3}$ B: $\frac{2}{M\cdot(2R)^3} \sim \frac{1}{4MR^3}$

Question 9: Two identical fish tanks are connected by a tube through which the fish can swim. If there are originally 100 fish in the first tank and 300 fish in the second tank, and if the fish swim around randomly, the Second Law of Thermodynamics tells us that:

A) After a certain time, there will always be 200 fish in each tank.
B) After a while, it is very likely that the number of fish on the two sides will be approximately equal.
C) If we look into one of the tanks after a long time, it is equally likely that we will find any number of fish from 0 to 400.
D) After a long time, the distribution of fish may be different from 100-300, but *on average,* the number of fish on one side will stay close to 100.
E) The fish, the water, and the tank will eventually become nothing more than a hot ball of plasma, the anonymous fragments of an aquarium that once was.

Question 10: In a spirited game of Electric Field Hockey, Chuck the Chicken attempts to score a goal (and obviously succeeds) by shooting the negatively charged puck toward a negatively charged infinite plane.



The magnitude of the electric field is E = 4 N/C. The puck has a charge -1 C and a mass of 250 grams. The distance between the puck and the net is x = 8 m.

Assuming Chuck succeeds, determine the ***magnitude of the initial velocity*** *of the puck and calculate* ***how long it takes*** *for the puck to travel into the net.*

We have $\vec{F} = m\vec{a}$, with:

$$a_x = \frac{F_x}{m} = 0$$

$$a_y = \frac{F_y}{m} = \frac{-E \cdot Q}{m} \qquad E = 4\,N/C \quad Q = 1C.$$

So: $x = v_x^0 t$ and $y = v_y^0 t + \frac{1}{2} a_y t^2$, where we have defined (v_x^0, v_y^0) to be the initial velocity. Since the angle is 45°, $v_x^0 = v_y^0$. Then, when the puck hits the net, we have:

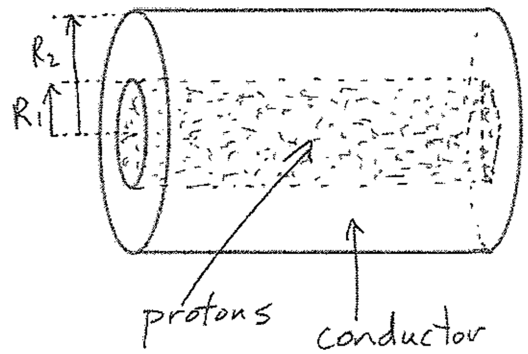
$$v_x^0 t_f = x_f = 8m \qquad (1)$$

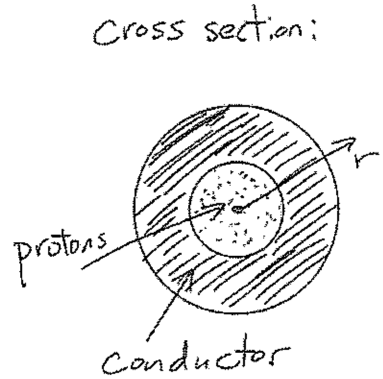
$$v_x^0 t_f - \frac{1}{2}\left(\frac{EQ}{m}\right) t_f^2 = y_f = 0 \qquad (2)$$

From (2), we have: $v_x^0 = \frac{1}{2}\left(\frac{EQ}{m}\right) t_f$. Plugging in to (1), we get: $\frac{1}{2}\left(\frac{EQ}{m}\right) t_f^2 = 8m$.

So: $t_f = \sqrt{\dfrac{16m \cdot 0.25kg}{4N/C \cdot 1C}} = 1s$ and $V_0 = \sqrt{(v_x^0)^2 + (v_y^0)^2} = \sqrt{2}\, v_x^0$

$$= \sqrt{2} \cdot \frac{1}{2}\left(\frac{EQ}{m}\right) t_f = 11.2\,m/s$$





Question 11:

At a particle accelerator, a beam of protons travels through a hollow conducting cylinder, as shown above. Determine the magnitude of the outward electric field, as a function of the distance r to the center of the beam, in the regions $0 < r < R_1$, $R_1 < r < R_2$, and $r > R_2$. Assume that the protons are spread throughout the interior of the cylinder, so that the charge density inside is a constant ρ.

Hint: assume the beam is infinitely long and consider cylindrical Gaussian surfaces of length ℓ and different radii. What is the flux through the curved parts? What is the flux through the flat ends of the cylinder?

First, the field inside the conductor $R_1 < r < R_2$ is zero.

Inside the tube, consider a Gaussian surface of length ℓ and radius r as shown.

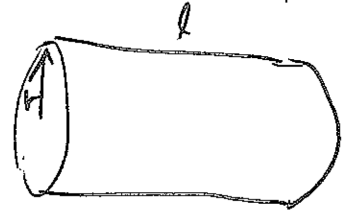
The volume of the tube is $(\pi r^2) \times \ell$ so the charge enclosed is $Q_{encl} = \rho \cdot (\pi r^2) \cdot \ell$. There is no flux through the ends, and the flux through the sides is

$$\text{Flux} = |\vec{E}| \cdot \text{Area} = |\vec{E}| \cdot 2\pi r \cdot \ell .$$

Using Gauss's Law: $|\vec{E}| \cdot 2\pi r \cdot \ell = \frac{1}{\varepsilon_0} \cdot Q_{encl} = \frac{1}{\varepsilon_0} \cdot \rho (\pi r^2) \ell$

$$\boxed{\Rightarrow |\vec{E}|_{in} = \frac{r}{2\varepsilon_0} \cdot \rho}$$

Outside the tube, we consider another surface:



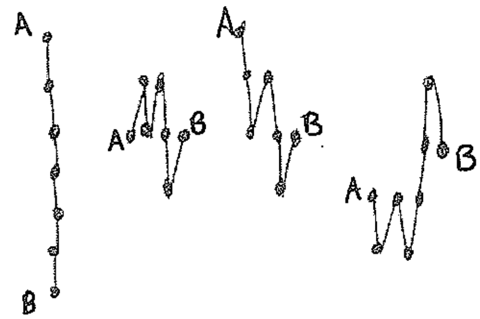
This time, the charge enclosed is always

$$Q_{encl} = \pi R_1^2 \cdot l \cdot \rho$$

The flux is $|\vec{E}| \cdot \text{Area} = |\vec{E}| \cdot 2\pi r \cdot l$. Using Gauss' Law, we get

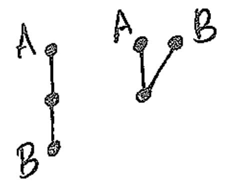
$$|\vec{E}| \cdot 2\pi r \cdot l = \pi R_1^2 \cdot l \cdot \rho \cdot \frac{1}{\varepsilon_0}$$

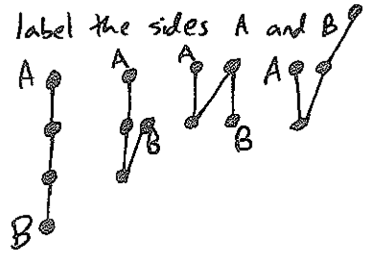
$$\boxed{\Rightarrow |\vec{E}|_{out} = \frac{R_1^2 \rho}{2 r \varepsilon_0}}$$



Question 12:

a) A long polymer molecule has a series of many "joints," each of which can be straight or bent 180° back on itself. The straight and the bent configurations both have the same energy. Draw all the possible configurations of molecules with 2 and 3 segments. Assume that the molecules have a definite orientation, so the two ends can be distinguished. (you can label the sides A and B as above)

2: 

3: 

b) When these molecules are observed in a medium where they are completely free to move around, they are never found stretched out to their maximum length, but instead they are bent up so that the total length is much shorter than the maximum possible. Explain qualitatively why this is the case. Estimate the chances that a given molecule will be found completely stretched out. If the polymer has 100 joints, describe a calculation that could be done (in principle) to estimate what the length of a typical configuration would be.

There are a very large number of possible configurations, but relatively few of these have the molecule at or nearly at its maximum length. Assuming all configurations are equally likely, we are therefore most likely to find a molecule in a configuration where it is much shorter than its maximum length.

The total number of configurations can be found by noting that at each joint, there are 2 possible configurations (straight or bent) so the total number of configs. for the whole molecule is $2 \times 2 \times 2 \times \ldots \times 2 = 2^{\#\text{ of Joints}}$.

So for a molecule with 100 joints, the chances that we'll find it fully stretched out are 1 in 2^{100}. Pretty slim.

To find the length of a typical configuration, we want to take the average length over all configurations:

$$\text{typical length} = \frac{\sum_{\substack{\text{all} \\ \text{configs.}}} (\text{length})}{2^{100}}$$

THIS WOULD GET YOU SERIOUS BONUS POINTS.

One way to do this is find the total number of configs. with length n (call this N_n), in terms of which

$$\text{avg} = \frac{\sum_n n \cdot N_n}{2^{100}} \qquad (*)$$

To find N_n, notice that if we represent a configuration by a list $(+1, +1, -1, -1, +1, -1, \ldots)$, the length is just the sum of the numbers: $L = N_1 - N_{-1}$.

straight

bent

$$\begin{aligned} L &= N_1 - N_{-1} \\ &= N_1 - (100 - N_1) \\ &= 2N_1 - 100 \end{aligned}$$

So the number of configurations with length $2N_1 - 100$ is the number of ways of choosing N_1 things out of 100

$$N_{2N_1-100} = \frac{100!}{N_1!(100-N_1)!}.$$

Now we can use this in our formula $(*)$, but actually getting an answer is not really going to fit on the rest of this page.

8:50