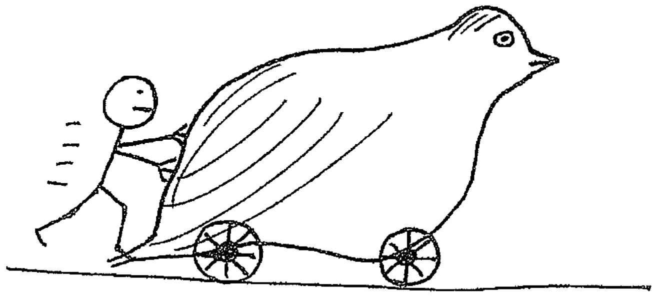
PHYSICS WORKSHEET

Question 1



A stick person pushes a giant wheeled bird. The net force on the bird divided by the bird's mass is a function $f(t)$ (that we know) so:

$$\frac{dv}{dt} = \underset{\text{acceleration}}{f(t)}.$$

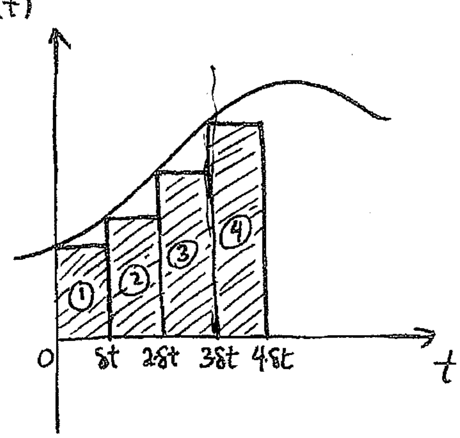
use $v(t+\delta t) \approx v(t) + \delta t\, f(t)$

a) If the bird's velocity is 0 at $t=0$, fill in the table below:

TIME	VELOCITY	ACCELERATION
0	0	$f(0)$
δt	$0 + \delta t\, f(0)$	$f(\delta t)$
$2\,\delta t$	$0 + \delta t\, f(0) + \delta t\, f(\delta t)$	$f(2\delta t)$
$3\,\delta t$	$0 + \delta t\, f(0) + \delta t\, f(\delta t) + \delta t\, f(2\delta t)$	$f(3\delta t)$

you can use the same method that we used for the drag ball!

b)



Find the following areas in the diagram at the left:

i) Area of box ①: $\delta t\, f(0)$

ii) Area of box ① + box ②: $\delta t\, f(0) + \delta t\, f(\delta t)$

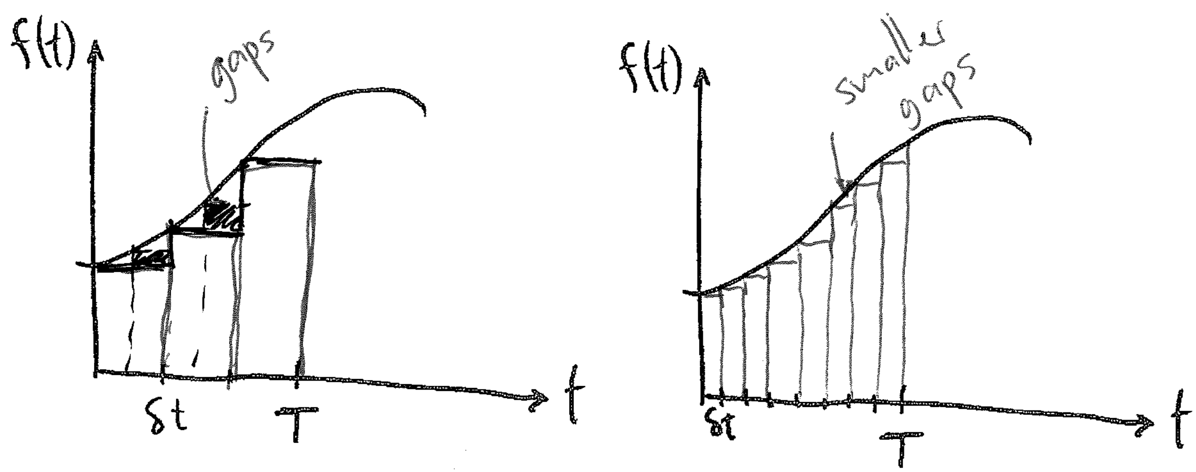
iii) Area of box ① + box ② + box ③: $\delta t\, f(0) + \delta t\, f(\delta t) + \delta t\, f(2\delta t)$

c) Compare your answers for the velocity in a) to your answers for the areas in b). Guess a formula for the change in velocity from time 0 to time t in terms of the area under the graph of f(t).

$V(t) - V(0) \approx$ sum of the areas of rectangles up to time t.

$\approx$ area under the graph of $f(t)$ from 0 to t.

d) To find the velocity at some time T accurately, we want to take δt very small. Explain (using the pictures below) why making δt small also means that the sum of rectangle areas more accurately approximates the area under the curve.



As δt gets smaller, the gaps get smaller, so the area of the rectangles is closer to the area under the curve.

e) How would your answer to c) change if we instead wanted to find the change in velocity from time t_1 to some later time t_2?

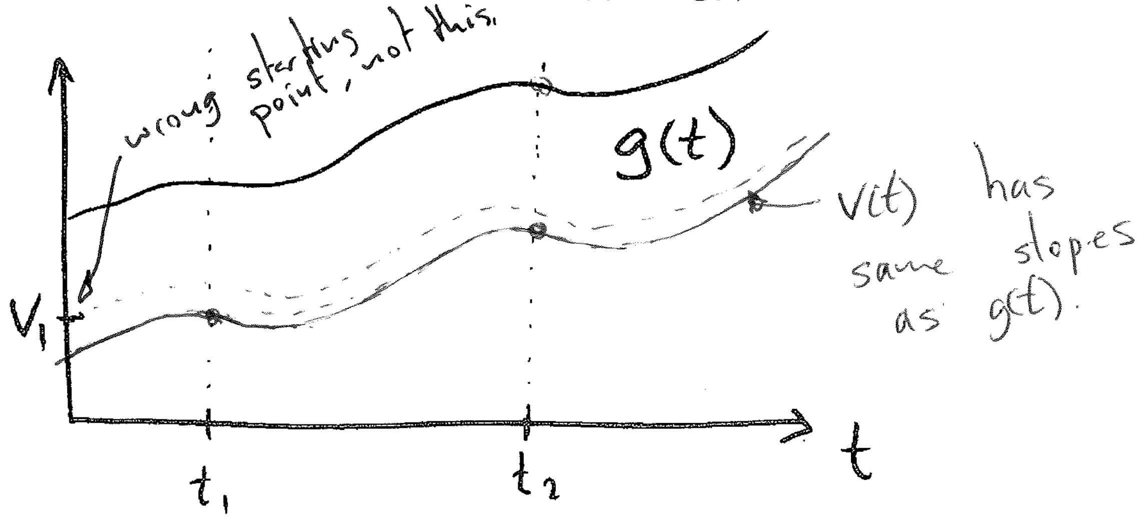
$v(t_2) - v(t_1) =$ area under the graph from t_1 to t_2.

f) If the velocity at time t_1 is v_1, and the and we are given that dv/dt = f(t), what is the velocity v_2 at time t_2 (based on your answer to e)? *(Your answer will include the phrase "the area under the graph of f(t)").*

v_2 = V_1 + {the area under the graph of f(t) from t_1 to t_2.}

Question 2

a) In the last problem, we knew that dv/dt = f(t) for some function f and we wanted to find v. Suppose we are able to find a function g(t) with dg/dt = f(t). What can we say about the two functions v(t) and g(t)?



b) If we are told that the velocity at time t_1 is v_1, sketch the function v(t) on the graph above.

c) Write a formula for v(t) in terms of g(t).

$$v(t) = g(t) + C$$

d) Write a formula for the velocity v_2 at time t_2 in terms of v_1 and the function g(t):

$$\left.\begin{aligned} V_1 &= v(t_1) = g(t_1) + c \\ V_2 &= v(t_2) = g(t_2) + c \end{aligned}\right\} \quad V_2 = V_1 + g(t_2) - g(t_1)$$

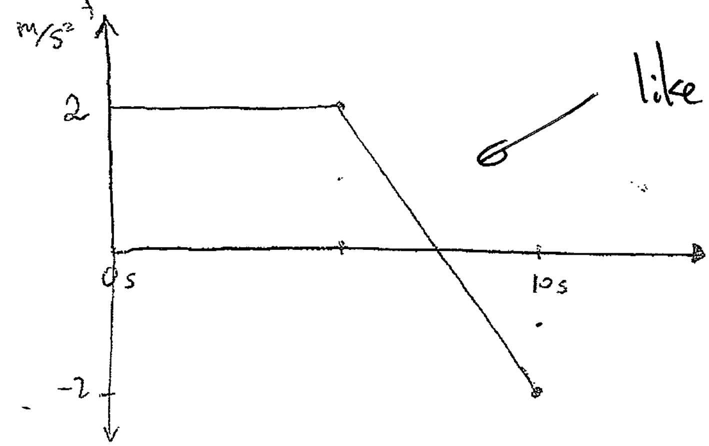
e) Your answers to 1f and 2d should be two different expressions for the velocity v_2. Since they must be equal, what can you say about the relation between the function $g(t)$ whose derivative is f and the area under the graph of f from t_1 to t_2?

$g(t_2) - g(t_1)$ = area under graph of f from t_1 to t_2

$$g(t_2) - g(t_1) = \int_{t_1}^{t_2} f(t)\, dt$$

$\boxed{\text{QUESTION 3}}$ In the following questions, an object's motion obeys $\frac{dv}{dt} = f(t)$ for the given $f(t)$ and $v(t=0) = 0$. Find $v(t=10s)$.

a) $f(t)$ is the function shown:



like clicker question

b) $f(t) = 3 m/s^3 \cdot t$

(try to use the method in question 2 for this one).

See notes.