

Exam answers:

MC: C E E D D

B E A E C

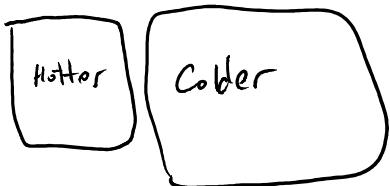
G C D C E

A A B A B

A C D B A

C C

Problem 1:



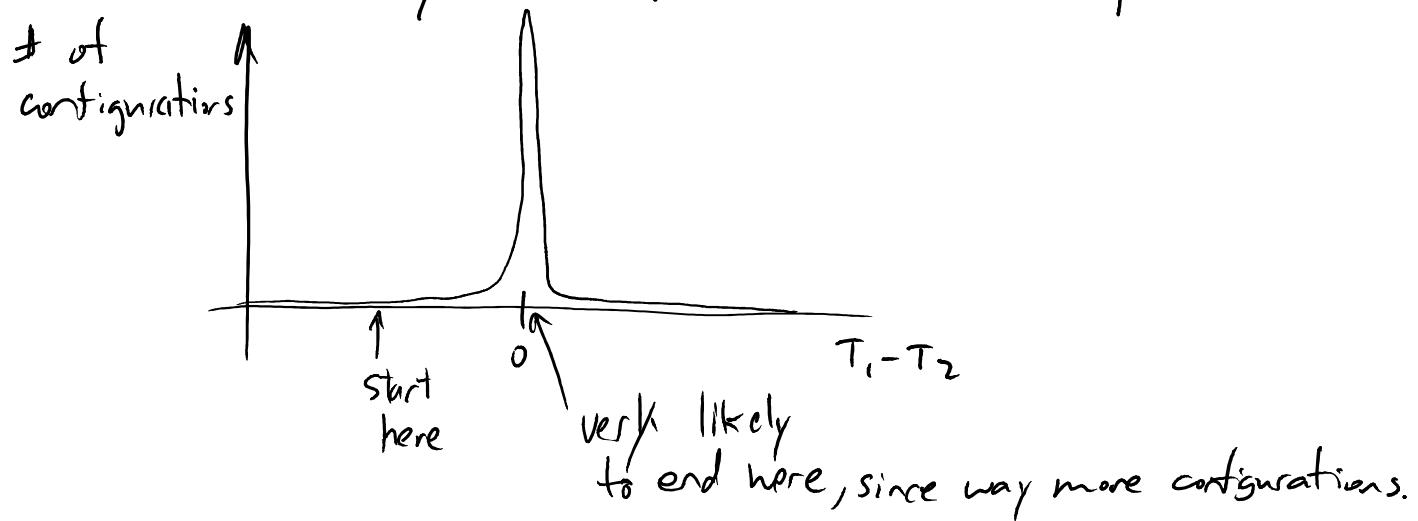
If the total system has a certain amount of

energy E_{TOT} the two parts have energy

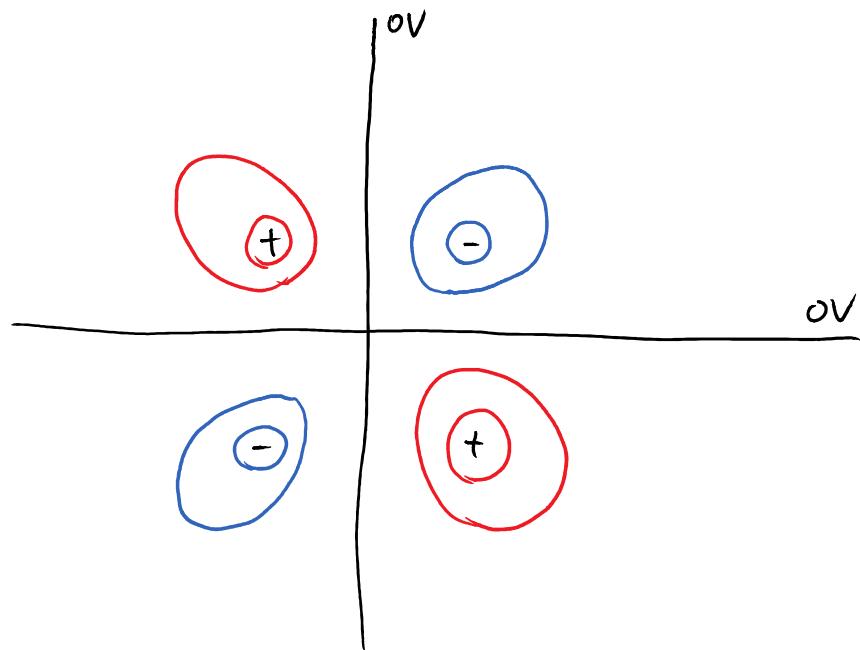
E_1 and $E_2 = E_{\text{TOT}} - E_1$. For each (E_1, E_2) there are a very large number of ways to distribute the energy within the two systems. This is measured by the entropy of the total system.

When the systems interact, energy can flow either way and this allows the full system to change between different configurations. But overall, there are far more configurations w. similar temperature. So it's almost certain that energy will flow

in such a way to equalize the temperatures



②

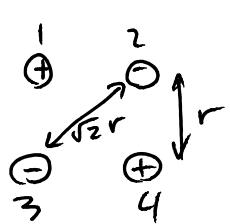


$$\text{Potential energy} = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$$

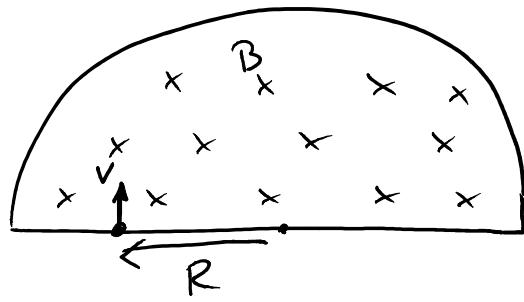
$$= kQ^2 \left\{ -\frac{1}{r} - \frac{1}{r} + \frac{1}{\sqrt{2}r} + \frac{1}{\sqrt{2}r} - \frac{1}{r} - \frac{1}{r} \right\}$$

$$= \frac{kQ^2}{r} \left\{ -4 + \sqrt{2} \right\}$$

$$= -4.65 \times 10^{-8} \text{ J}$$



a)



The particles travel on circular paths at constant velocity, so:

$$a = \frac{v^2}{R}$$

The acceleration is due to the magnetic field:

$$a = \frac{F}{m} = \frac{qvB}{m}$$

$$\text{So: } \frac{qvB}{m} = \frac{v^2}{R} \Rightarrow v = \frac{qRB}{m}.$$

The time to go half a circle is $t = \frac{\pi R}{v} = \frac{\pi m}{qB}$, which is independent of v and R .

b) The time for a full circle is $2.0 \times 10^{-6} \text{ s}$, so for half a circle, it takes $1.0 \times 10^{-6} \text{ s}$. From part a)

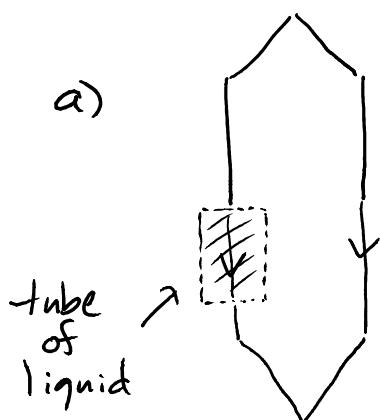
$$1.0 \times 10^{-6} \text{ s} = \frac{\pi \cdot m}{q \cdot B}$$

$$\Rightarrow B = \frac{\pi m}{q \cdot (1.0 \times 10^{-6} \text{ s})} = \frac{3.14 \times 1.67 \times 10^{-27} \text{ kg}}{1.6 \times 10^{-19} \text{ C} \times 1.0 \times 10^{-6} \text{ s}} = 3.28 \times 10^{-2} \text{ T}$$

c) Each time, the atom's energy increases by

$E = q \cdot \Delta V = e \cdot 50 \text{ kV} = 50 \times 10^3 \text{ eV}$. To obtain a final energy of 500 MeV, we need $500 \text{ MeV} / 50 \times 10^3 \text{ eV} = 10^4$ accelerations

Problem 4

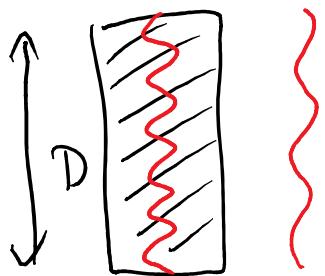


For this setup, the key point is that the larger index of refraction in the drink means that v is smaller, and since frequency will be the same everywhere, the wavelength $\lambda = \frac{v}{f}$ will be shorter in the liquid. This means that the number of wavelengths along the two paths will differ, and the difference will depend on how much liquid is in the tube. When we have a difference of a whole $\frac{1}{2}$ of wavelengths, there will be constructive interference, while if the difference is an integer $+ \frac{1}{2}$ wavelength, we'll have destructive interference. As liquid is added, we go between these possibilities, so the image gets brighter or darker.

b) Suppose the depth of liquid is D . The wavelength in the liquid is

$$\lambda = \frac{v}{c} = \frac{c}{n f} = \frac{\lambda}{n}$$

$$\lambda = \frac{v}{f} = \frac{c}{nf} = \frac{\gamma}{n}$$



The number of wavelengths in the liquid is then $N_1 = \frac{D}{\lambda/n}$. The number of wavelengths in the same distance on the other path is:

$$N_2 = \frac{D}{\lambda}. \text{ So the difference is:}$$

$$N_1 - N_2 = \frac{D}{\lambda} (n - 1)$$

When this increases by $\frac{1}{2}$, we'll go from constructive to destructive interference.

So the change in liquid height will be:

$$\frac{\delta D}{\lambda} (n - 1) = \frac{1}{2}$$

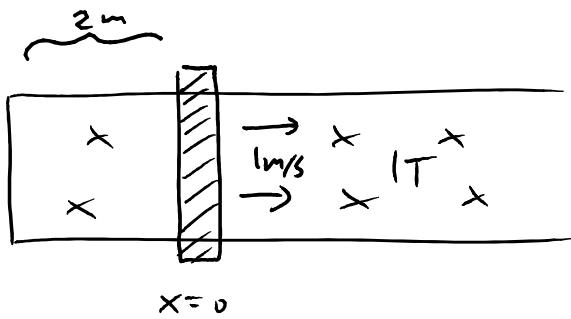
We are given that $\lambda = 500\text{nm}$, and can calculate that $\delta D = \frac{0.5 \times 10^{-9}\text{m}^3}{\pi \cdot (0.01\text{m})^2}$

$$\delta D \leftarrow \delta V = \text{Area} \times \delta D$$

$$\Rightarrow 0.5 \times 10^{-9}\text{m}^3 = \pi \cdot (0.01\text{m})^2 \times \delta D$$

$$\text{So: } n = 1 + \frac{\lambda}{2 \cdot \delta D} = 1.157$$

(5)



The bar comes to rest because the changing flux through the rectangular loop induces a current counterclockwise in the loop by Faraday's Law (so the induced current produces a flux that counters the change). There is then a magnetic force to the left on the upward moving charges in the bar, so this slows the motion.

b) Let $x(t)$ and $v(t)$ be the bar's position & velocity as a function of time. Then:

$$\Phi(t) = L(2m + x) \cdot B \quad L=1\text{m}$$

$$\Rightarrow \frac{d\Phi}{dt} = L \cdot B \cdot v$$

$$\text{The induced current is } I = \frac{\epsilon}{R} = \frac{1}{R} \cdot \frac{d\Phi}{dt} = \frac{LB}{R} \cdot v$$

$$\text{The force on the bar is } F = ILB = -\frac{L^2 B^2}{R} \cdot v$$

Newton's Law gives:

$$F = ma \Rightarrow -\frac{L^2 B^2}{R} \cdot v = m \cdot \frac{dv}{dt}$$

$$\text{Thus: } \frac{dv}{dt} = -\frac{L^2 B^2}{mR} \cdot v$$

Solving, we get

$$V(t) = V_0 e^{-\frac{L^2 B^2}{mR} t} \quad \text{where } V_0 = 1 \text{ n/s}$$

To find $x(t)$ we integrate:

$$\frac{dx}{dt} = V_0 e^{-\frac{L^2 B^2}{mR} t}$$

$$\Rightarrow x(t) = C - \frac{V_0 m R}{L^2 B^2} e^{-\frac{L^2 B^2}{mR} t}$$

$$x(t) = 2m \text{ at } t=0 \quad \text{so: } C = 2m + \frac{V_0 m R}{L^2 B^2}.$$

$$\text{Thus } x(t) = 2m + \frac{V_0 m R}{L^2 B^2} \left(1 - e^{-\frac{L^2 B^2}{mR} t} \right)$$

$$\begin{aligned} \text{At } t = \infty \quad x(t) &= 2m + \frac{V_0 m R}{L^2 B^2} \\ &= 3m \end{aligned}$$

