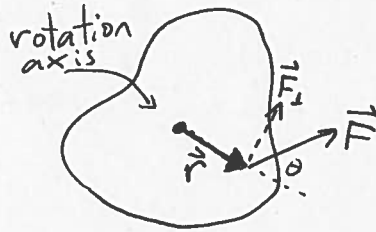


LAST TIME:

TORQUE

gives rise to

ANGULAR ACCELERATION



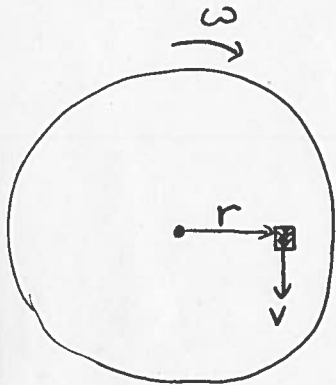
$$\tau = |\vec{r}| |\vec{F}_\perp|$$

$$= |\vec{r}| |\vec{F}| \sin\theta$$

$\tau = I\alpha$ (if I constant. Generally $\tau = \frac{dL}{dt}$ where $L =$

WORKSHEET TP DEMO.

CLICKER 2
CLICKER 3



For rotating body, regular speed of some part related to angular speed by

$$v = \omega \cdot r$$

Why: for 2π rotation, this part travel distance $2\pi r$

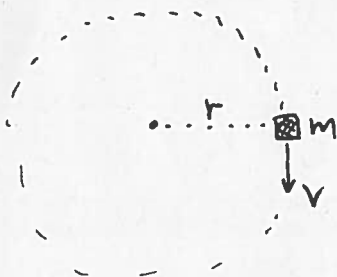
$$\therefore \text{distance traveled} = r \cdot \Delta\theta$$

$$\therefore \frac{\text{distance traveled}}{\Delta t} = r \frac{\Delta\theta}{\Delta t}$$

$$\Rightarrow v = \omega \cdot r$$

CLICKER 4

How much angular momentum does this part carry?

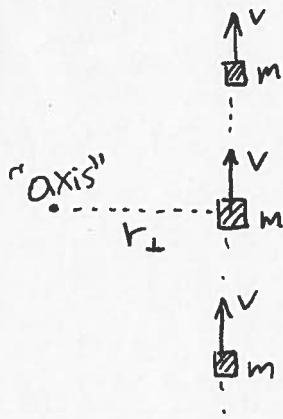


$$I = mr^2$$

$$\text{so } L = (mr^2)\omega$$

$$= mr^2 \cdot \frac{v}{r}$$

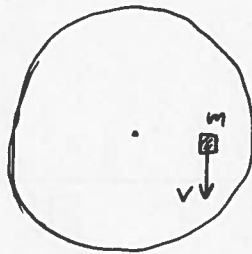
$$\Rightarrow \boxed{L = mvr} \leftarrow \text{Mark's initials!!}$$



If object not constrained to move around rotation axis, its angular momentum must still be conserved,

so
$$L = m v r_{\perp}$$

ROTATIONAL ENERGY: rotating object has kinetic energy even if its center of mass is fixed.



Add up K.E. for all the parts

$$\begin{aligned}
 \text{K.E.} &= \sum_{\text{bits}} \frac{1}{2} m v^2 \\
 &= \sum_{\text{bits}} \frac{1}{2} m r^2 \cdot \omega^2 \\
 &= \frac{1}{2} M \left[\sum_{\text{bits}} \frac{m}{M} r^2 \right] \omega^2 \\
 &= \frac{1}{2} I \omega^2
 \end{aligned}$$

Annotations: "mass of bit" points to m in the first term; "total mass" points to M in the third term; "this is the average r^2 " points to the bracketed sum in the third term.

Total kinetic energy of a moving rotating object:

$$E_{\text{kin}} = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2$$

