

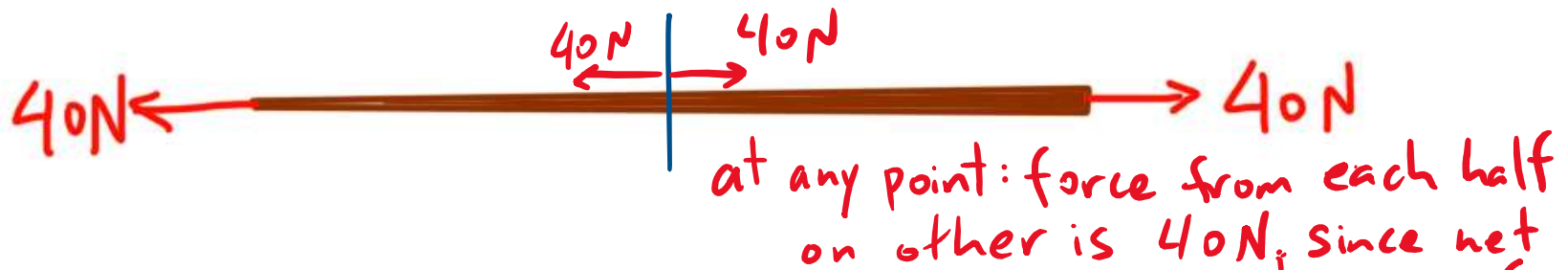
## Learning Goals:

- For static systems with multiple parts, to use Newton's second and third laws to related the various internal forces
- For a multipart system subject to temperature changes, to write an expression describing the change in length of each part due to the temperature change and the changing forces from adjacent parts
- For a multipart system subject to temperature changes to calculated the final lengths and forces for each part of the system



A copper wire of varying thickness is pulled at each end with a force of 40N. Which of the following statements is true?

- A) The tension force is a constant 80N all along the wire, and the stress is also constant along the wire.
- B) The tension is a constant 40N all along the wire, and the stress is also constant along the wire.
- C) The stress is constant throughout the wire but the tension varies.
- D) The tension is a constant 80N all along the wire, but the stress varies along the wire.
- E) The tension is a constant 40N all along the wire, but the stress varies along the wire.



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- E) The tension is a constant 40N all along the wire, but the stress varies along the wire.

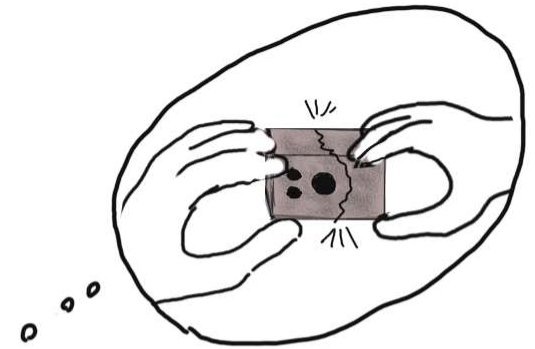
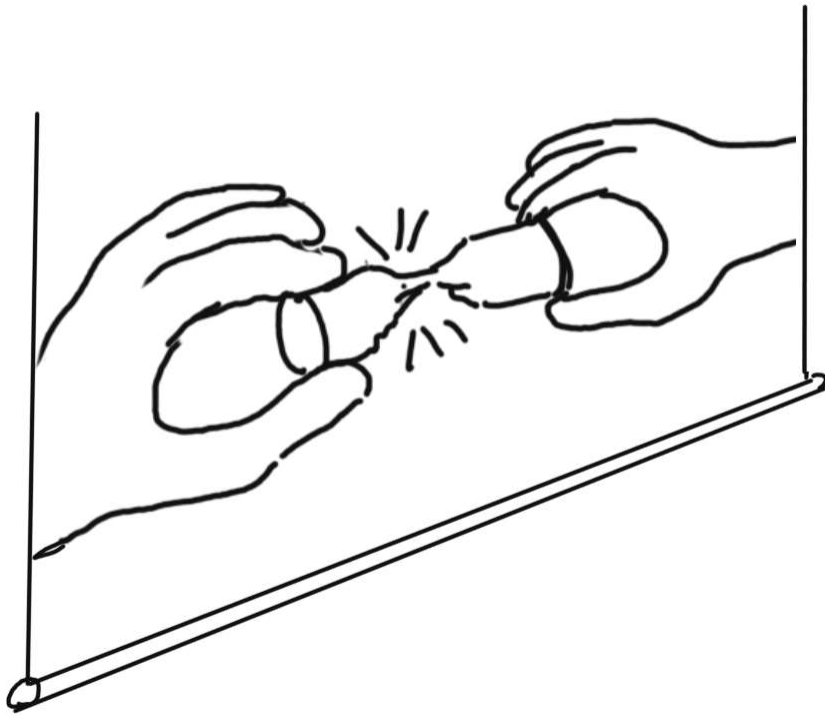
stress

$$= \frac{F}{A}$$

varies since

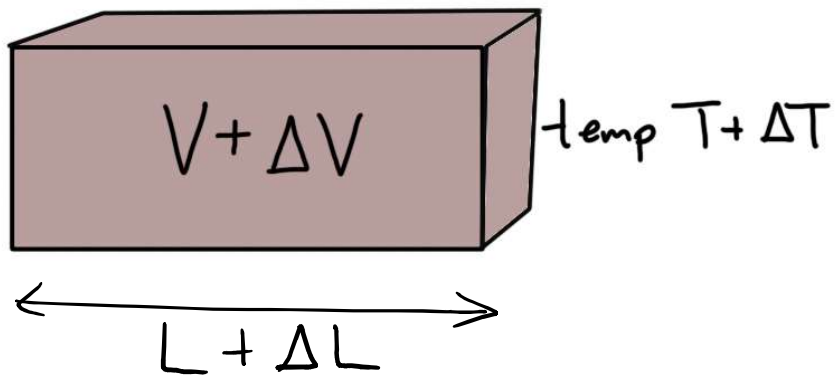
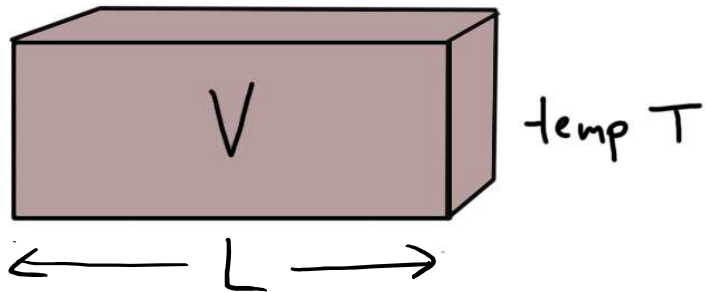
A

varies



Last time in  
physics 157...

Thermal expansion:

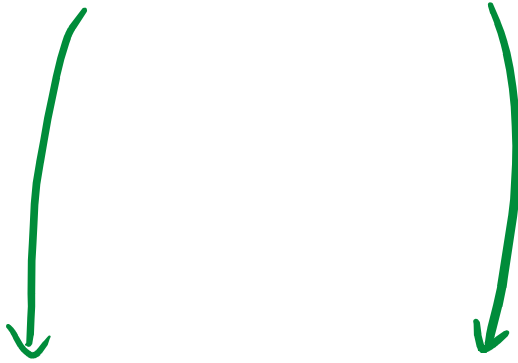
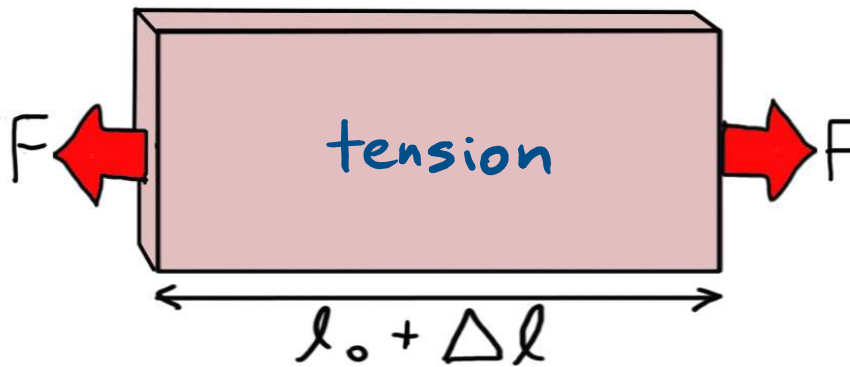
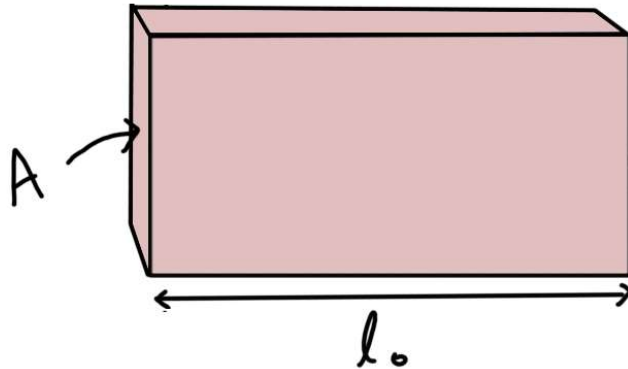
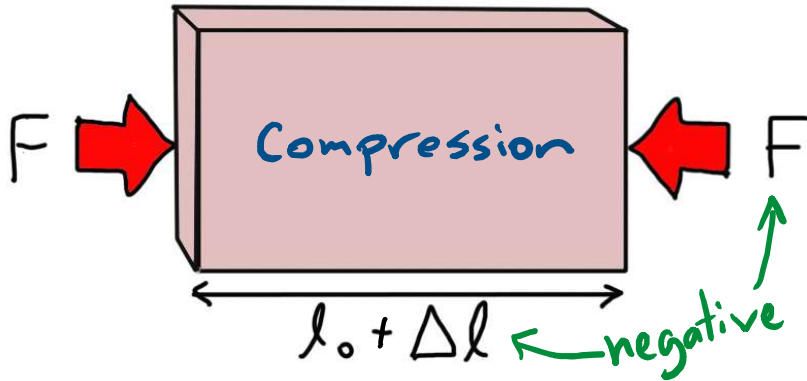


$$\Delta L = \alpha L_0 \Delta T$$

$$\Delta V = \beta V_0 \Delta T$$

$$\beta = 3\alpha \text{ for solids}$$

# STRESS & STRAIN

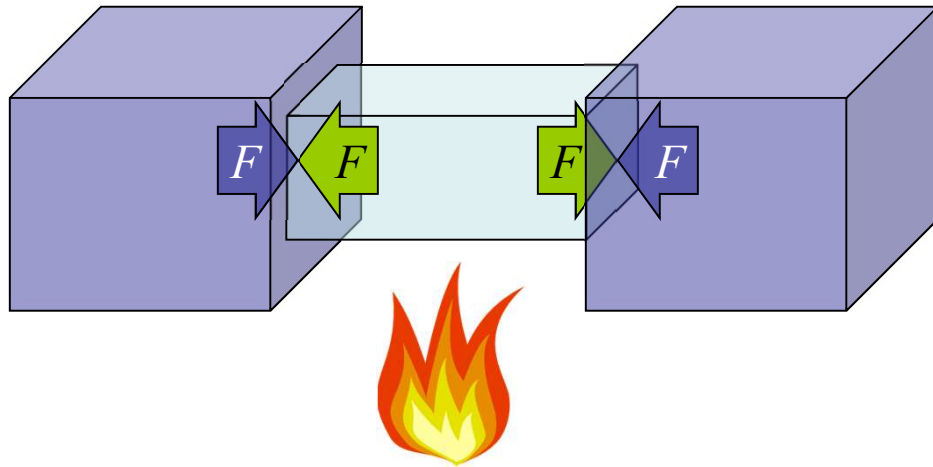


$$\frac{F}{A} = Y \frac{\Delta l}{l_0}$$

sign of  $F$ :

- positive for tension  
pulling
- negative for compression  
pushing

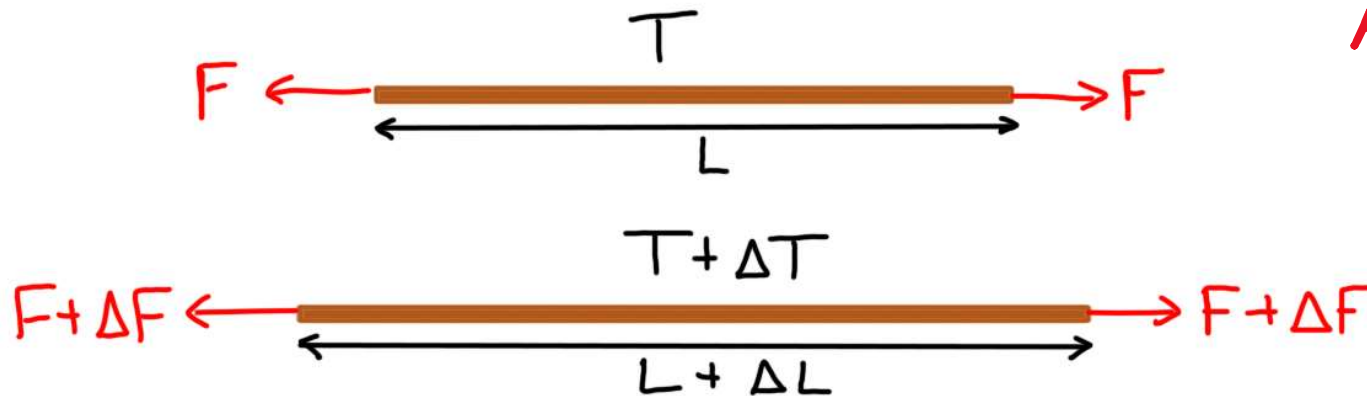
THERMAL STRESS : forces on a material due to surrounding materials preventing thermal expansion/contraction



\* Ignore the "thermal stress" formula from the textbook \*

$$\Delta L = \alpha L \Delta T$$

$$\frac{F}{A} = Y \frac{\Delta L}{L}$$



A copper wire under a tension force of  $F$  and at temperature  $T$  initially has a length  $L$ . If we heat up the wire by  $\Delta T$  and also change the tension force by  $\Delta F$ , by how much does the length of the wire change?

*Hint: treat the change in length from thermal expansion and the change in length from the force increase separately*

**Click A when you have an answer, B if you are stuck**



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Thermal expansion:

$$(\Delta L)_T = \alpha L \Delta T$$

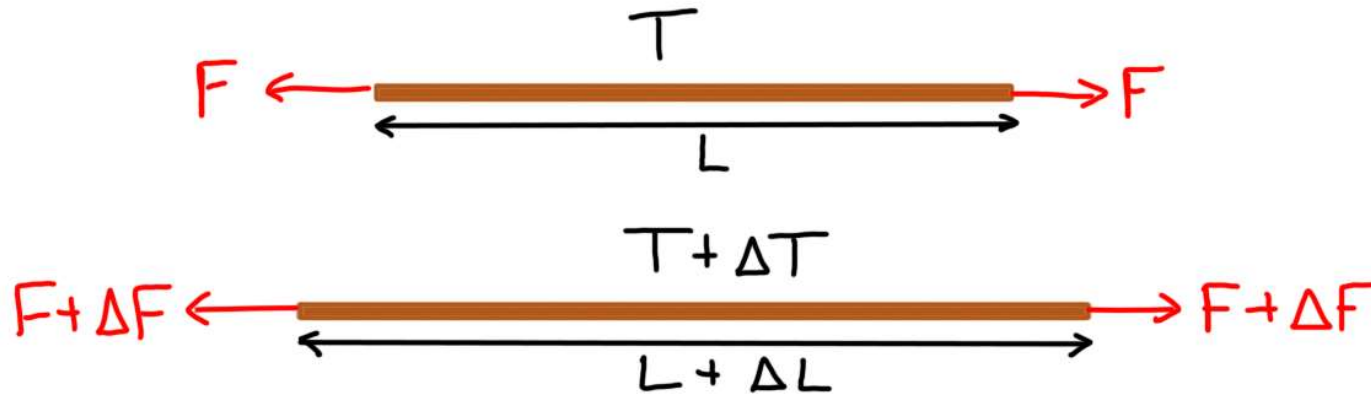
Mechanical stretching:

$$\frac{\Delta F}{A} = Y \frac{(\Delta L)_F}{L} \Rightarrow \Delta L_F = \frac{\Delta F}{A} \cdot \frac{L}{Y}$$

Total expansion:

$$\Delta L = (\Delta L)_T + (\Delta L)_F$$
$$\Rightarrow \Delta L = \alpha L \Delta T + \frac{L}{Y} \cdot \frac{\Delta F}{A}$$

# NET CHANGE IN LENGTH



$$\Delta L = \Delta L_T + \Delta L_F$$

from stress/strain formula

$$\Delta L_T = \alpha L \Delta T$$

$$\Delta L_F = \frac{1}{Y} L \frac{\Delta F}{A}$$

This applies to each part of a system

A harder problem:

## Stressed Rods

A compound bar consisting of a copper rod with a length of 1 m and cross-section area of  $2.00 \text{ cm}^2$  is placed end to end with a steel rod with length 1m and cross-section area  $2.00 \text{ cm}^2$ . The compound rod is placed between two rigid walls. Initially there is no stress in the bars at room temperature  $20^\circ \text{ C}$ .

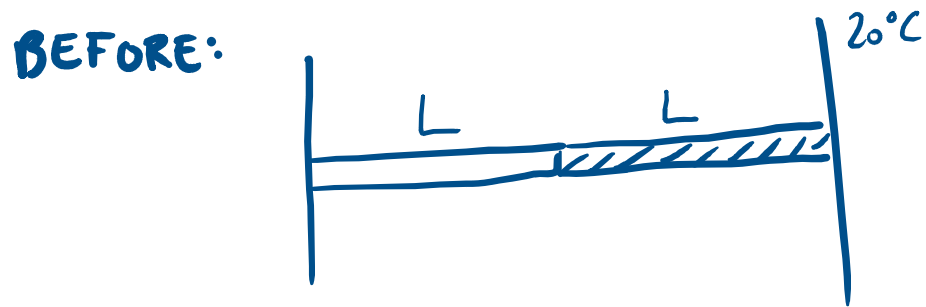
Find the force on each wall at  $40^\circ \text{ C}$ .

$$\alpha_{\text{steel}} = 12 \times 10^{-6} \text{ K}^{-1}, \alpha_{\text{copper}} = 17 \times 10^{-6} \text{ K}^{-1},$$
$$Y_{\text{steel}} = 200 \times 10^9 \text{ N m}^{-2}, Y_{\text{copper}} = 110 \times 10^9 \text{ N m}^{-2}$$

**STEP 1** : visualize what will happen. Draw a before/after picture.  
Give names to known + unknown quantities + label diagram.  
Understand which quantities are changing and which quantities are fixed.

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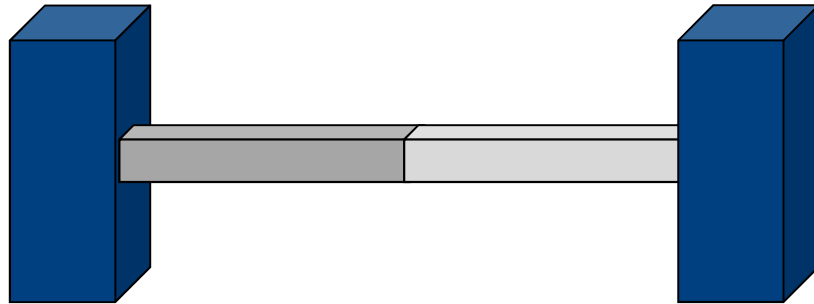


**Clicker:** As the system is heated, we expect that

A) Both rods will increase in length

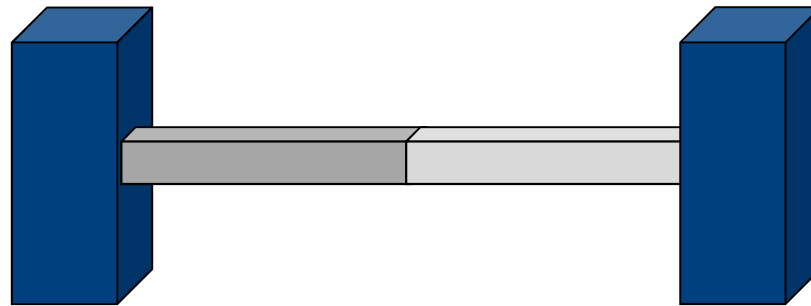
B) Both rods will stay the same length

C) One rod will get longer and the other rod will get shorter



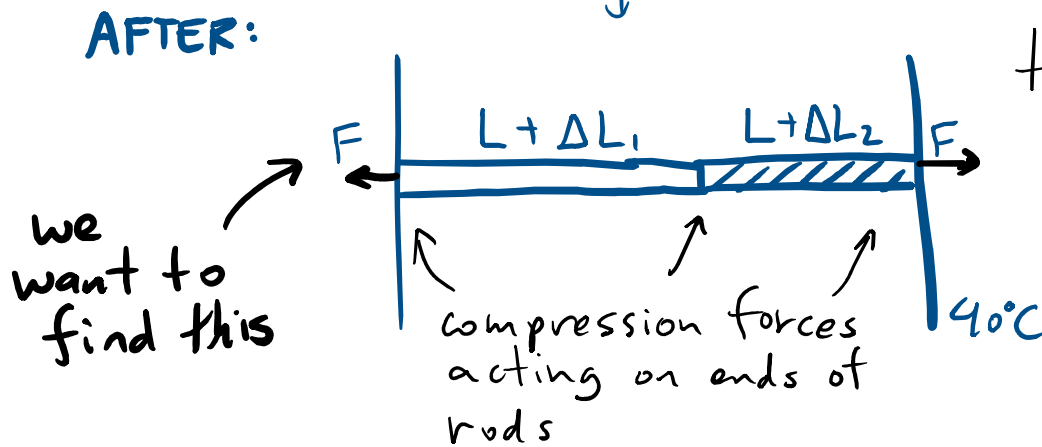
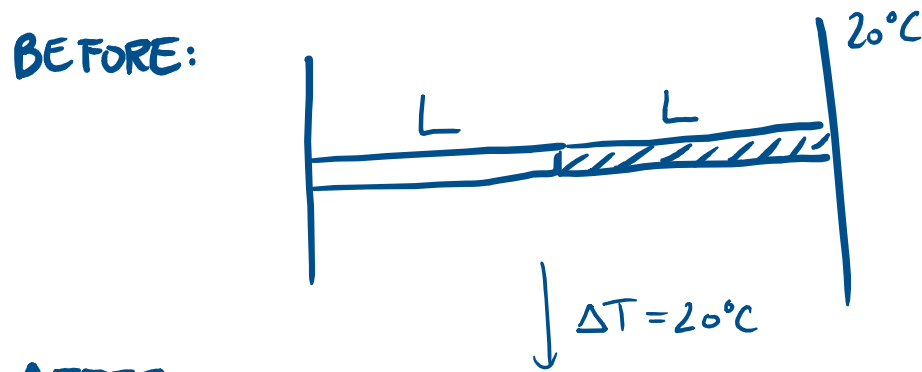
**Clicker:** As the system is heated, we expect that

- A) Both rods will increase in length
- B) Both rods will stay the same length
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rigid walls, so  
B or C  
but different  
materials so  
will likely  
change relative  
length

**STEP 1** : visualize what will happen. Draw a before/after picture.  
Give names to known + unknown quantities + label diagram.  
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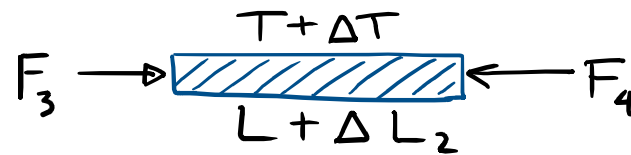
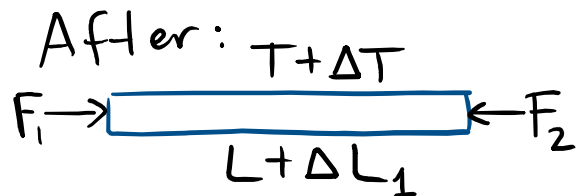
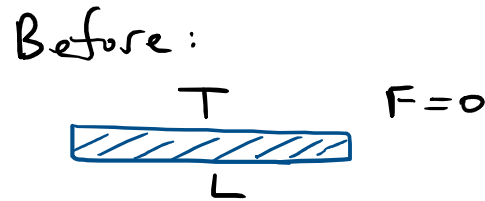
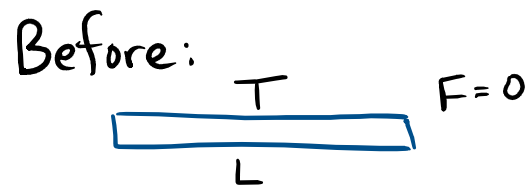
total length is fixed

$$\Delta L_1 + \Delta L_2 = 0$$

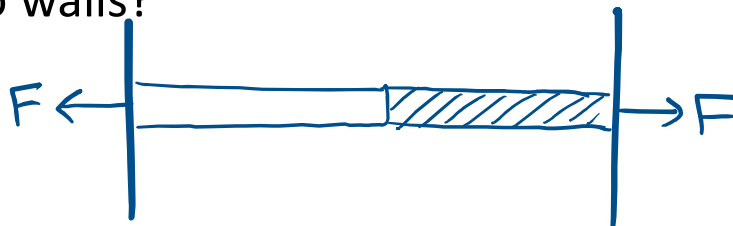


**STEP 2:** Isolate the parts of the system. For each part, draw before/after free body diagrams making use of Newton's Laws to relate forces.

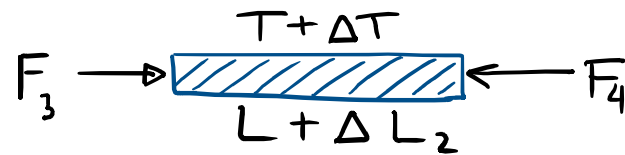
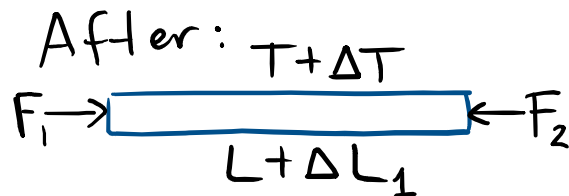
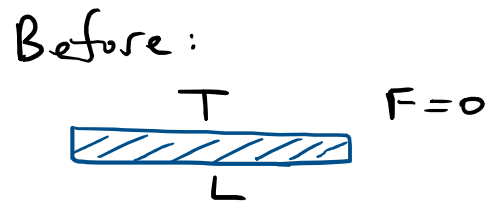
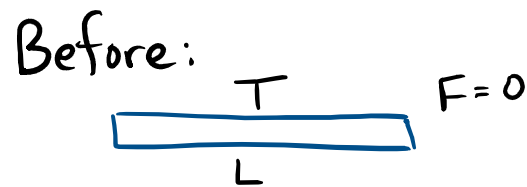
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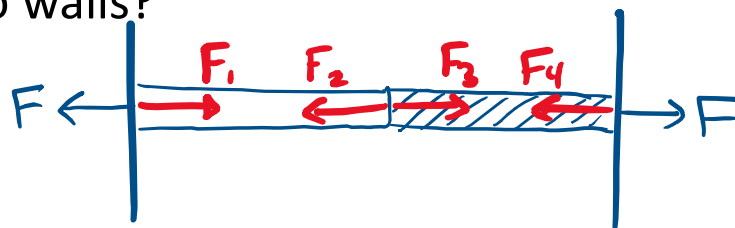
**Question:** What are  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_4$  in terms of  $F$ , the magnitude of the forces on the two walls?



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**Question:** What are  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_4$  in terms of  $F$ , the magnitude of the forces on the two walls?



$F_1 = F$  : Newton's 3rd.

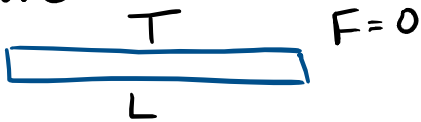
$F_2 = F_1$  : left bar not accelerating

$F_3 = F_2$  : Newton's 3rd

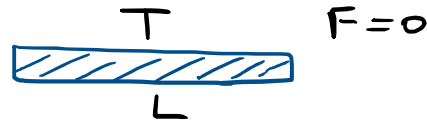
$F_4 = F_3$  : right bar not accelerating

**STEP 3:** For each part, write an equation relating the change in length to the changes in temperature and forces.

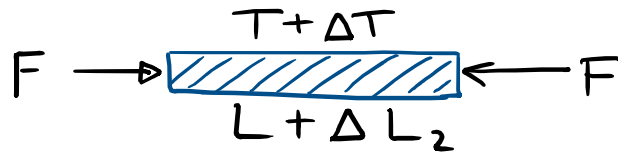
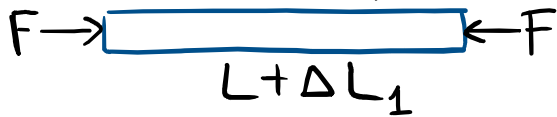
Before:



Before:



After:  $T+\Delta T$



$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_1}$$

$$\Delta L_2 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_2}$$

check that signs make sense with our interpretation of  $F$  as a positive number

STEP 4: collect equations + solve for unknowns

$$\Delta L_1 + \Delta L_2 = 0 \quad (1)$$

$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_1} \quad (2)$$

$$\Delta L_2 = \alpha_2 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_2} \quad (3)$$

STEP 4: collect equations + solve for unknowns

$$\Delta L_1 + \Delta L_2 = 0 \quad (1)$$

$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_1} \quad (2)$$

$$\Delta L_2 = \alpha_2 L \Delta T - \frac{F}{A} \cdot \frac{L}{Y_2} \quad (3)$$

Plug (2) and (3) into (1):

$$\alpha_1 L \Delta T + \alpha_2 L \Delta T - \frac{F}{A} \frac{L}{Y_1} - \frac{F}{A} \frac{L}{Y_2} = 0$$

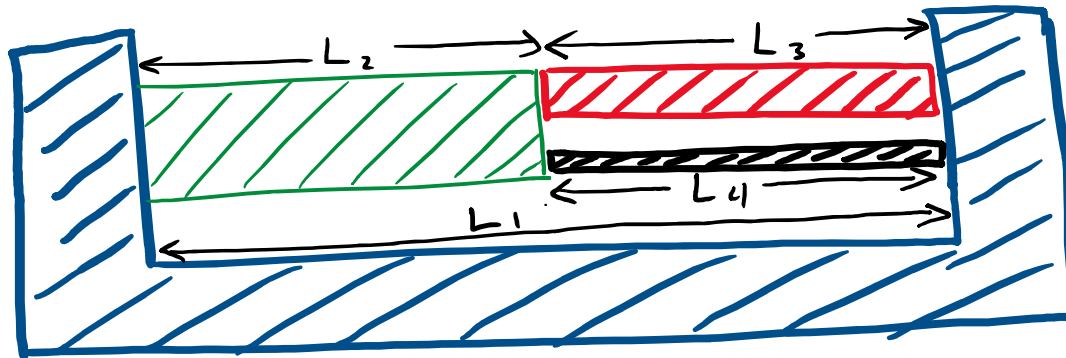
Isolate terms with F:

$$\frac{F \cdot L}{A} \left( \frac{1}{Y_1} + \frac{1}{Y_2} \right) = (\alpha_1 + \alpha_2) L \Delta T$$

Solve:

$$F = \frac{(\alpha_1 + \alpha_2) \Delta T}{\left( \frac{1}{Y_1} + \frac{1}{Y_2} \right)} = 8.2 \times 10^3 \text{ N}$$

## EXTRA QUESTION:



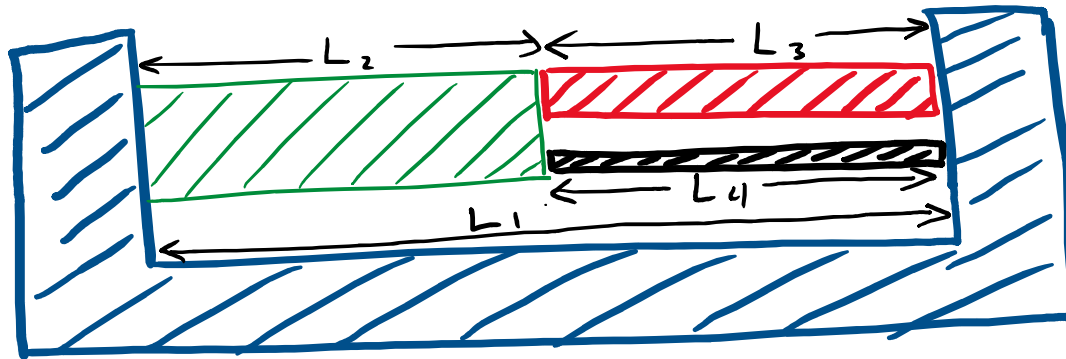
- 1) If the system shown is heated, what constraints must be satisfied by the four quantities  $\Delta L_1$ ,  $\Delta L_2$ ,  $\Delta L_3$ , and  $\Delta L_4$ ?
- 2) What relation do we have between the compressive forces  $F_2$ ,  $F_3$ , and  $F_4$  on the green, red, and black objects?

Click A if you are done number 1

Click B if you are done number 1 and 2

Click C if you are stuck

## QUESTION:



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**Extra Clicker:** If 0.2mm diameter nylon fishing line is good for catching fish up to 2kg, what thickness of line would you need to catch a 50kg fish?

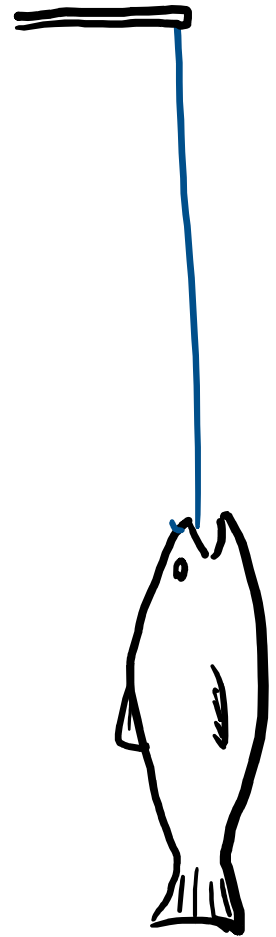
- A) 0.5mm
- B) 1mm
- C) 2mm
- D) 5mm
- E) 300mm

**EXTRA:** By roughly how much would 1m of 0.2mm diameter line be stretched by a 2kg fish?

( $Y_{\text{nylon}} = 3 \text{ GPa}$ )

$G = \text{giga} = 10^9$

$$\frac{F}{A} = Y \frac{\Delta L}{L_0}$$



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- A) 0.5mm
- B) 1mm**
- C) 2mm
- D) 5mm
- E) 300mm

*F is 25x before  
to get equivalent "safe" stretching,  
need A 25x before  
so diameter 5x before.*

$$\Delta L \approx \frac{1\text{m}}{3 \times 10^9} \cdot \frac{20\text{N}}{\pi \cdot (10^{-4}\text{m})^2} \approx 0.2\text{m}$$

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