

Learning Goals:

- For an object made of some material, to calculate the changes in length or volume that material undergoes in response to changes in temperature and external forces (stress).
- To explain why the change of length of an object due to thermal expansion is proportional to its initial length.
- For systems consisting of two different materials, to quantitatively analyze effects resulting from the different expansion rates of different parts.
- To explain why the fractional change in volume of an object for a small change in temperature is three times the fractional change in length

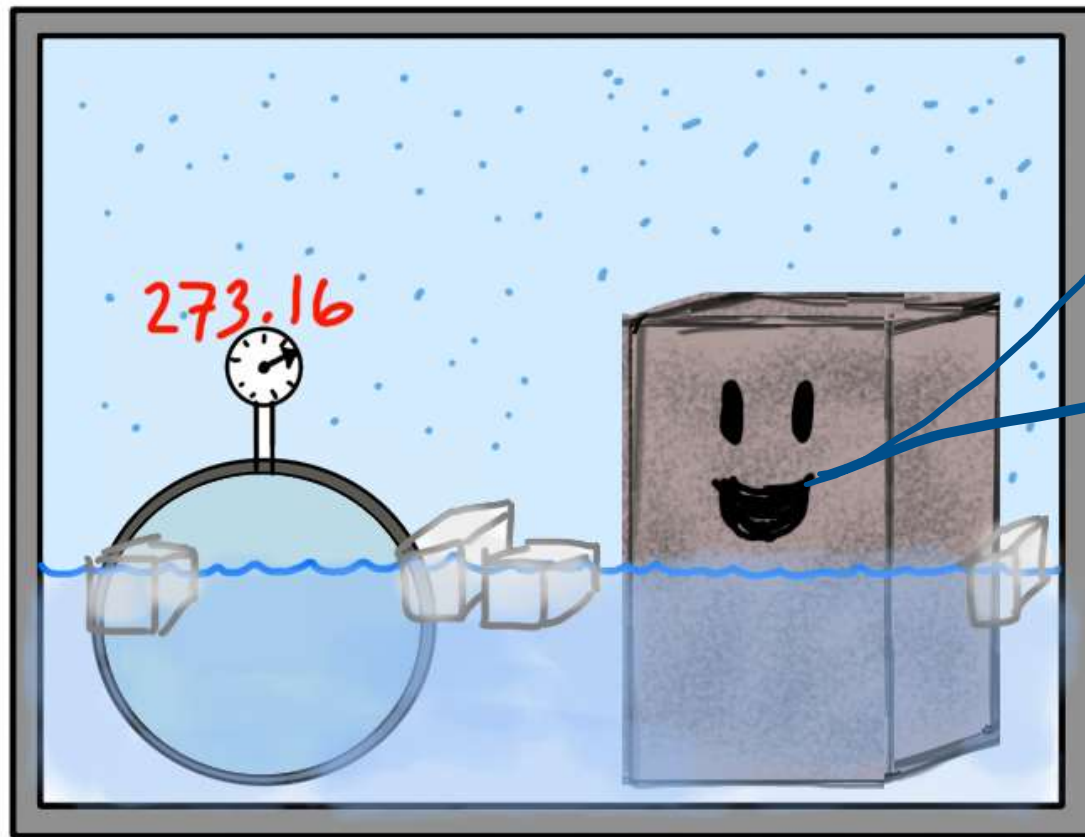
Office hours today: 3:30 - 5:30 Hennings 420

Clicker: A steel ball does not quite fit through a hole in a copper plate. If $\alpha_{\text{steel}} < \alpha_{\text{copper}}$, we could help the ball fit through the hole by

- A. Heating the system
- B. Cooling the system
- C. Either A or B will work
- D. Neither A nor B will work

$$\Delta L = \alpha L_0 \Delta T$$

EXTRA: does the hole get larger or smaller when we heat the system? Why?



Last
time in
Physics 157...

Define Kelvin scale by:

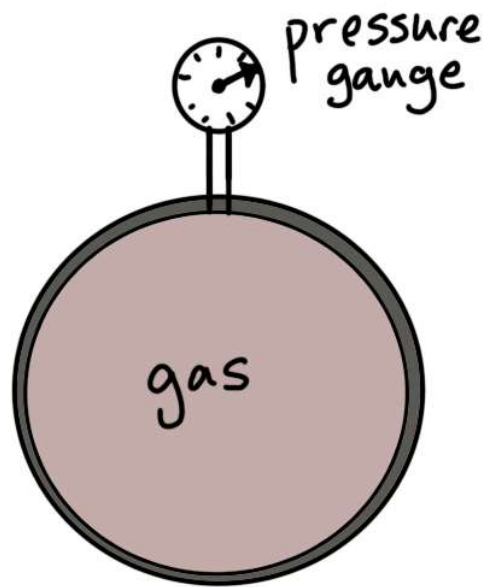
$$T = \text{const.} \times P$$

and

↑
pressure
↑
depends on
particular thermometer

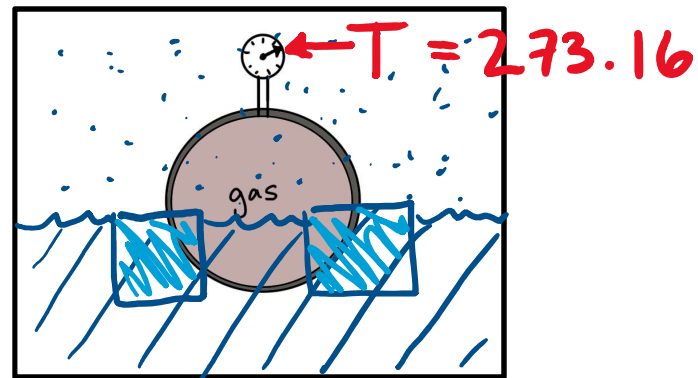
$$T = 273.16 \text{ K}$$

at triple point of water

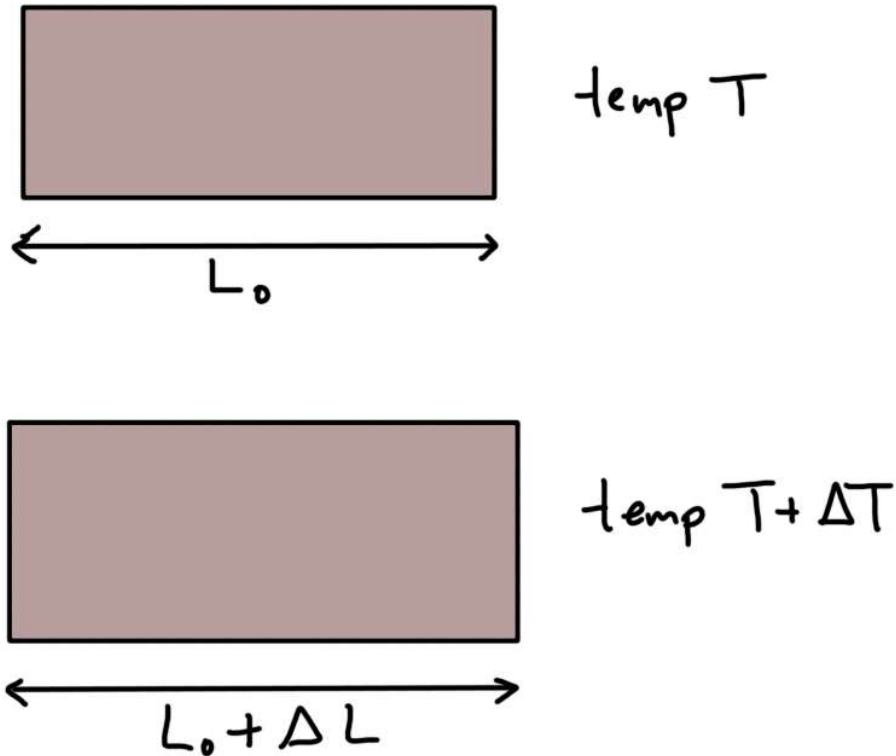


constant
volume
gas thermometer:

$$T_C = T_K - 273.15$$



Thermal expansion:



coefficient of linear expansion: a basic property of a material

$$\Delta L = \alpha L_0 \Delta T$$

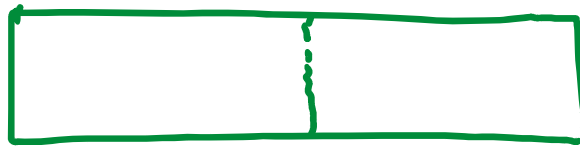
- assumes $\frac{\Delta L}{L}$ is small
- α can depend on T

$$\Delta L = \alpha L_0 \Delta T$$

Discussion question: why is the change in length of an object proportional to its initial length L_0 ? E.g. why does a steel rod that starts out twice as long expand twice as much?

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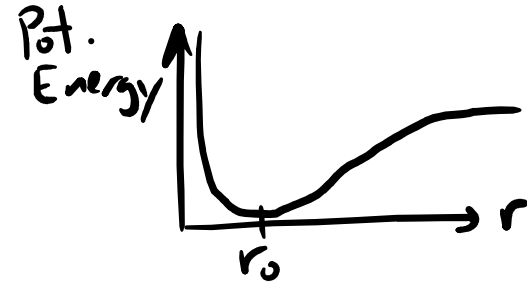
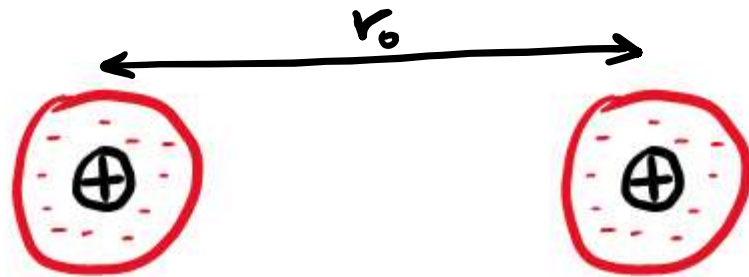


↑ ↑
each half expands
independently, same
amount as original object

total expansion is double

Why do materials usually expand when heated?

Nearby atoms in solid:



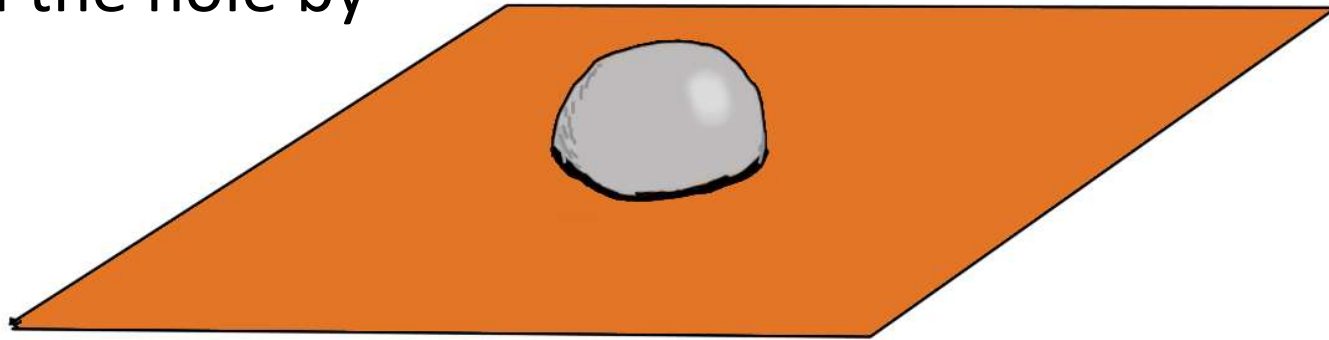
Lowest energy



Add energy

More higher energy configurations with $r > r_0$ than $r < r_0$.

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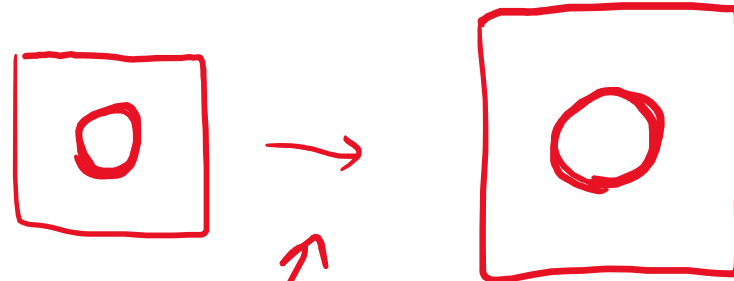
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EXTRA: does the hole get larger or smaller when we heat the system? Why?

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ball + hole both expand,
but hole expands more
since $\alpha_{\text{Cu}} > \alpha_{\text{steel}}$



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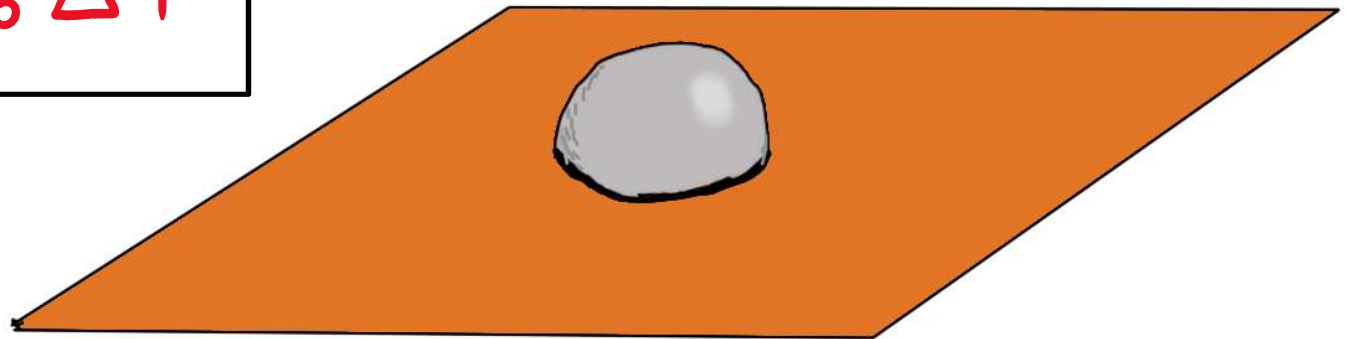
hole grows in proportion to
plate (same as if hole were filled)

If the radius of the ball at $T = 20^\circ\text{C}$ is 1.001cm and the radius of the hole is 1.000cm, to what temperature must we heat the system before the ball falls through?

We have: $\alpha_s = 1.2 \times 10^{-5} \text{ K}^{-1}$ and $\alpha_c = 8 \times 10^{-5} \text{ K}^{-1}$

Discuss a strategy for solving this. What should be true about ΔL_{ball} relative to ΔL_{hole} ?

$$\Delta L = \alpha L_0 \Delta T$$



Strategy: ① understand what happens to each part

② understand how the parts are related

①

hole expands: $\Delta L_{\text{hole}} = \alpha_{\text{Cu}} L_0^{\text{Cu}} \Delta T$ ← unknown

ball expands: $\Delta L_{\text{ball}} = \alpha_{\text{S}} L_0^{\text{S}} \Delta T$ ✓

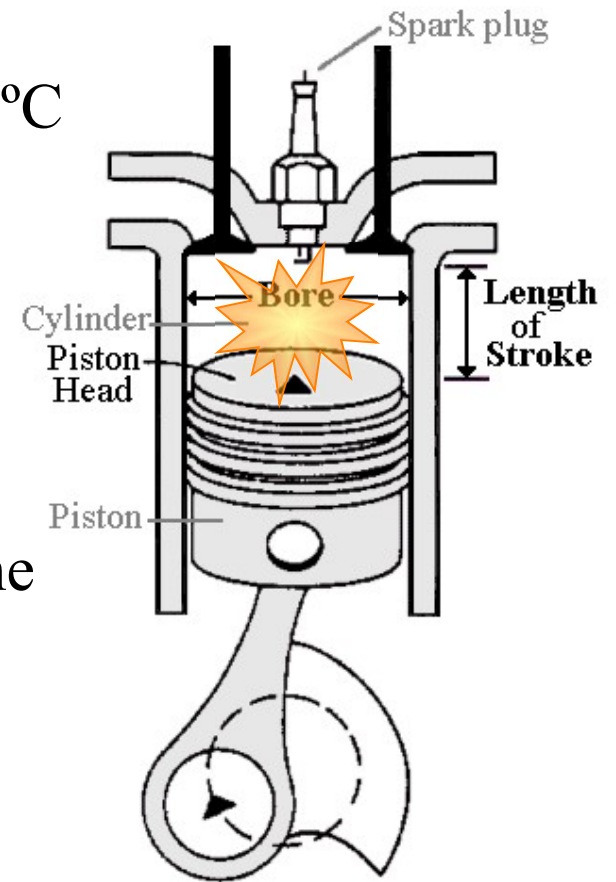
② We need $\Delta L_{\text{hole}} = \Delta L_{\text{ball}} + 0.001 \text{ cm}$
in order for the ball to fall through

③ Rest is math: $\alpha_{\text{Cu}} L_0^{\text{Cu}} \Delta T = \alpha_{\text{S}} L_0^{\text{S}} \Delta T + 0.001 \text{ cm}$
solve for ΔT

Clicker: In some car engines, the piston is aluminum ($\alpha = 2.4 \times 10^{-5}$), while the cylinder is cast iron ($\alpha = 1.2 \times 10^{-5}$). If the engine needs to operate between 0°C and 120°C , which of these is **not** a good design:

- A) The piston barely fits in the cylinder at 120°C
- B) The piston barely fits in the cylinder at 0°C

EXTRA: what do we need to worry about if the engine gets too hot? Too cold?



$$\Delta L = \alpha L_0 \Delta T$$

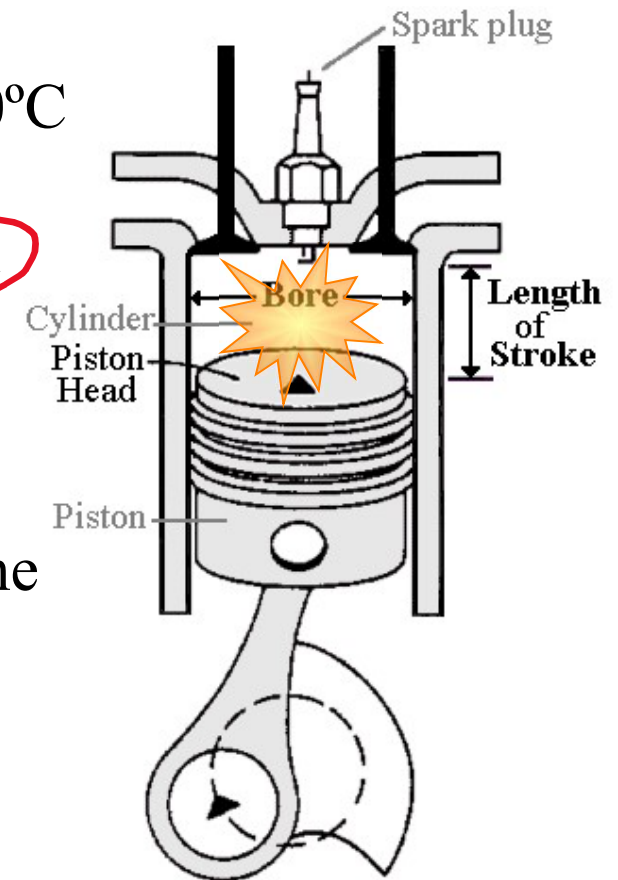
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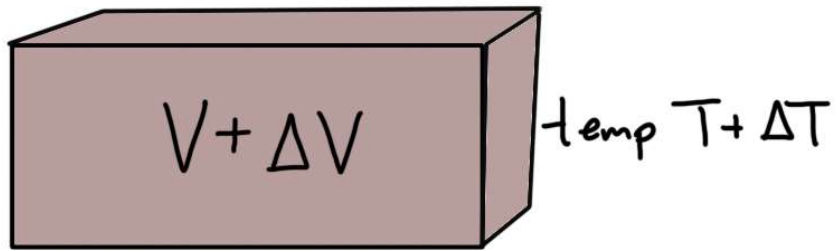
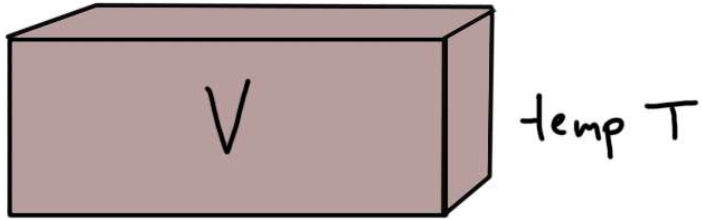
piston expands more than cylinder as engine heats up. Wouldn't be able to move at higher temperatures

EXTRA: what do we need to worry about if the engine gets too hot? Too cold?



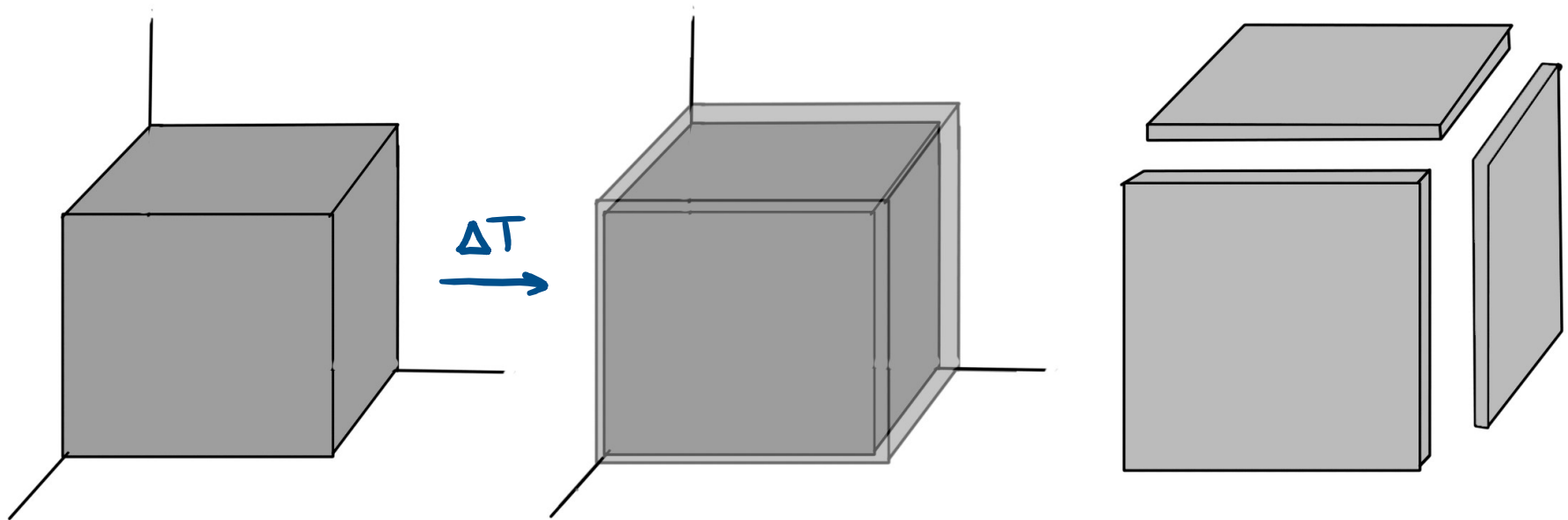
$$\Delta L = \alpha L_0 \Delta T$$

Volume expansion:



$$\Delta V = \beta V_0 \Delta T$$

also applies to liquids

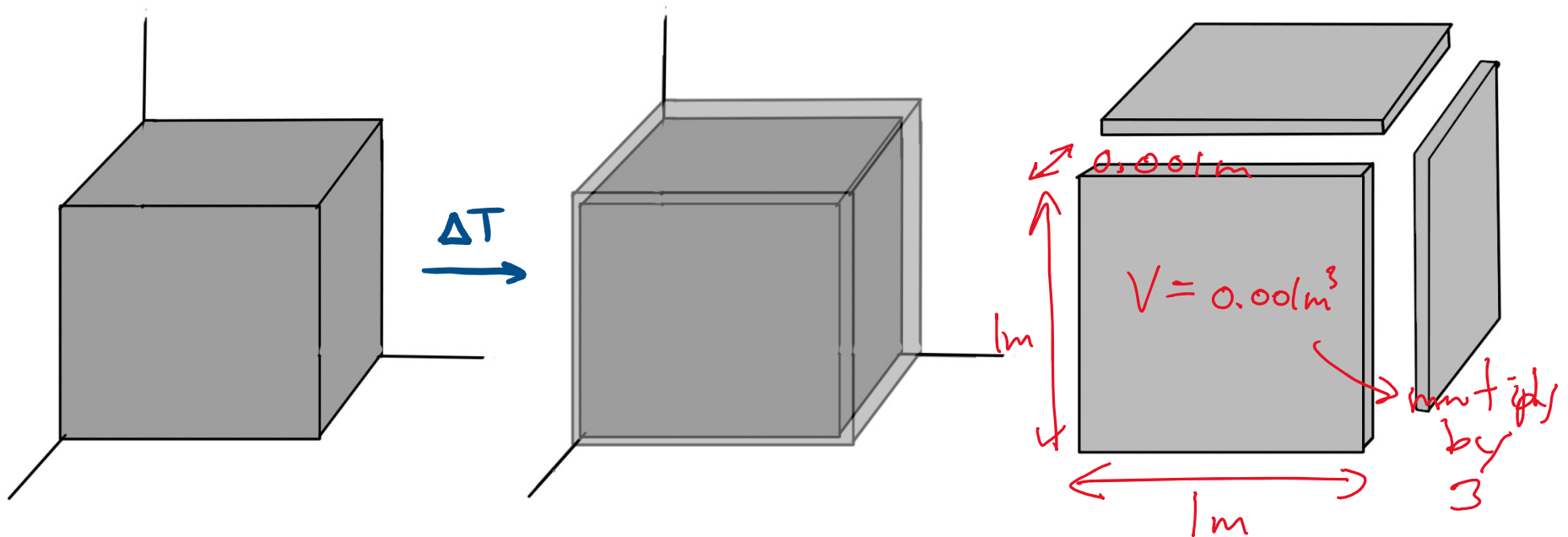


Clicker: When heated, each side of a 1m cube of material expands by 0.001m. The extra volume (shown in the third picture) after the expansion is approximately

A) 0.000000001m^3 B) 0.00001m^3 C) 0.001m^3 D) 0.003m^3

E) There is not enough information

look at the picture and use geometry to solve this!



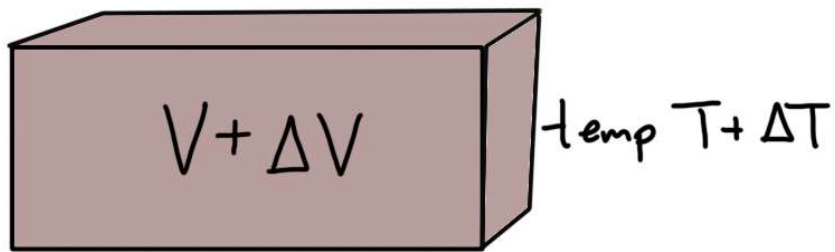
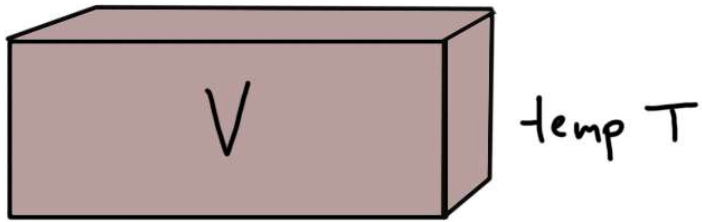
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Volume expansion:



$$\Delta V = \beta V_0 \Delta T$$

$$\beta = 3\alpha \text{ for solids}$$

also applies to liquids

Mathematical derivation:

original volume: L^3

new volume $(1.001 \times L)^3 \approx 1.003 L^3$

so 0.3% bigger

$$\text{generally: } (L + \Delta L)^3 = L^3 + \underbrace{3L^2\Delta L + 3L(\Delta L)^2 + (\Delta L)^3}_{\Delta V}$$

$$\frac{\Delta V}{V} = 3 \cdot \frac{\Delta L}{L} + 3 \left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta L}{L}\right)^3$$

this means
 $\beta = 3\alpha$

these are negligible compared to
the first term if $\frac{\Delta L}{L}$ is small