SO FAR: Understand motion/rotations of 1 object.


isolated: Energy, momentum, angular momentum CONSERVED

generally: $\frac{d\vec{p}}{dt} = \vec{F}$

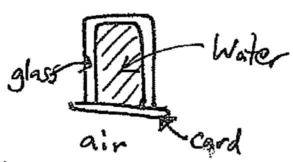
$\frac{d\vec{L}}{dt} = \vec{\tau}$

these rules apply to the bits that matter is made of.

TODAY: $10^{23}$ objects

- use what we know to understand complicated systems (solids/liquids/gases)

e.g. force from a gas



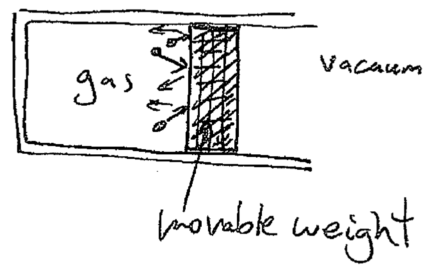

air can hold up the card, even with water pushing down.

(Q: what forces are on the card?)

- Where does this force come from?
- What microscopic properties (variables) of the air does it depend on?
- Can we estimate the force in terms of the microscopic properties?

* Force caused by collisions transferring momentum from gas molecules to object.

Magnitude of this force: $\frac{dp}{dt}$ for some test object acted on by only this force:




$$|\text{Force}| = \left|\frac{dp_{weight}}{dt}\right| \underset{\text{momentum conservation}}{=} \left|\frac{dp_{gas}}{dt}\right| = \left(\begin{array}{c}\#\text{ collisions}\\ \text{per}\\ \text{time}\end{array}\right) \times \left(\begin{array}{c}\text{average}\\ \Delta p \text{ per}\\ \text{collision}\end{array}\right)$$

depends on density of gas, (#/volume) area of surface, speed of atoms

depends on avg speed of atoms, mass of atoms.

Double density → double # of collisions/time, same $\Delta p$ per collision → double force

Double avg. speed → double avg $\Delta p$ per collision, double # collisions per time → 4× force

Double mass of atoms. → double avg $\Delta p$, same # of collisions/time → double force.

Double area → same avg $\Delta p$, double # collisions per time. → double force.

Which in words is

$$\text{Force} = (\text{const}) \times \begin{pmatrix}\text{mass of}\\ \text{atoms}\end{pmatrix} \times (\text{density}) \times \begin{pmatrix}\text{avg}\\ \text{speed}\end{pmatrix}^2 \times (\text{area})$$

Define pressure as

$$P \equiv \frac{F}{A} = \overbrace{\frac{1}{3}\,\frac{N}{V}\, m\, v_{avg}^2}^{\text{microscopic}}$$

macroscopic

density of gas

mass of molecules

average speed of molecules

Notice that

$$P = \frac{2}{3}\,\frac{N}{V}\left(\frac{1}{2}\, m\, v_{avg}^2\right)$$

$$\Rightarrow \boxed{PV = \frac{2}{3}\, N\, \epsilon_{avg}}$$

average kinetic energy per atom

This looks like the ideal gas law:

$$PV = nRT \qquad \text{or} \qquad PV = N k_B T$$

$n$: # moles

$R$: $8.31 \ J/\text{mol}\cdot K$

$N$: # atoms

$k_B$: Boltzmann constant $1.38 \times 10^{-23} \ J/K$

and emperical equation valid for many situations. (Low P high T).

Agrees with microscopic picture if

$$\underset{\text{macro}}{T} = \frac{2}{3 k_B} \underset{\text{micro.}}{\epsilon_{avg}}$$

The temperature is proportional to the average translational kinetic energy of molecules/atoms.