LAST TIME:

rotational motion +

CONSERVATION OF ANGULAR MOMENTUM

for a rigid body rotating about axis of symmetry

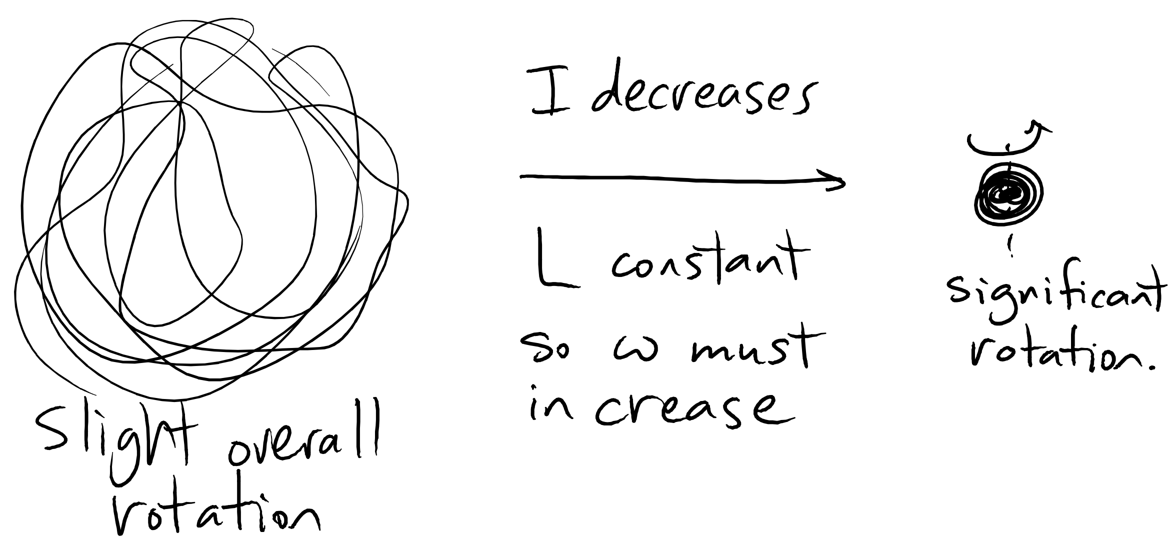
$$L = I\omega$$

- L ← Angular momentum
- I ← moment of inertia: $M \times (\text{average } R^2)$, R = distance to axis
- ω ← angular velocity $\frac{d\theta}{dt}$

For a system with rotational symmetry about an axis, L is conserved.

For non-rigid object, I can change

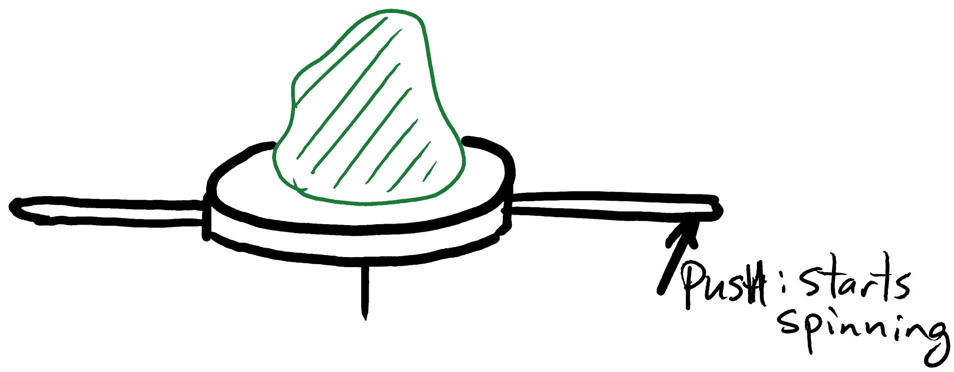
e.g. gravitational collapse



Rotational Newton's Laws

① L doesn't change for isolated object

② External influences can change angular momentum



can define TORQUE τ by rate of change of angular momentum for some standard object.

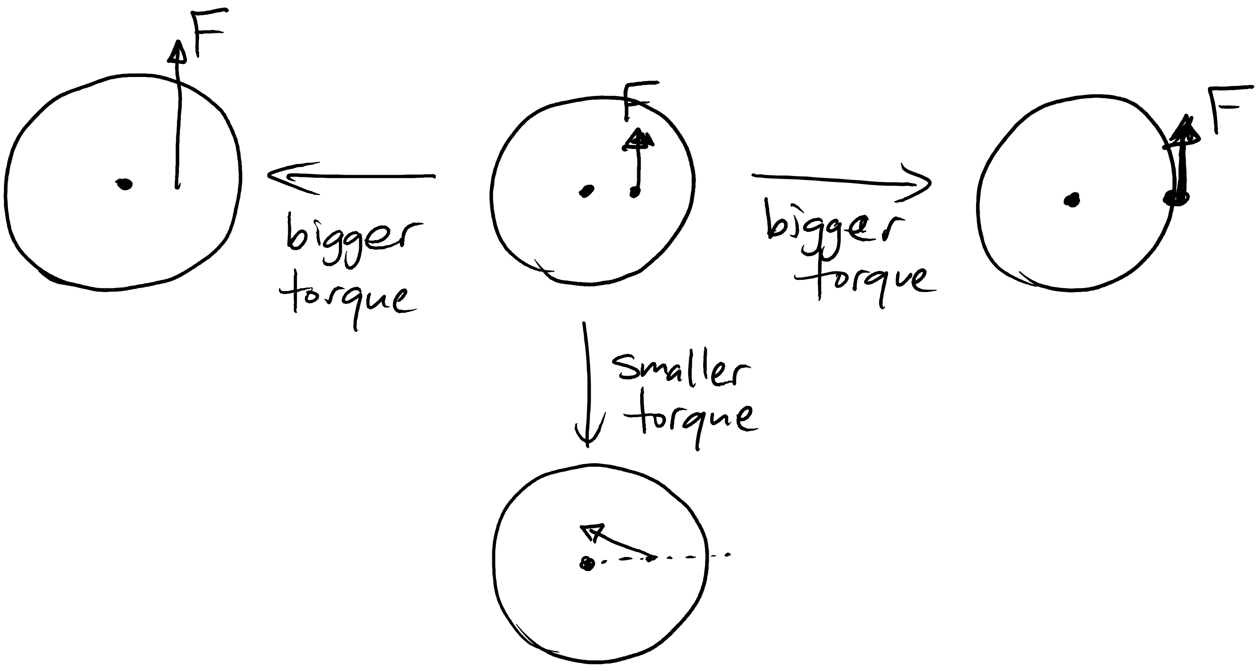
* Same external influence will produce same change in angular momentum for any body *

$$\tau = \frac{dL}{dt}$$ (like $F = \frac{dp}{dt}$)

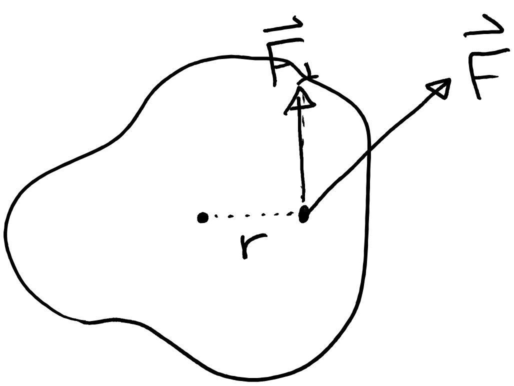
OR $$\tau = I\alpha$$ (like $F = ma$)

for fixed I

τ is determined by forces acting on an object + ★where they act★



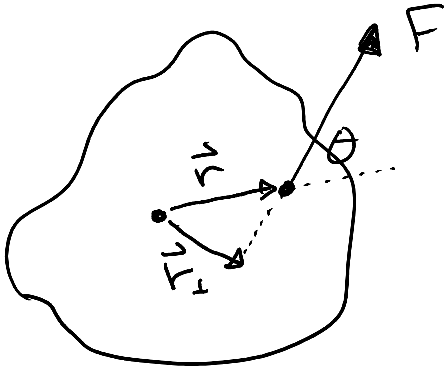
Generally: Torque produced by force perpendicular to the direction from the rotation axis



$$\tau = |F_\perp| \cdot r$$

OR

$$\tau = |\vec{F}| \cdot r_\perp$$



OR

$$\tau = |\vec{F}|\,|\vec{r}|\,\sin\theta$$

(θ: angle between $\vec{F}$ and $\vec{r}$)

Net torque: calculate τ from each force

- all torques that want to make object spin in same direction have same sign.

Solving rotational dynamics problems just like regular dynamics:

- Draw diagram w. forces
- Find net torque
- Find $\alpha = \frac{\tau}{I}$ (if I constant)
- This gives $\frac{d\omega}{dt}$

  use
  - Euler method
  - guess & check
  - antidifferentiation
  - integration

- to solve for $\omega(t)$, $\theta(t)$