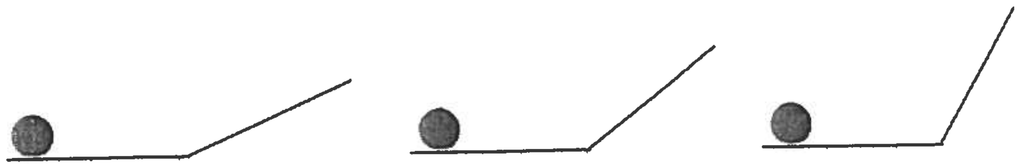
Question 1: All the balls in the pictures below have equal mass and are moving to the right with equal speed. Each ball rolls up a ramp and then back down. Which ball will reach the highest vertical position?

Energy conservation

$\Rightarrow E_{initial} = E_{final}$

a) b) c)



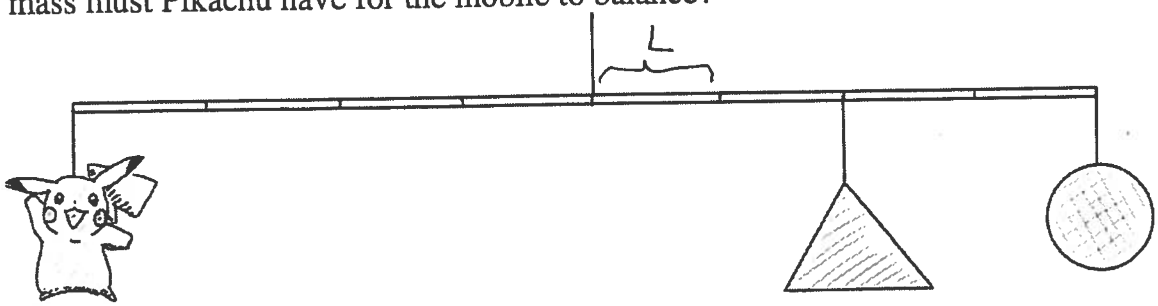
Final energy $= mgh_f$

$\therefore h_f$ same for all

since $E_{initial}$ same

d) They will all reach the same height.

Question 2: In the picture below, if the triangle has mass 6kg and the circle has a mass 3kg, what mass must Pikachu have for the mobile to balance?



A) 3kg B) 6kg C) 7.5kg D) 9kg E) 18kg

balanced $\Rightarrow$ net torque $= 0 \Rightarrow \sum F_i \cdot r = 0$

$$6kg \cdot 2L + 3kg \cdot 4L = M_p \cdot 4L$$

$$\Rightarrow M_p = 6kg$$

Question 3:

Two protons are travelling at large speeds v and 2v. We can say that the momentum of the faster proton is

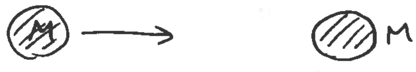
$$P_1 = \gamma_v m v$$

A) Less than double the momentum of the slower proton
B) Double the momentum of the slower proton
C) More than double the momentum of the slower proton
D) Any of the above could be true, depending on what v is.

$$P_2 = \gamma_{2v} m (2v)$$

$$\frac{P_2}{P_1} = 2 \cdot \frac{\gamma_{2v}}{\gamma_v} > 2$$

Question 4: A ball of putty with mass M and total energy 2 M c² collides with a stationary ball of putty of mass M. If the two merge to form a single putty-ball, we can say that its mass is



Inelastic ⇒ some K.E. lost to mass energy ∴ mass > $2Mc^2$

A) Less than 2M
B) Equal to 2M
C) Greater than 2M but less than 3M
D) Equal to 3M
E) Greater than 3M

OR: in other frame

(M) → ← (M)

(M) ∴ M′ > 2M

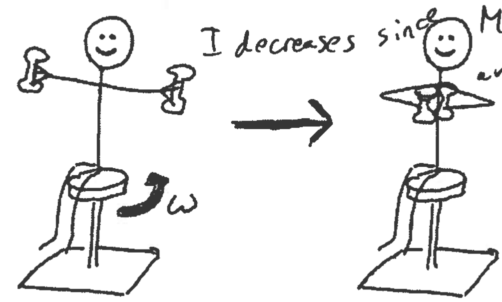
moving since p conserved

TOTAL ENERGY = $3Mc^2$

Mass energy + Kin energy $= 3Mc^2$

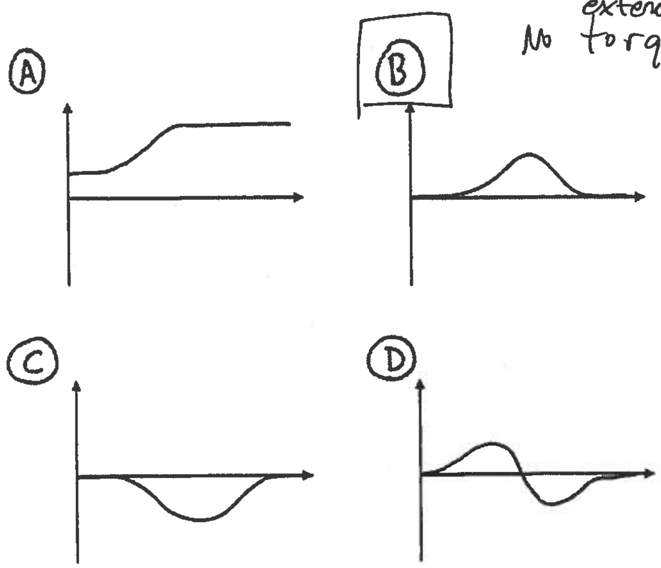
∴ Mass energy $< 3Mc^2$

Question 5: Jay-Z is spinning around on a frictionless stool, holding weights with his arms extended. He then pulls the weights in towards his body and holds them there. Which of the following best represents a graph of Jay-Z's angular acceleration during this process if we assume his angular velocity is initially positive?

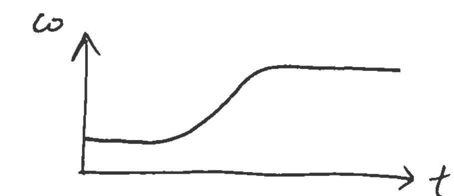


I decreases since M same, avg R^2 ↓

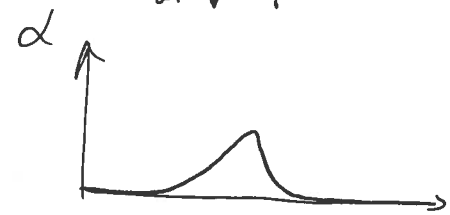
No external torque ⇒ $L = I\omega$ conserved ⇒ ω↑ when I↓



angular velocity:



$\alpha = \frac{d\omega}{dt}$ ↓ slope



(E) $\alpha = 0$ for all t

Question 6: A spring is extended from its equilibrium length. Assuming no atoms are lost in the process, the stretched spring is

A) more massive than it was initially
B) less massive than it was initially
C) the same mass as it was initially

M = total energy of object in rest frame

need to add energy to stretch it

Question 7: Two balls of putty fly through the air and collide, sticking to each other to make a single ball of putty. During the collision, how many of:

always conserved if no external forces

i) momentum ✓ ii) mechanical energy ✗ iii) total relativistic energy ✓

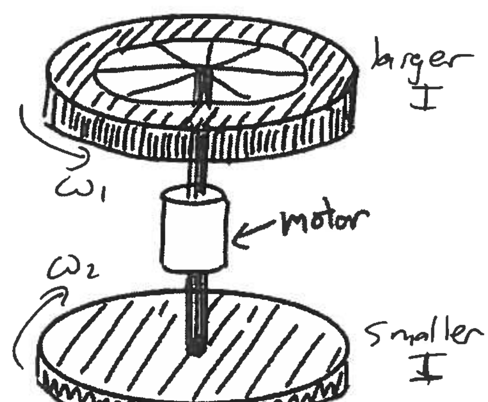
Some is lost to heat / (mass energy)

always conserved for isolated system.

must be conserved?

A) 0
B) 1
C) 2
D) 3

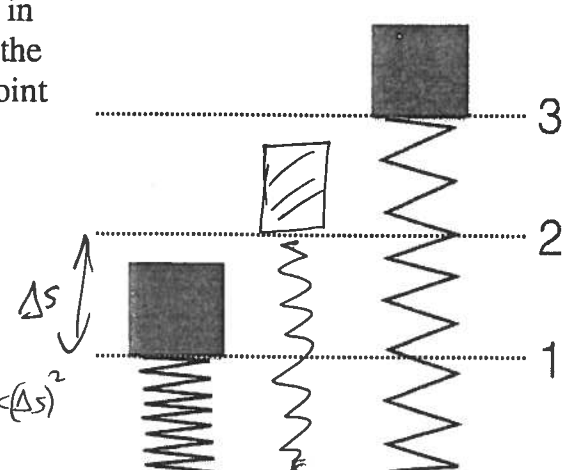
Question 8: The object shown consists of a solid disk and a ring both with the same mass and radius, connected by a motor that makes the two ends spin relative to one another. If the object is floating in outer space with no initial angular momentum and the motor is turned on, we will find that



A) The ring and disk will spin in opposite directions, with the ring spinning faster.
B) The ring and disk will spin in opposite directions, with the disk spinning faster.
C) The ring and disk will spin in opposite directions, each with the same angular speed.
D) Neither the ring nor the disk are able to spin, since there is no external torque on the system.

Net L remains zero, so $I_1\omega_1$ equal in magnitude to $I_2\omega_2$ $\therefore$ ω_2 larger

Question 9: The picture on the left shows a block at rest in position 1, compressing a spring. You lift the block past the point 2 where the spring is at its equilibrium length, to point 3 shown in the picture on the right.



The work done by you moving the block from 1 to 2 is

Energy at 1: $\frac{1}{2}k(\Delta s)^2$

Energy at 2: $mg\Delta s$

Energy at 3: $2mg\Delta s + \frac{1}{2}k(\Delta s)^2$

A) the same as
B) more than
C) less than

the work done moving the block from 2 to 3.

$$W_{1-2} = E_2 - E_1 = mg(\Delta s) - \frac{1}{2}k(\Delta s)^2$$

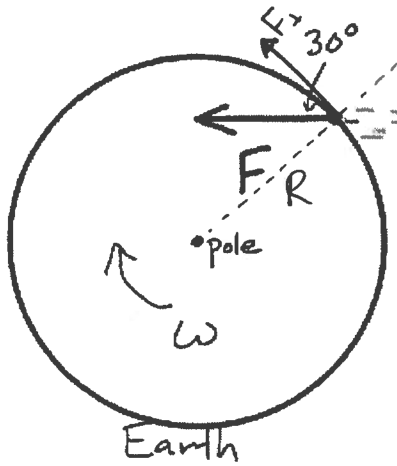
$$W_{2\to 3} = E_3 - E_2 = mg(\Delta s) + \frac{1}{2}k(\Delta s)^2$$

OR: $W = F\Delta x$

for $1\to 2$ F_g + F_{spring} are in opposite directions

for $2\to 3$ same direction

Question 10: A group of students from the Science One class of 2078 are sitting around one day complaining that there aren't enough hours in a day to get all their work done. So they decide to make the day longer. Using the newly built Lunar Force Cannon, the students are able to exert a constant force on the Earth as shown in the diagram. After one year, the Earth's rotation slows down enough so that the day is 25 hours long. How much force is the force cannon exerting on the Earth ($M=6.0\times10^{24}$kg, $R=6.4\times10^{6}$m)? **(7 points)**



The torque caused by the force F is

$$\tau = F_{\perp} \cdot R$$
$$= F \cdot \cos(30°) \cdot R$$
$$= \frac{\sqrt{3}}{2} FR.$$

This causes angular acceleration

$$\alpha = \frac{\tau}{I} = \frac{\sqrt{3}}{2} \frac{F \cdot R}{\frac{2}{5} M R^2} = \frac{5\sqrt{3}}{4} \frac{F}{M \cdot R}.$$

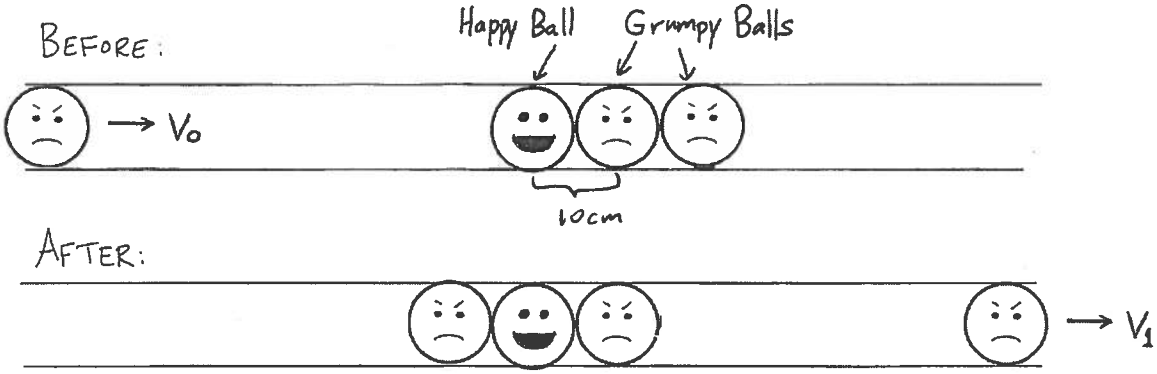
The change in ω in $t = 1$ year is:

$$\Delta\omega = \alpha \Delta t \quad \text{(since } \alpha \text{ is constant)}$$
$$= -\frac{5\sqrt{3}}{4} \frac{F}{M \cdot R} \cdot t$$

We have $\omega_0 = \frac{2\pi}{24 \text{ hours}} = \frac{2\pi}{24 \cdot 3600\ s}$ $\quad \omega_f = \frac{2\pi}{25 \cdot 3600\ s}$ so

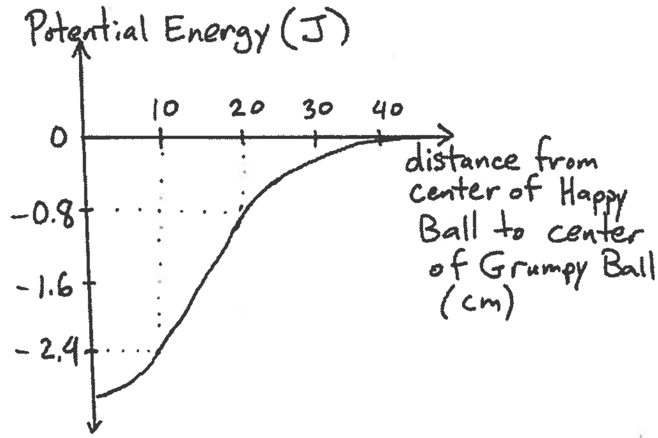
we need

$$F = \frac{4}{5\sqrt{3}} \cdot \frac{M \cdot R \cdot (\omega_0 - \omega_f)}{t} = \frac{4}{5\sqrt{3}} \times \frac{6\times10^{24}\text{kg} \cdot 6.4\times10^{6}\text{m}}{365 \times 24 \times 3600\ s} \times \Delta\omega = 1.6 \times 10^{18}\ N$$



Question 11: Grumpy Balls have a lower potential energy the closer they are to Happy Ball, as show in the graph below.

A Grumpy Ball with initial speed $v_0 = 1$m/s comes in from the left as shown in the top picture. It collides with the other balls, causing the ball on the right to move off with speed v_1. If all the balls have diameter 10cm and mass 1kg, and if 0.1J is lost to heat during the inelastic collision, what is the final speed of the Grumpy Ball in the second picture? *(Note: Happy Ball is fixed in place, so momentum isn't conserved here.)* **(7 points)**



Energy is conserved, so $E_{before} = E_{after}$.

$$E_{before} = \underbrace{\tfrac{1}{2}mv_0^2}_{\text{Kinetic}} + U_{10cm} + U_{20cm}$$

$$E_{after} = \tfrac{1}{2}mv_1^2 + U_{10cm} + U_{10cm} + E_{thermal}$$

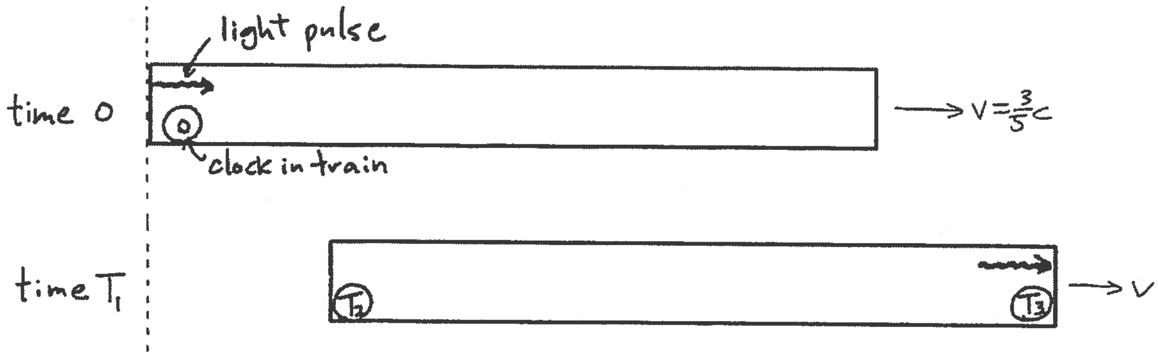
$$E_{before} = E_{after} \Rightarrow \tfrac{1}{2}mv_0^2 + U_{10cm} + U_{20cm} = \tfrac{1}{2}mv_1^2 + U_{10cm} + U_{10cm} + E_{thermal}$$

$$\Rightarrow \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}mv_0^2 + (U_{20cm} - U_{10cm}) - E_{thermal}$$

$$\Rightarrow \tfrac{1}{2}(1kg)v_1^2 = \tfrac{1}{2}(1kg)v_0^2 + (-0.8J + 2.4J) - 0.1J$$

$$\Rightarrow v_1^2 = (4\,m^2/s^2)$$

$$\Rightarrow v_1 = 2m/s$$



Question 12: The pictures above show a pulse of light leaving from one end of a moving train of (proper) length 30m and arriving at the other end. Observers on the train and on the track both agree that the light leaves the back of the train at time 0. **(5 points total)**

a) At what time T_1 do observers on the track see the light hit the front of the train? Explain. (*Hint: the train is moving.*)

The length of the train in this frame is $\frac{L_{proper}}{\gamma} = \frac{4}{5} \cdot 30\text{m}$

$= 24\text{m}$

Location of light pulse: $c \cdot t$

Location of front of train: $24\text{m} + \frac{3}{5}ct$

Equal when: $ct = 24\text{m} + \frac{3}{5}ct$

$\Rightarrow \frac{2}{5}ct = 24\text{m}$

$\Rightarrow t = \frac{60\text{m}}{c} = 2 \times 10^{-7}\text{s}$ $\quad \therefore T_1 = 2 \times 10^{-7}\text{s}$

b) What is the time that is observed to pass on the clock at the back of the train during this time? This is the time T_2 on the back clock in the second picture. Explain.

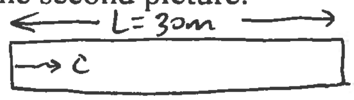
During this time, the back clock on the train is observed to run slow, due to time dilation

$$\therefore T_2 = \frac{T_1}{\gamma}$$

$$= \frac{4}{5} \cdot 2 \times 10^{-7}\text{s}$$

$$= 1.6 \times 10^{-7}\text{s}$$

c) How much time do observers in the train measure for light to reach the front of the train? Explain. (*Hint: imagine you are in the train.*) This is the time T_3 on the front clock in the second picture.



The time is just $\frac{30m}{c} = 1.0 \times 10^{-7}$ s

d) The relativity of simultaneity tells us that the front and back clocks in the train will be observed to read different times. Using your answers to b) and c), how much earlier is the time on the front clock?

We have $T_2 - T_3 = 1.6 \times 10^{-7}\,s - 1.0 \times 10^{-7}\,s$

$= 6 \times 10^{-8}\,s$

The front clock reads a time that is $6 \times 10^{-8}\,s$ earlier.