## Physics 157 oscillations worksheet

A cylinder with gas at constant temperature $T$ has a piston of mass M which can move freely up and down. We would like to calculate the oscillation frequency of the piston if it is displaced up or down.
use: $\mathrm{M}=200 \mathrm{~kg}, \mathrm{~g}=10 \mathrm{~m} / \mathrm{s}^{2}, \mathrm{P}_{0}=100 \mathrm{kPa}, \mathrm{A}=0.03 \mathrm{~m}^{2}$,
 $\mathrm{T}=300 \mathrm{~K}, \mathrm{nR}=5 \mathrm{~J} / \mathrm{K}$
a) Draw a free body diagram for the piston showing the vertical forces. Calculate the magnitude of the net upwards force on the object as a function of the height $h$ of the piston.

Your answer should be a function of $h$.
c) Sketch a graph of the net upward force on the piston vs the height of the piston
d) What is the equilibrium height of the piston?
e) What is the oscillation frequency $\omega$ ?
f) Extra: if we have larger amplitude oscillations, how do you think the displacement as a function of time would differ from a pure sinusoidal function?

Hint: if the restoring force as a function of position is $F(x)$, then the equilibrium position can be found using $F\left(x_{\text {eq }}\right)=0$. The effective $k$ for the system is then the magnitude of the slope of the graph of $F(x)$ at this equilibrium position, that is

$$
k=\left|F^{\prime}\left(x_{\text {eq }}\right)\right|
$$

The angular frequency is related to this by $\omega=(\mathrm{k} / \mathrm{m})^{1 / 2}$

