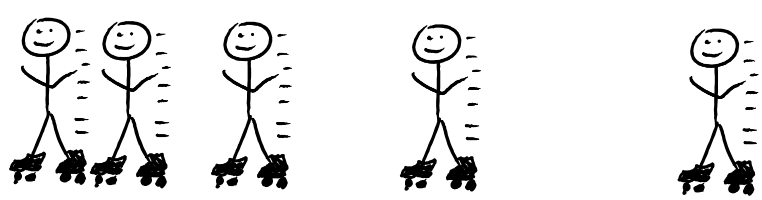
PHYSICS:

Our goal:
- observe the universe
- figure out the rules (represent by mathematical model)

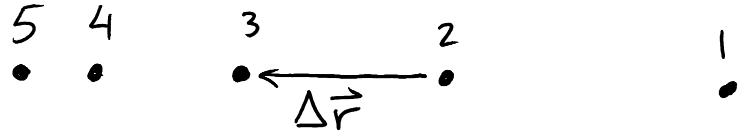
Start simple: rules for motion + interaction of solid objects → Need to represent motion mathematically

Exercise: make a motion diagram

e.g. object moving across surface w. friction



Particle model



(average) velocity between 2 and 3:

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$

$\Delta\vec{r}$ → vector from initial point to final point

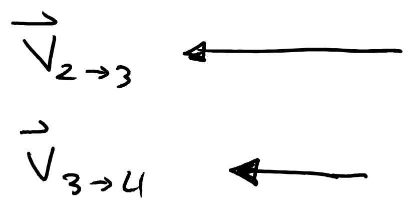
Δt → time elapsed

Acceleration: $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$

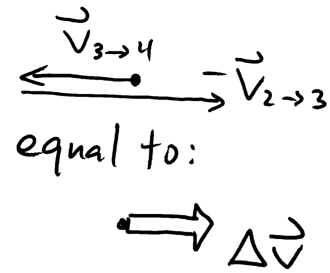
← $\Delta \vec{v}$: difference between velocity vectors (later minus earlier)

← Δt: time step

e.g. acceleration at point 3:



$\Delta \vec{v} \approx \vec{v}_{3\to 4} - \vec{v}_{2\to 3}$



Acceleration is to the right.

Q: What was the POSITION of the object at the chosen time?

Need to introduce coordinates to describe locations of objects.

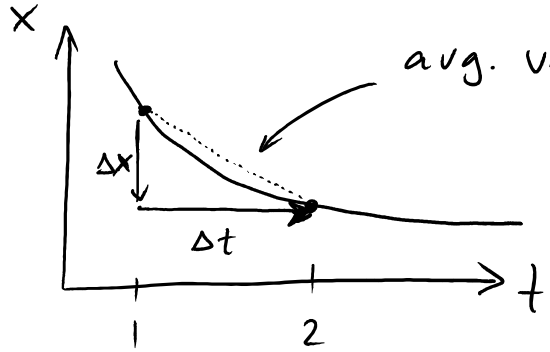
CHOOSE $x=y=z=0$ point

CHOOSE $+x$ direction, $+y$ direction

→ velocities, lengths, accelerations don't depend on our choices

Represent motion of objects by $(x(t), y(t), z(t))$

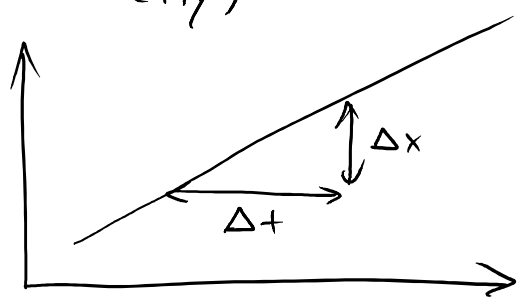
Represent visually via graph:



avg. velocity from 1 - 2

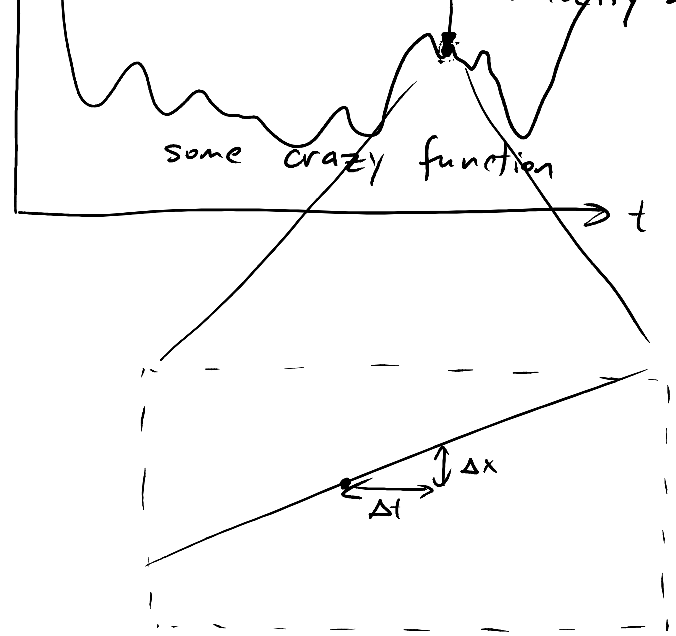
$$\frac{\Delta x}{\Delta t} = \text{slope of dotted segment}$$

Uniform motion:
(constant velocity)



$\frac{\Delta x}{\Delta t}$ same for any Δt

what is the instantaneous velocity at this point



any <u>smooth</u> function looks like a straight line if we zoom in enough

∴ define v as the slope of this line

$$V_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \equiv \frac{dx}{dt}$$

↑ defined as

DERIVATIVE OF $x(t)$