Learning goals:

- Give an example of a material where the thermal expansion coefficient has a significant temperature dependence and describe some consequences of this
- Define the concepts of stress and strain and explain how the Young's modulus of a material is defined
- To argue that the Young's modulus depends only on the material and not on the shape or size
- To calculate the fractional change in length of an object given its dimensions, the Young's modulus, and the forces applied
- For static systems with multiple parts, to use Newton's second and third law to related the various internal forces
- to calculate the mechanical forces (thermal stress) on a constrained system arising from changes in temperature.

Homework sessions with TAs:

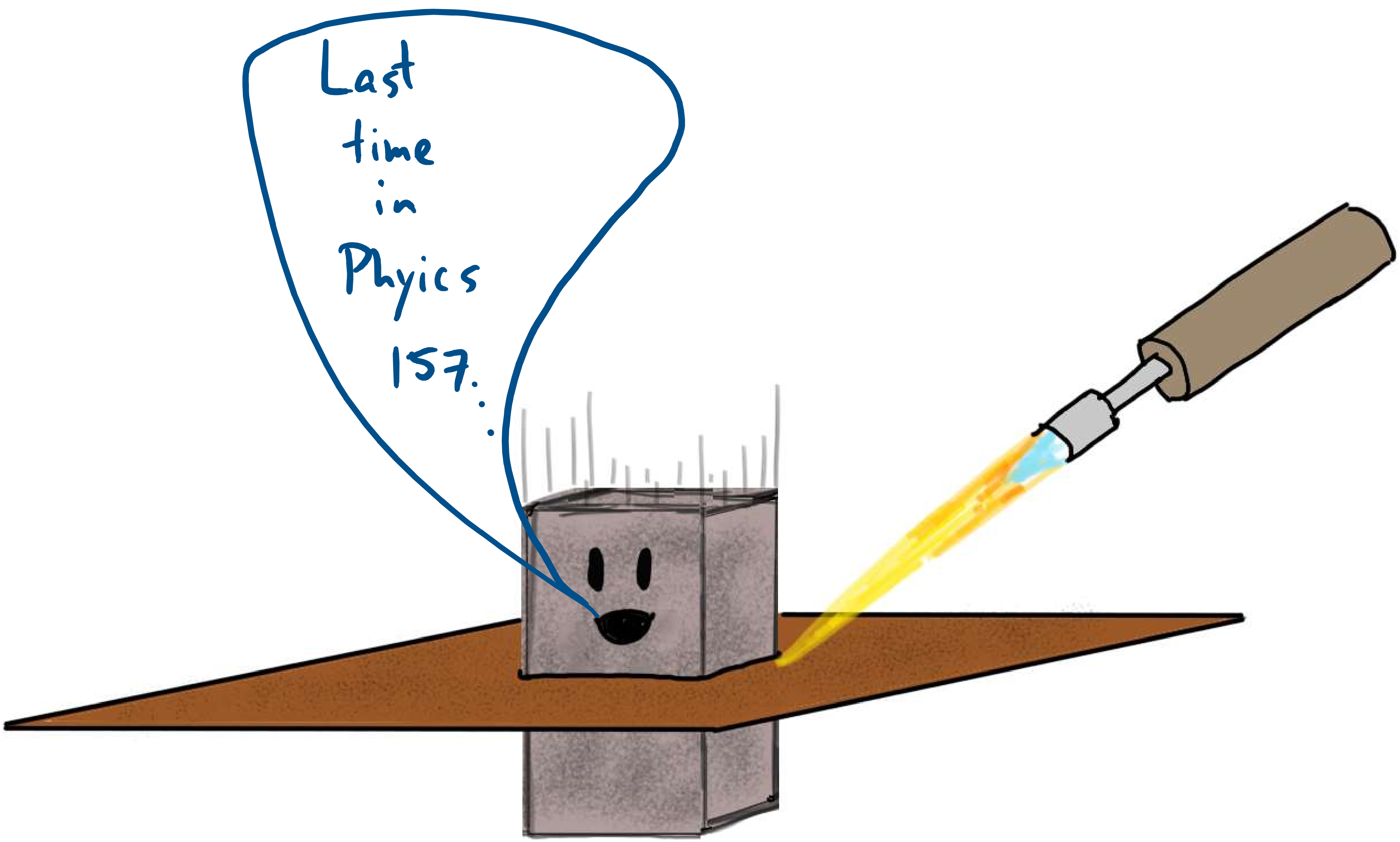
Monday 5-7pm in Hennings 200

Tuesday 5-7pm in Hennings 202

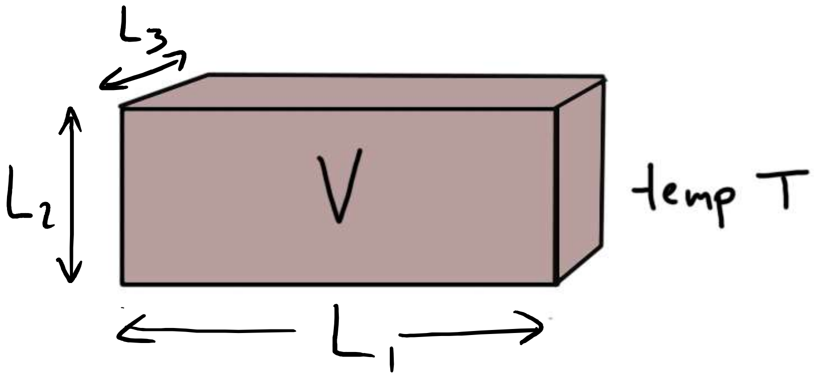
Piazza: an online Q+A forum for questions about homework, course material, etc...

Office hours: Today: 3-5pm

Hennings 420

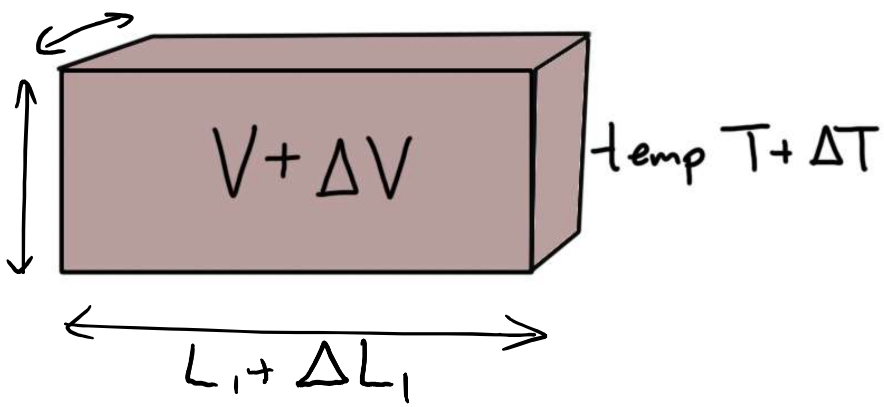

Last time in Phyics 157...

Thermal expansion:



$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

applies to each dimension

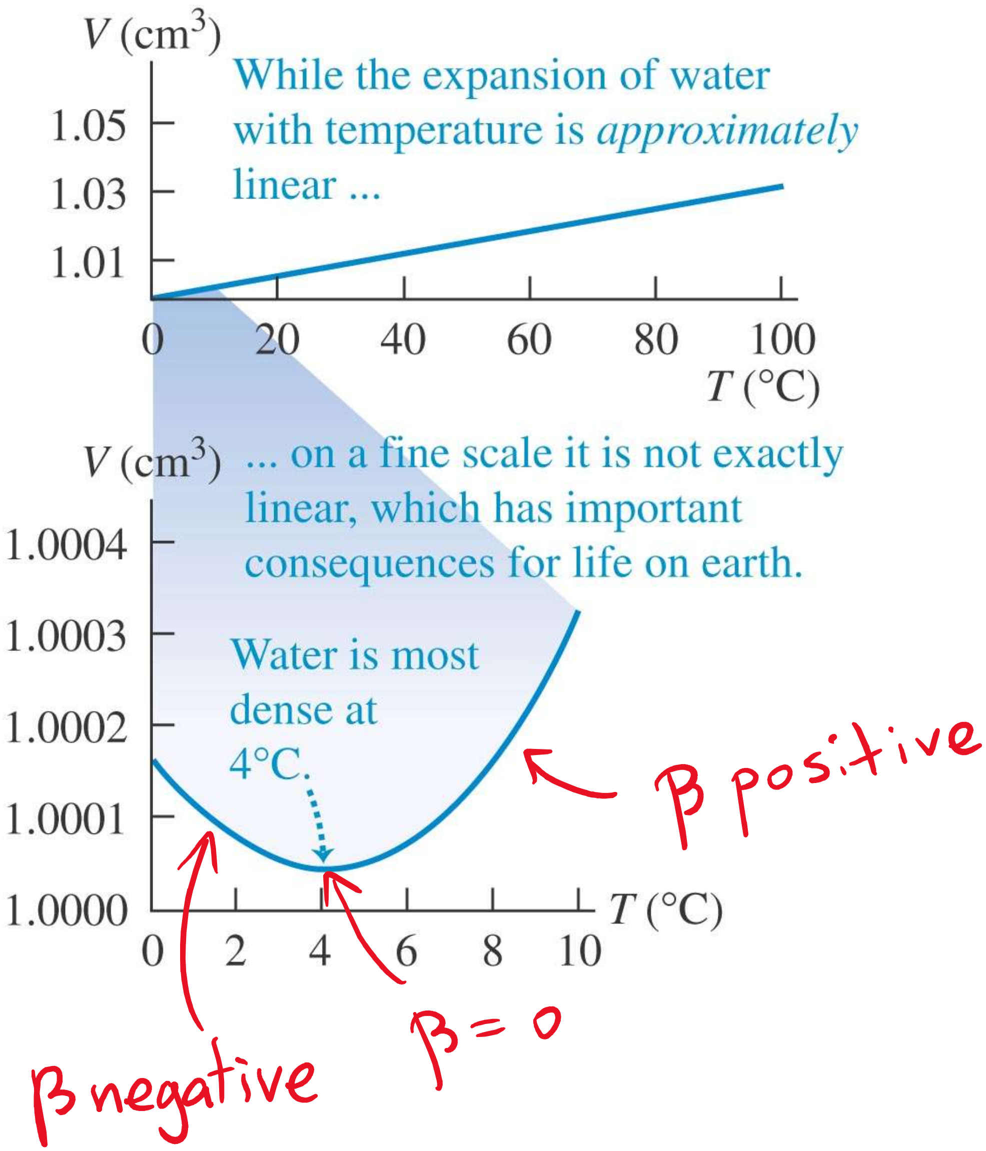


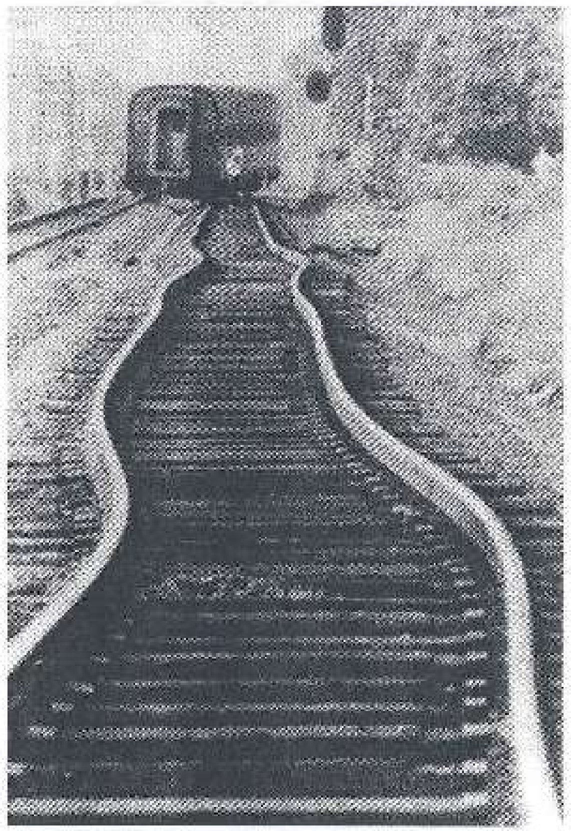
$$\boxed{\Delta V = \beta V_0 \Delta T}$$

$\beta = 3\alpha$ for solids

also applies to liquids

- Water, a special example



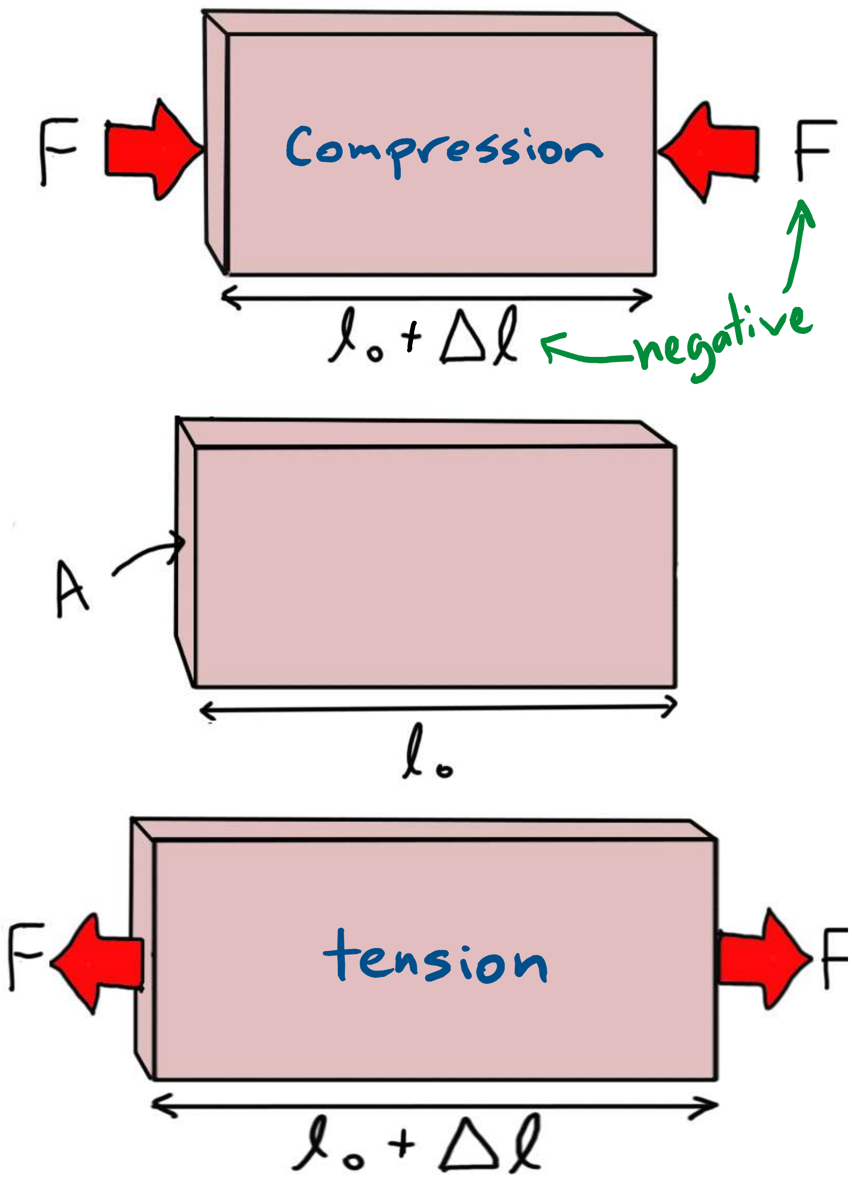


Today: thermal expansion

↓

forces

STRESS & STRAIN



$$\frac{F}{A} = Y \frac{\Delta l}{l_0}$$

stress
(units of pressure)

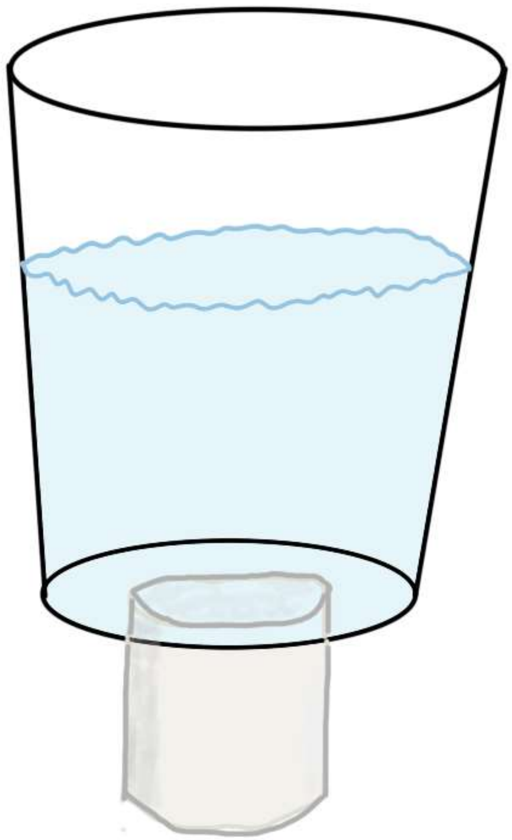
Young's modulus
=
a basic property of a material
(resistance to squishing)

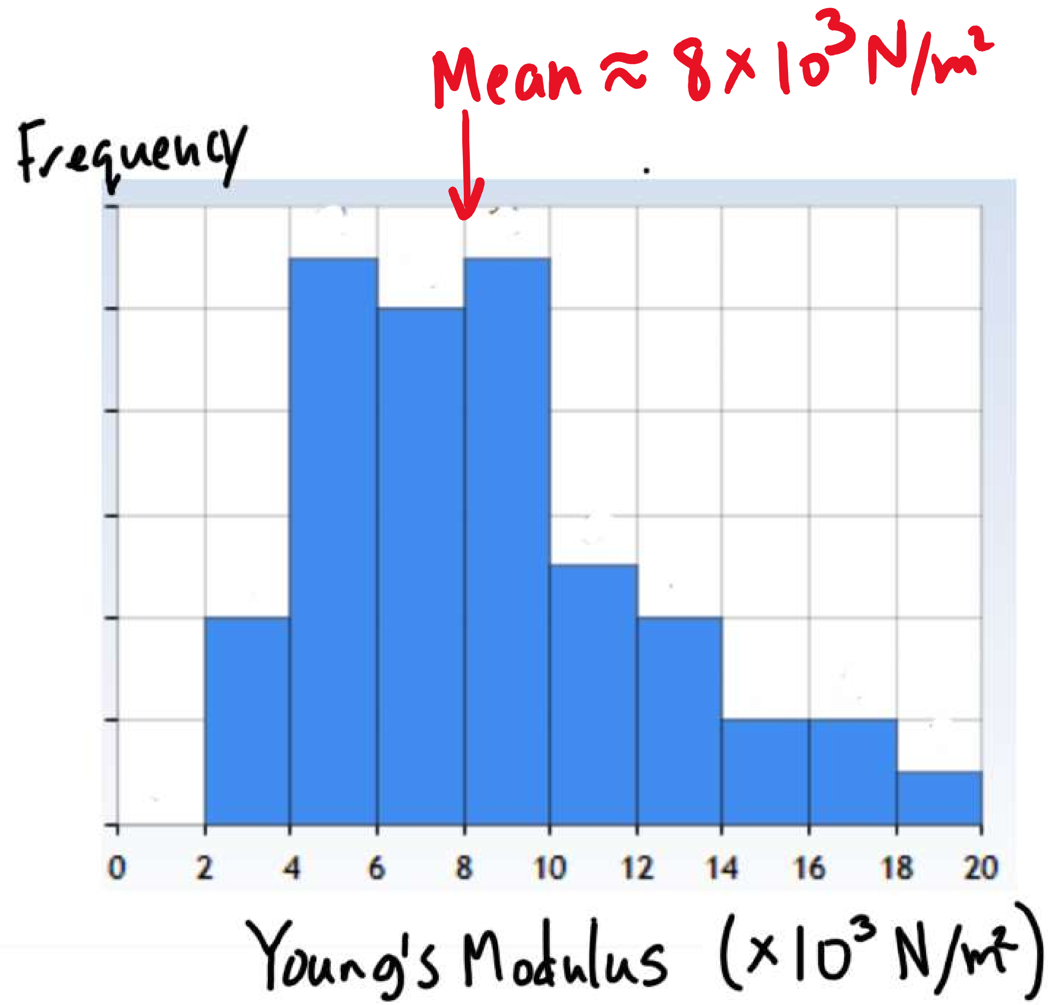
strain

like $F = k\Delta x$ for spring

Young's modulus of a marshmallow

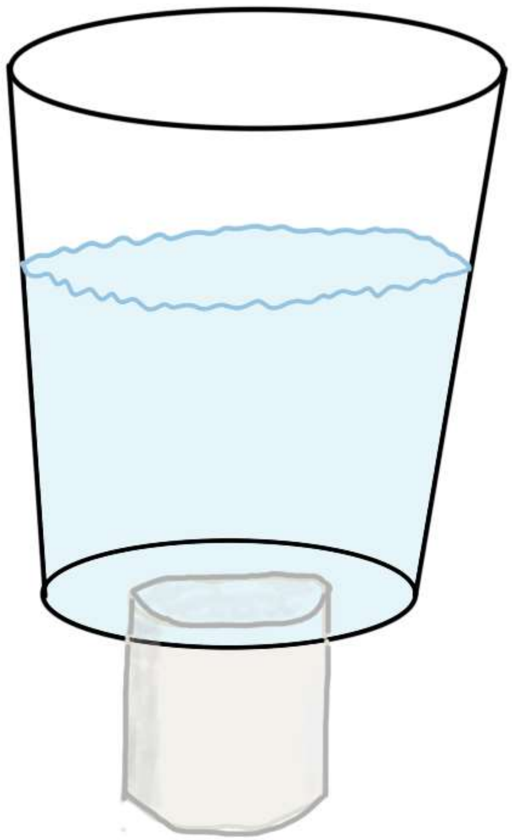
$$\frac{F}{A} = Y \frac{\Delta L}{L_0}$$





Young’s modulus of a marshmallow

$$\frac{F}{A} = Y \frac{\Delta L}{L_0}$$



$$\frac{\Delta L}{L_0} \approx 0.1 - 0.2$$

$$F \approx 1\,N$$

$$A \approx 5\ cm^2 = 5 \times (10^{-2} m)^2$$

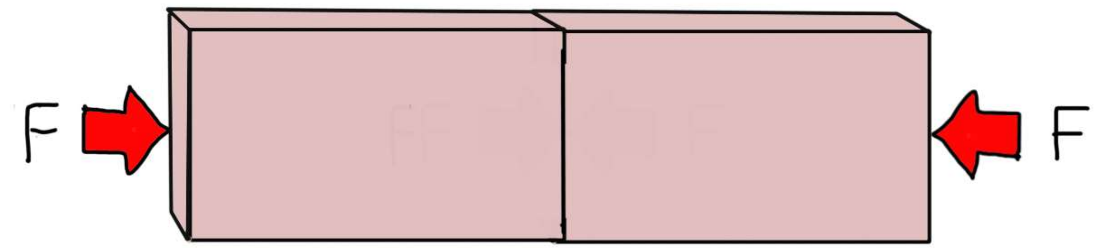
gives $Y \approx 10^4\ N/m^2$

Clicker: Suppose you repeated the measurement of Y for a mini-marshmallow. In this case, we would expect a value of Y that is

A. Significantly higher
B. Significantly lower
C. About the same

$$\boxed{\frac{F}{A} = Y \frac{\Delta L}{L_0}}$$

Clicker: In the top picture, the force on the right brick from the left brick has magnitude



A) 0

B) F

C) 2 F

All forces shown have the same magnitude.

EXTRA: can you come up with a sharp argument for your answer?

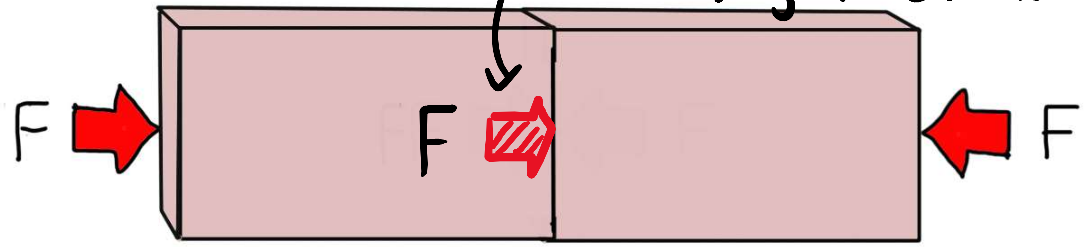
Clicker: In the top picture, the force on the right brick from the left brick has magnitude

A) 0

B) F

C) 2 F

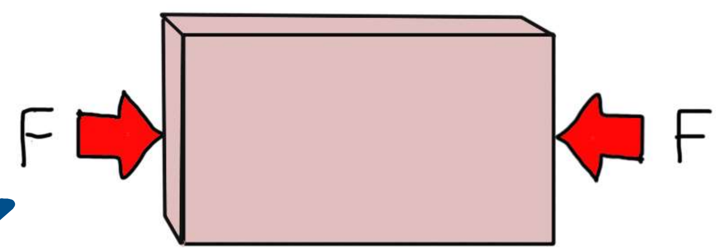
force from left brick on right brick

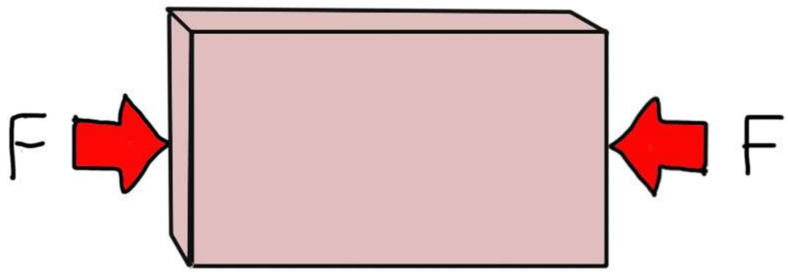


- bricks not moving so net force on right brick must be zero, so force of left brick on right exactly opposes force from right

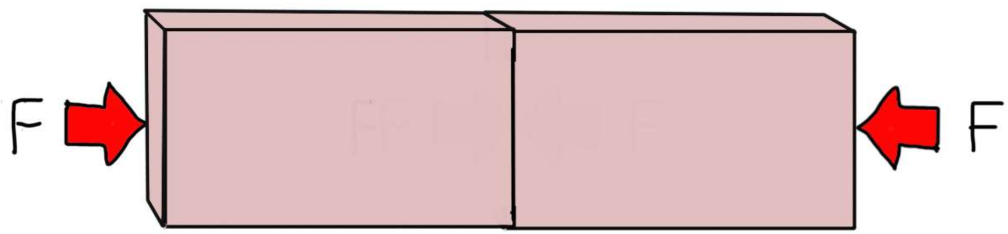
All forces shown have the same magnitude.

Each half above has same compression as this brick

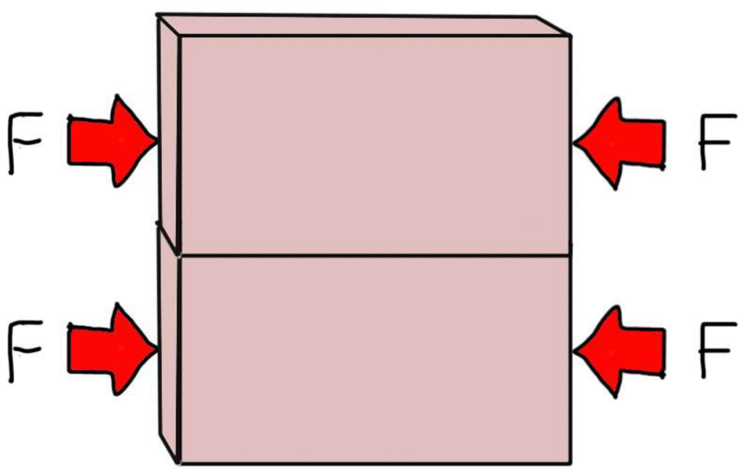




$$Y = \frac{F/A}{\Delta L / L_0}$$



same F, A
double L_0, double ΔL
$\therefore$ same Y



double F, double A
same ΔL, L
$\therefore$ same Y

Y is the same in all cases
→ it only depends on the material

Young Modulus of Various Materials

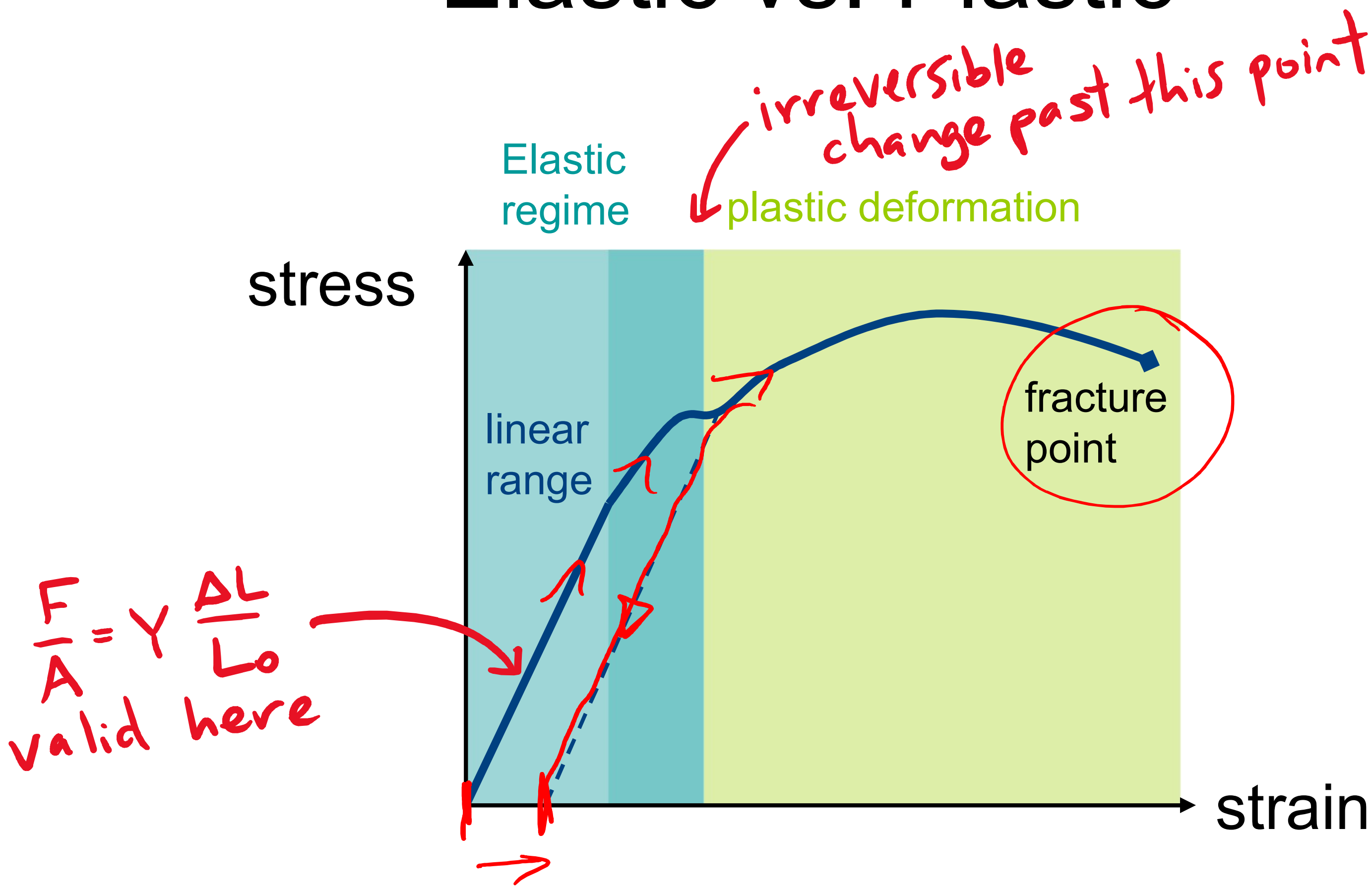
Material	Young's Modulus, Y (Pa)
Aluminum	7.0×10^{10}
Brass	9.0×10^{10}
Copper	11×10^{10}
Crown glass	6.0×10^{10}
Iron	21×10^{10}
Lead	1.6×10^{10}
Nickel	21×10^{10}
Steel	20×10^{10}
Marshmallow	8×10^{3}

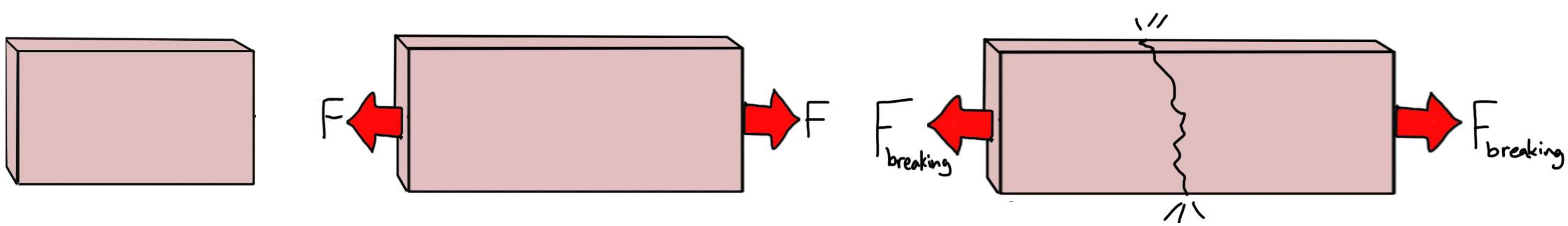
units of pressure:

eg. Pressure of 0.01% of Y on ends will give 0.01% compression

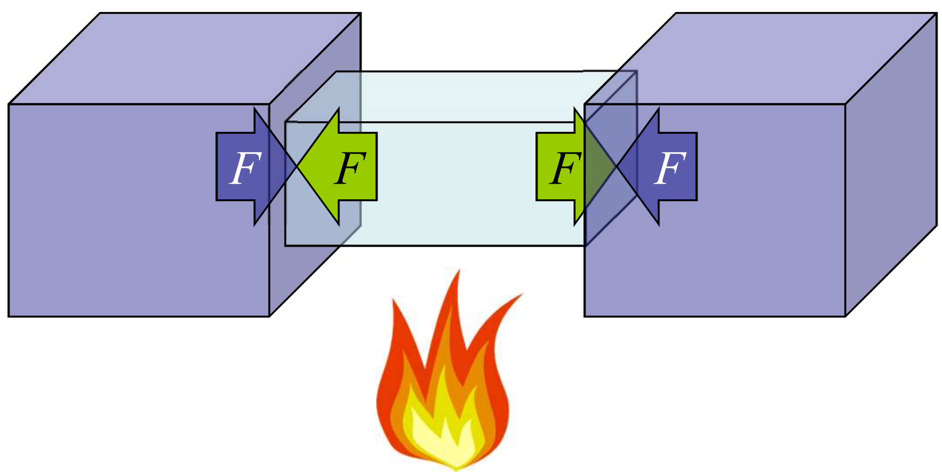
$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

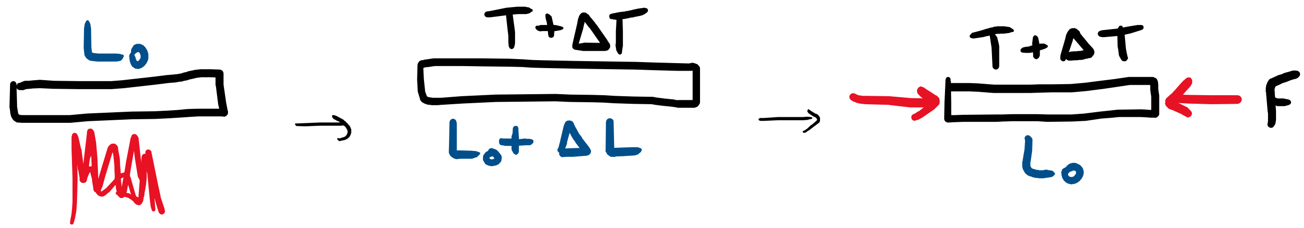
Elastic vs. Plastic





THERMAL STRESS : forces on a material due to surrounding materials preventing thermal expansion/contraction





A steel rod of length L_0 is heated by temperature ΔT.

How much stress (force per unit area) is required to compress the rod back to its original length?

Write an answer in terms of ΔT and the parameters Y, α, L_0 for the rod.

Click A if you have an answer, B if you are stuck.

thermal expansion:

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

stress vs strain:

$$\boxed{\frac{F}{A} = Y \frac{\Delta L}{L_0}}$$

A steel rod of length L_0 is heated by temperature ΔT and expands. How much stress (force per unit area) is required to compress the rod back to its original length?

Write an answer for the magnitude of F/A in terms of Y, α, L_0, and ΔT.

Thermal Expansion

$$\Delta L_{th} = \alpha L_0 \Delta T$$

want $\Delta L_F = -\Delta L_{th}$

$$= -\alpha L_0 \Delta T$$

A) $Y \alpha L_0 \Delta T$

B) $Y \alpha \Delta T$

C) $Y L_0 \Delta T$

D) $\alpha L_0 \Delta T$

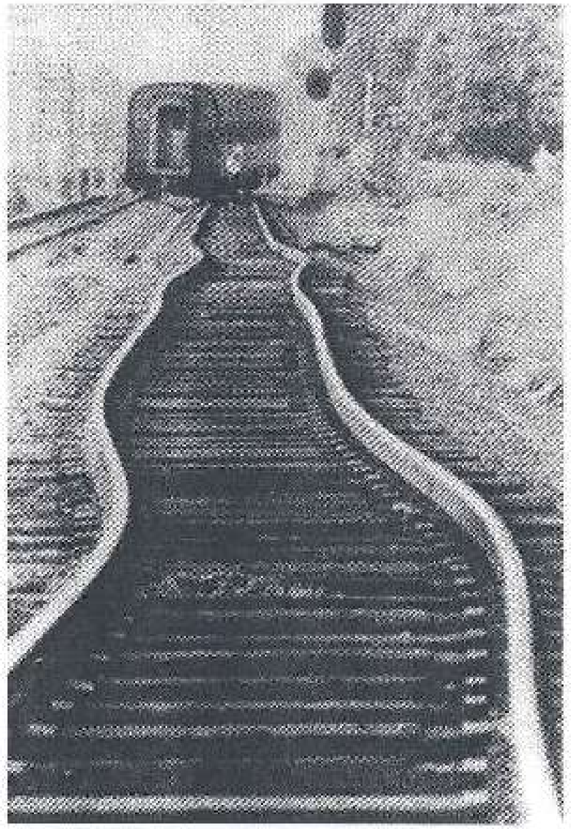
E) $Y \alpha L_0$

$$\frac{F}{A} = Y\left(\frac{\Delta L_F}{L_0}\right)$$

$$= Y(-\alpha \Delta T)$$

− sign means compressive force

$$\left|\frac{F}{A}\right| = Y \alpha \Delta T$$



Clicker: 10m long steel train rails are laid end to end on a winter day (0 ºC). If the engineer forgot to leave gaps for thermal expansion, roughly how much force is generated at the ends of each rail due to thermal stress when the temperature reaches 30 ºC?

Cross sectional area of rail: 0.01m²

$Y_{steel} = 20 \times 10^{10}\ Pa$

$\alpha_{steel} = 1.2 \times 10^{-5}\ K^{-1}$

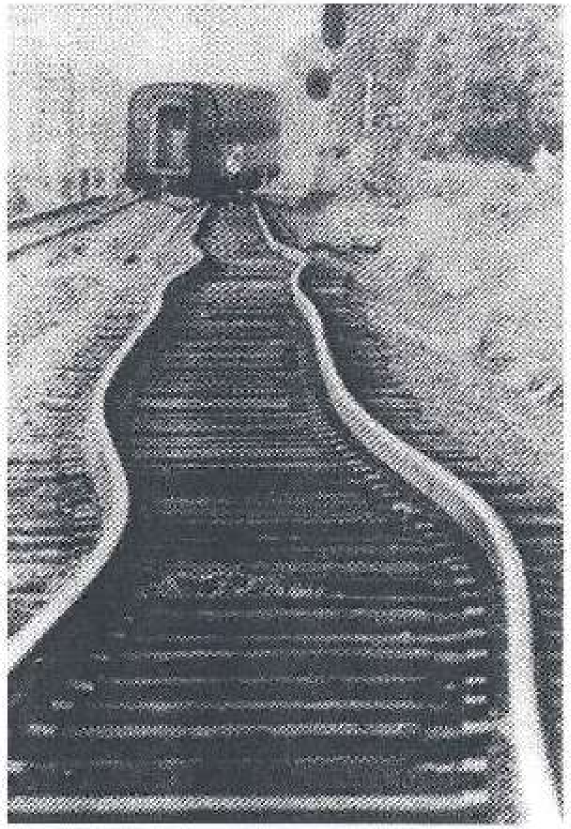
A) 700 N B) 7,000 N C) 70,000 N

D) 700,000 N E) 7,000,000 N

previous result:

$$\frac{F}{A} = Y\alpha \Delta T$$

EXTRA: How much gap should have been left?



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A) 700 N B) 7,000 N C) 70,000 N

D) 700,000 N E) 7,000,000 N

$F \approx 10^{-2} \times 2\times10^{11} \times 1.2\times10^{-5} \times 30 \approx 7\times10^{5}$

EXTRA: How much gap should have been left? $\Delta L = \alpha L_0 \Delta T$

$\approx 3.6\ mm$


fully loaded
Boeing 737
AIR CANADA
rail

Extra Clicker: Do you expect that the Young's modulus you measured for a marshmallow is higher or lower than for steel?

A. Higher
B. Lower
C. Could be higher or lower depending on the relative dimensions of the steel/marshmallow

$$\boxed{\frac{F}{A} = Y\frac{\Delta L}{L_0}}$$

Clicker: Do you expect that the Young's modulus you measured for a marshmallow is higher or lower than for steel?

Y only depends on what the object is made of, not its size

A. Higher

B. Lower

$\frac{F}{A} = Y \frac{\Delta L}{L}$: Y bigger if it takes more force to give same change in L.

C. Could be higher or lower depending on the relative dimensions of the steel/marshmallow

Y has units of pressure: roughly, the pressure required to produce a significant fractional change in length.