Name:
Student number:

Physics 157 Tutorial – week of November 5th

In this tutorial, you'll get some practice working with the mathematics behind oscillations. For an object undergoing simple harmonic motion, the displacement (i.e. position relative to the equilibrium position) is given by:

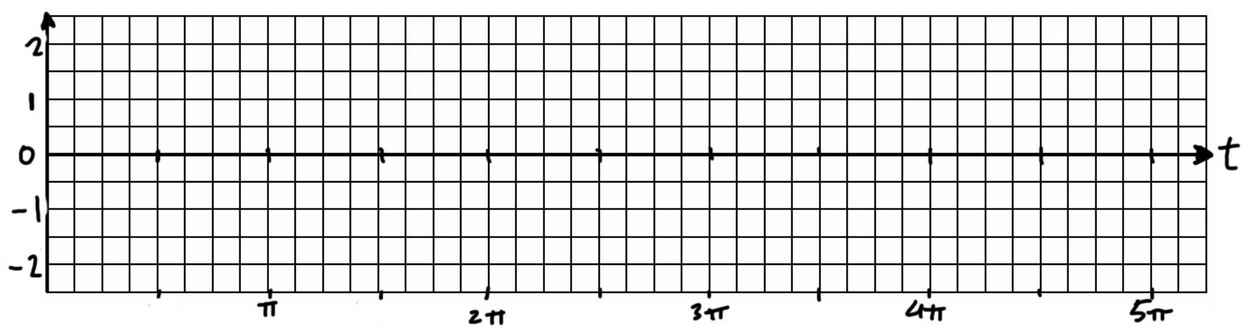
$$x(t) = A \cos(\omega t + \phi)$$

where the constant A is the amplitude, ω is the angular frequency (equal to 2π times the usual frequency, or 2π divided by the period, and ϕ is the "phase." For this worksheet, we'll omit units, with the understanding that all displacements are in centimeters, all times are in seconds and all frequencies are in inverse seconds.

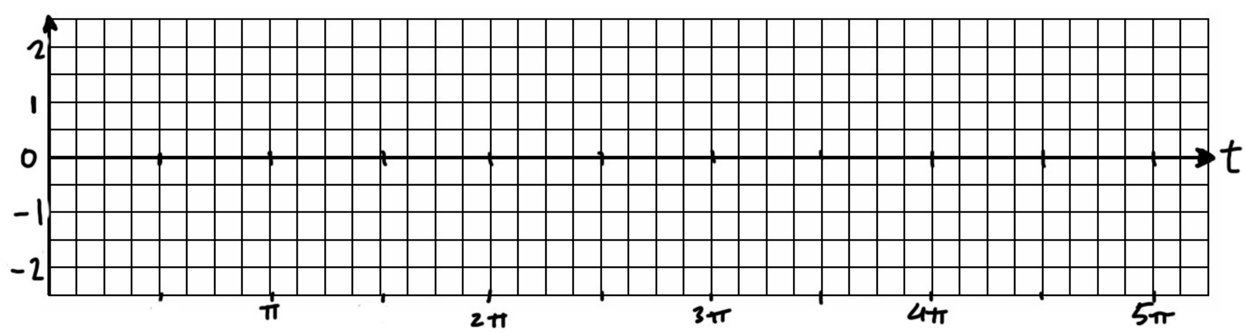
1) First, let's remember what effect changing the various parameters has. For each function, write the amplitude, angular frequency, period ($T = 2\pi/\omega$) and phase. Sketch the displacement as a function of time for the following functions. **Check your answers on the back page.**

(Hint: remember that cos(t) is zero at ±π/2, ±3π/2, ±5π/2, … and maximum at 0, ±2π, ±4π…)

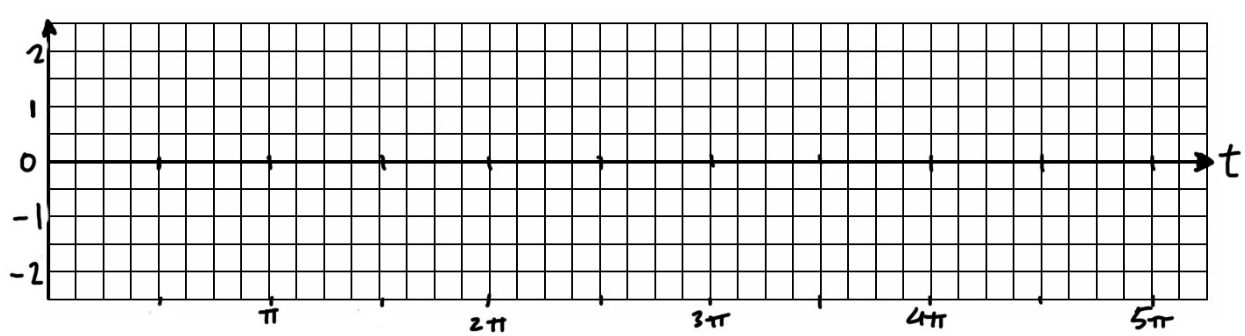
a) $x(t) = \cos(t)$: A = ω = T = ϕ =



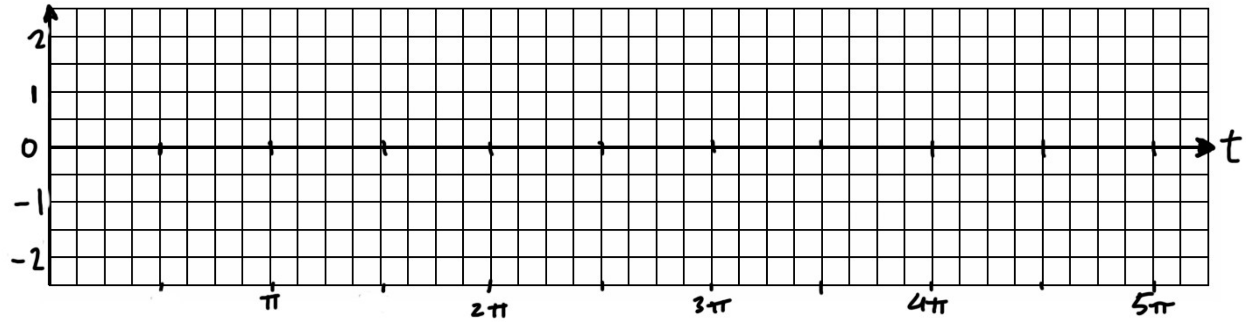
b) $x(t) = 2\cos(t)$: A = ω = T = ϕ =



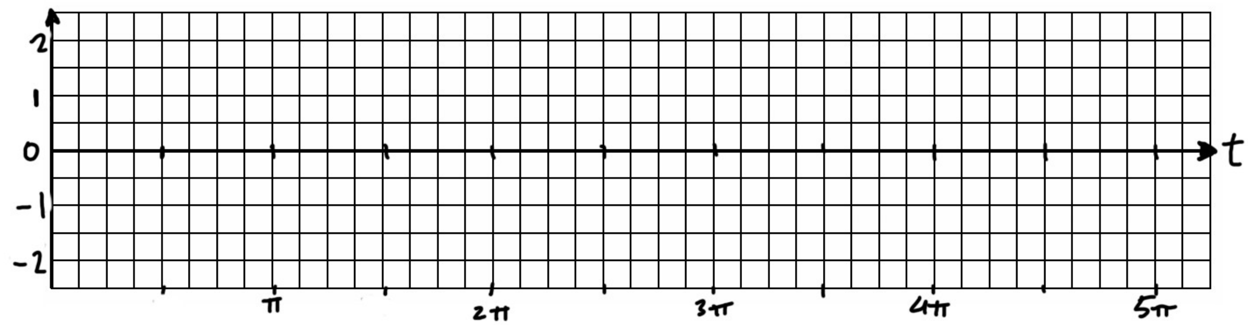
c) $x(t) = \cos(2t)$: A = ω = T = ϕ =



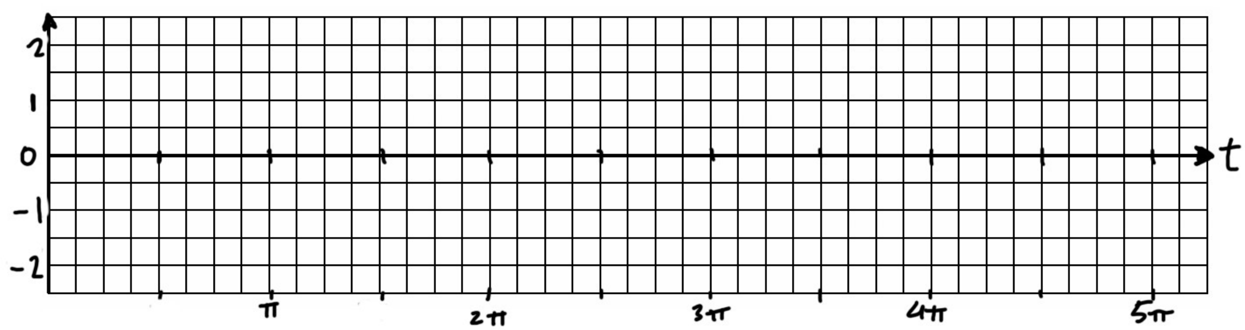
d) x(t) = cos(t + π/2): A = ω = T = ϕ =



e) x(t) = 2 cos(t - π/2) : A = ω = T = ϕ =

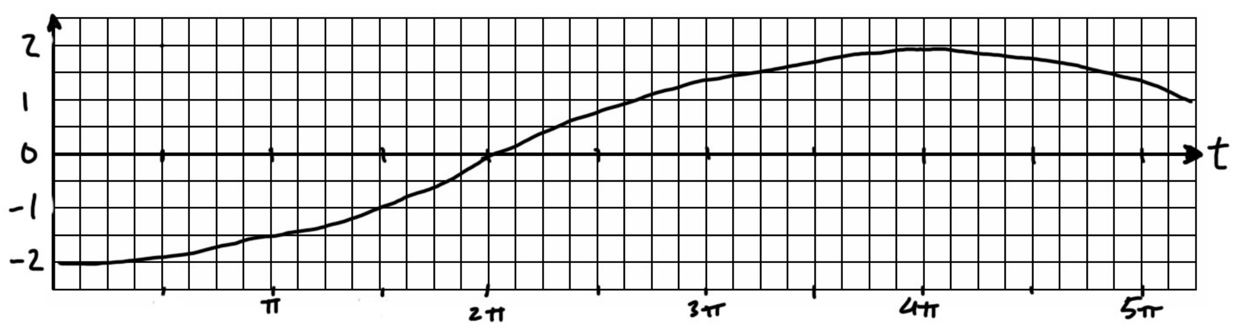


f) x(t) = 2 cos(t/2 - π) : A = ω = T = ϕ =

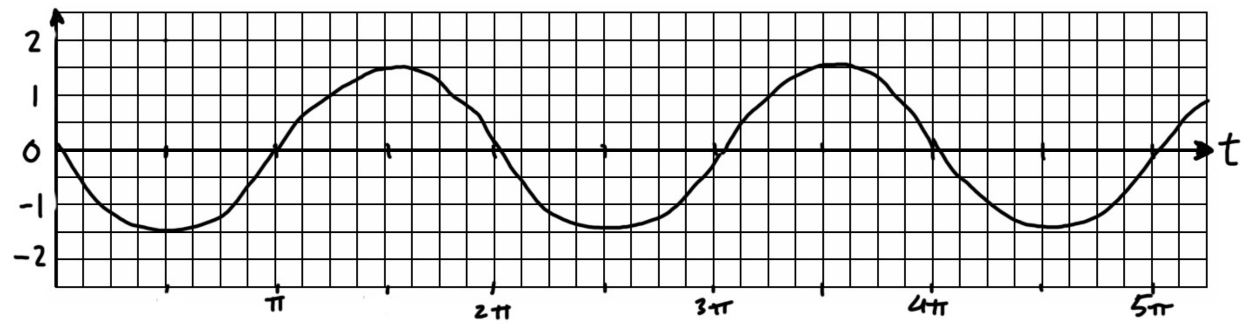


2) Now, let's go the other way. Write down the sinusoidal function corresponding to each of the following graphs. (*Hint: first figure out the amplitude, then the period (time from maximum to the next maximum), then the angular frequency (equal to* 2 π / T), then the phase ϕ (choose this so the cosine gives the right value for t=0). **Check your answers on the back page.**

a) x(t) =

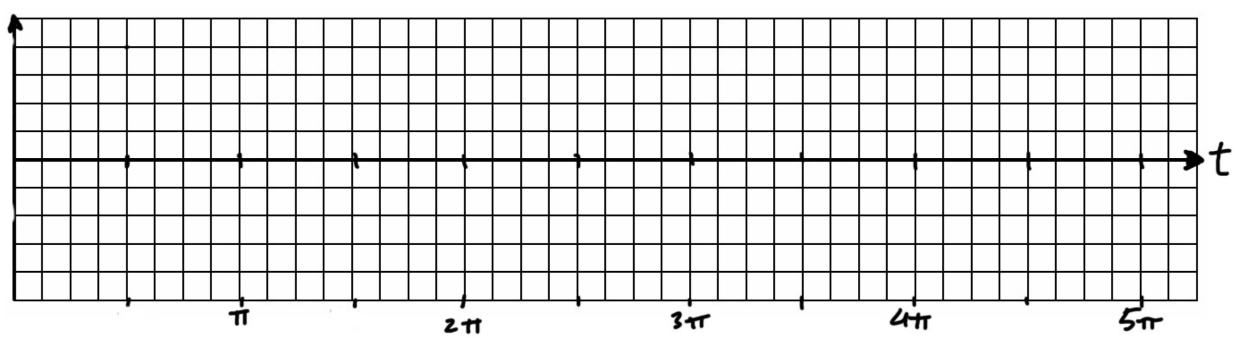


b) x(t) =

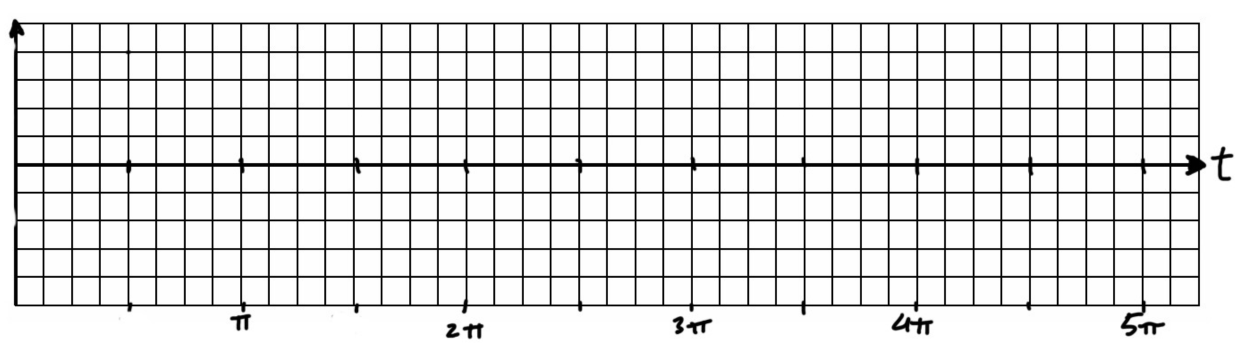


3 a) Now, starting from a graph of displacement vs time, let's understand what the velocity vs time and acceleration vs time graphs look like. Let's take the same x(t) as in 2b). Sketch the velocity as a function of time for this oscillator, recalling that the velocity at any time is equal to the slope of the displacement graph at that time (since v = dx/dt). You don't need to label the vertical axis.

Hint: In the graph above, indicate by +, -, and 0 the places on the t axis where the slope takes its largest positive values, its largest magnitude negative values and zero values. Draw these on the t axis below and use these to help you sketch the velocity.



b) Next, sketch the acceleration vs time graph, recalling that the acceleration at any time is equal to the slope of the velocity graph at that time (since a = dv/dt). You don't need to label the vertical axis.



c) Now, compare your acceleration graph to the original displacement graph. How are these related to each other mathematically?

4) Let's do the same thing with calculus. Hopefully, your answer for 2b) was

$$x(t) = 3/2 \cos(t + \pi/2)$$

a) By taking the derivative of this, what is v(t)?

$v(t) =$

b) By taking the derivative of v(t), what is a(t)?

$a(t) =$

Does the match what you sketched above? Does it explain your observations in 3c)?

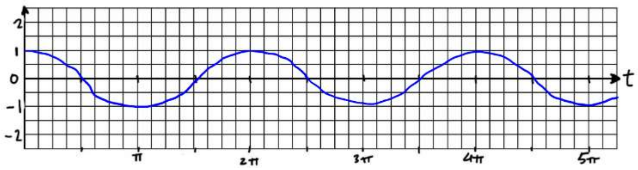
5) Going back to the general oscillating function $x(t) = A \cos(\omega t + \phi)$, what is the velocity v(t) and acceleration $a(t) = d^2x/dt^2$?

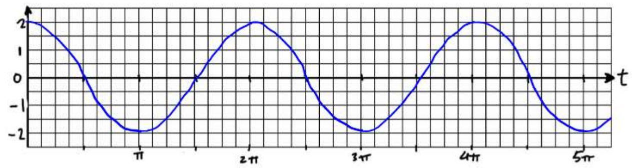
$a(t) =$

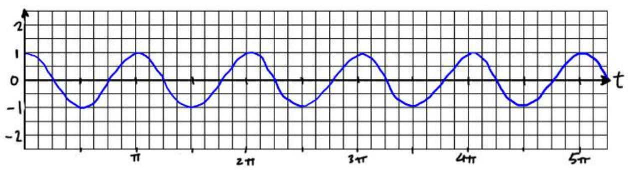
You should find that $d^2x/dt^2 = - C x(t)$ for some constant C. What is C in terms of A, ω, and ϕ?

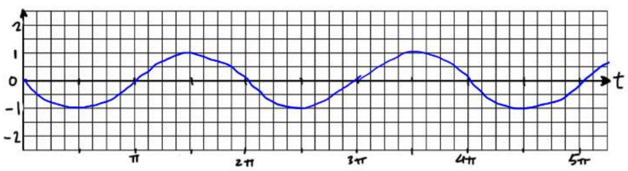
In physics, simple harmonic motion occurs for an object when there is a force in the opposite direction to the displacement which is proportional to the magnitude of the displacement. Can you explain how this leads to an equation like $d^2x/dt^2 = - C x(t)$?

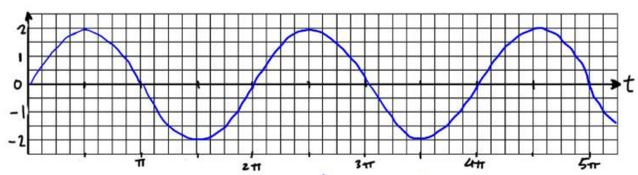
Q1 answers:

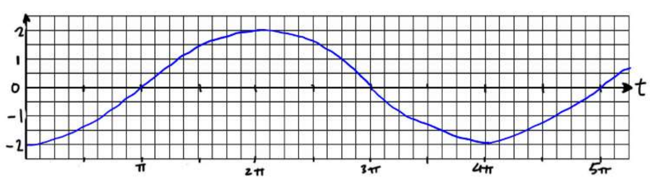












Q2 answers:

a) 2 cos(t/4 + π) OR 2 cos(t/4 - π)   (these are the same)

b) 3/2 cos(t + π/2)