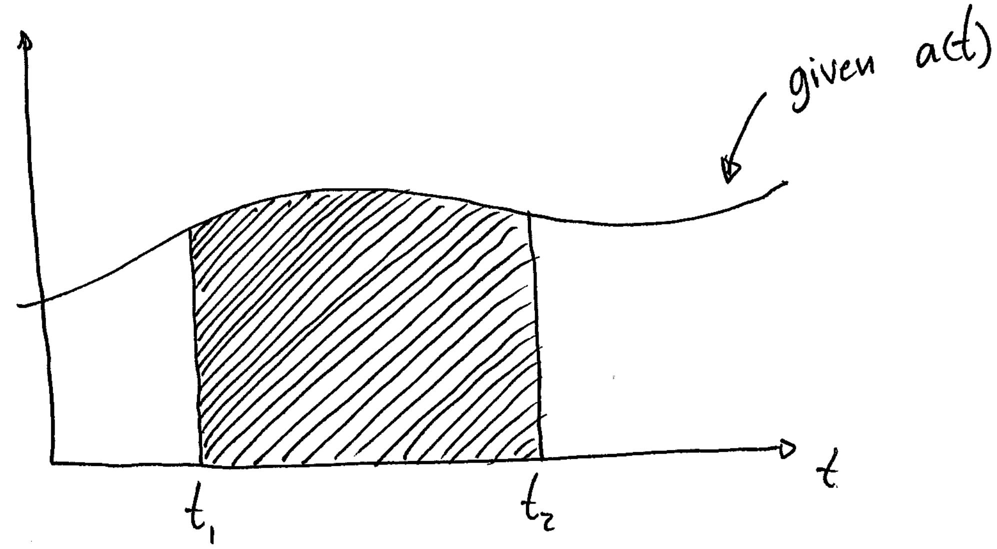
LAST TIME:



$$v(t_2) = v(t_1) + \text{area under } a(t) \text{ from } t_1 \text{ to } t_2.$$
$$= v(t_1) + \int_{t_1}^{t_2} a(t)\,dt$$

ALSO:

We know that $\frac{d\,v(t)}{dt} = a(t)$

We're told that $\frac{d\,g(t)}{dt} = a(t)$ and $v(t_1) = V_1$.

We guess that $v(t) = g(t) + C$

$$v(t_1) = g(t_1) + C = V_1$$

from initial conditions

$$\Rightarrow C = V_1 - g(t_1)$$

Then

$$v(t) = g(t) + [V_1 - g(t_1)]$$

EXAMPLE:

$$\frac{dv}{dt} = 2\,\mathrm{m/s^3} \cdot t \qquad v(0) = 10\,\mathrm{m/s}$$

A function whose derivative is $2\,\mathrm{m/s^3} \cdot t$ is $1\,\mathrm{m/s^3} \cdot t^2$.

this is g(t)

We can check: $\frac{d}{dt}\left(1\,\mathrm{m/s^3} \cdot t^2\right) = 2\,\mathrm{m/s^2} \cdot t$

So we get:

$$v(t) = 1\,\mathrm{m/s^3} \cdot t + C$$

Plug in $t = 0$:

$$v(t=0) = C = 10\,\mathrm{m/s}$$

Finally:

$$\boxed{v(t) = 10\,\mathrm{m/s} + 1\,\mathrm{m/s^3} \cdot t^2}$$

We can do the same thing to get $x(t)$, since we now know $\frac{dx}{dt} = 10\,\mathrm{m/s} + 1\,\mathrm{m/s^2} \cdot t$.

We need to know a $x(t)$ for some time though.

KINEMATICS EQUATIONS FROM HIGH SCHOOL:

Assume constant acceleration a.

$$\frac{dv}{dt} = a.$$

Suppose we know x_0 and v_0.

The velocity is

$$\left.\begin{aligned} v(t) &= at + c \\ v(t=0) &= v_0 = c \end{aligned}\right\} \Rightarrow v(t) = at + v_0$$

The velocity can be written as

$$\frac{d\,x(t)}{dt} = at + v_0$$

The position is

$$\left.\begin{aligned} x(t) &= v_0 t + \frac{1}{2}at^2 + c \\ x(t=0) &= x_0 = c \end{aligned}\right\} x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

The high school equations are derived assuming that acceleration is constant. Those equations are not general.