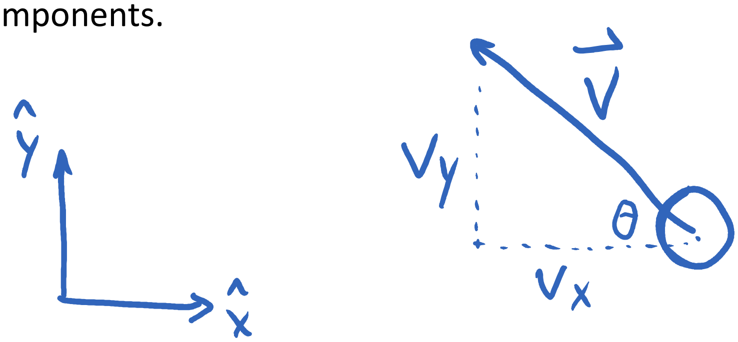
Problem 2: A projectile with air drag:

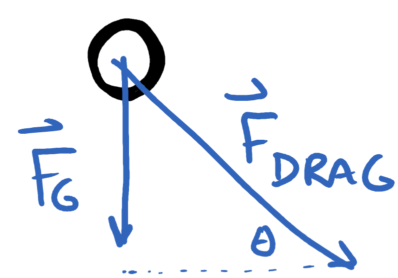
A person throws a ball with initial velocity (v_x,v_y). We would like to predict the trajectory of the ball. To do this, we need to know that the drag force acts in the direction opposite to the ball's velocity, and has a magnitude that is some function F(v), where v is the ball's speed. For this question, we will assume that $F(v) = D\,m\,v^2$, where m is the mass and D is a number.

Step 1: Draw a picture of the ball, showing coordinate axes, the velocity vector and its components.



Step 2: Identify the forces on the ball. Draw another diagram of the ball showing these forces.

Gravity: $\vec{F}_G = -mg\,\hat{y}$ Drag: $\vec{F}_D = -mD\cdot(v_x^2+v_y^2)\,\hat{v}$



this is $\cos\theta$ (we used the 1st picture)

For the next part, it will be useful to break the forces into components:

$$F_x = \left|\vec{F}_{DRAG}\right|\cos\theta = mD(v_x^2+v_y^2)\cdot\frac{v_x}{\sqrt{v_x^2+v_y^2}}(-1)$$

$$= -mDv_x\sqrt{v_x^2+v_y^2}$$

$$F_y = -mg - mDv_y\sqrt{v_x^2+v_y^2}$$

this is $|\vec{F}_D|\sin\theta$

check your answer with an instructor!

Step 3: Write down the components of the equation $\frac{d\vec{v}}{dt} = \frac{1}{m}\vec{F}_{NET}$:

$$\frac{dv_x}{dt} = \frac{1}{m}F_x = -Dv_x\sqrt{v_x^2+v_y^2}$$

$$\frac{dv_y}{dt} = \frac{1}{m}F_y = -g - Dv_y \cdot \sqrt{v_x^2+v_y^2}$$

Also, fill in the right hand sides here:

$$\frac{dx}{dt} = v_x \qquad \frac{dy}{dt} = v_y$$

Step 4: Use these equations to make our predictions.

Fill in the numerical values for the positions and velocities at t=0 and the drag coefficient D, as measured in class:

$t=0$:

$x = 3.5\,m$

$y = 2.1\,m$

$v_x = -6.6\,m/s$

$v_y = 0.7\,m/s$

Drag coefficient $D = 0.21\,m^{-1}$

Using this data, predict x ,y, v_x, and v_y at t = 0.01s and (if you have time) t=0.02s:

$t = 0.01s$:

$x = 3.434m$

$y = 2.107m$

$v_x = -6.6m$

$v_y = 0.7m$

Use

$x(0.01s) \approx x(0s) + 0.01s \cdot v_x(0s)$

$v_x(0.01s) \approx v_x(0s) + 0.01s \cdot a_x(0s)$

$\frac{F_x}{m}$

etc...

$t = 0.02s$:

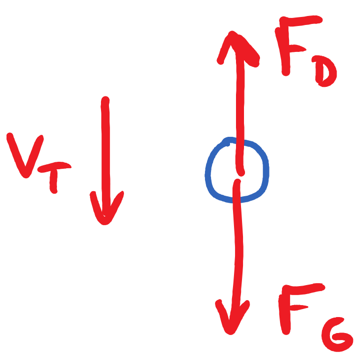
$x = 3.369m$

$y = 2.113m$

$v_x = -6.419 m/s$

$v_y = 0.483 m/s$

Extra: how could we measure the drag coefficient D? HINT: what happens after an object has been falling for a time?



At terminal velocity; acceleration is zero, so

$$|F_D| = |F_G|$$

$$\Rightarrow m D v_T^2 = mg$$

$$\Rightarrow D = \frac{g}{v_T^2}$$

measure teminal velocity.