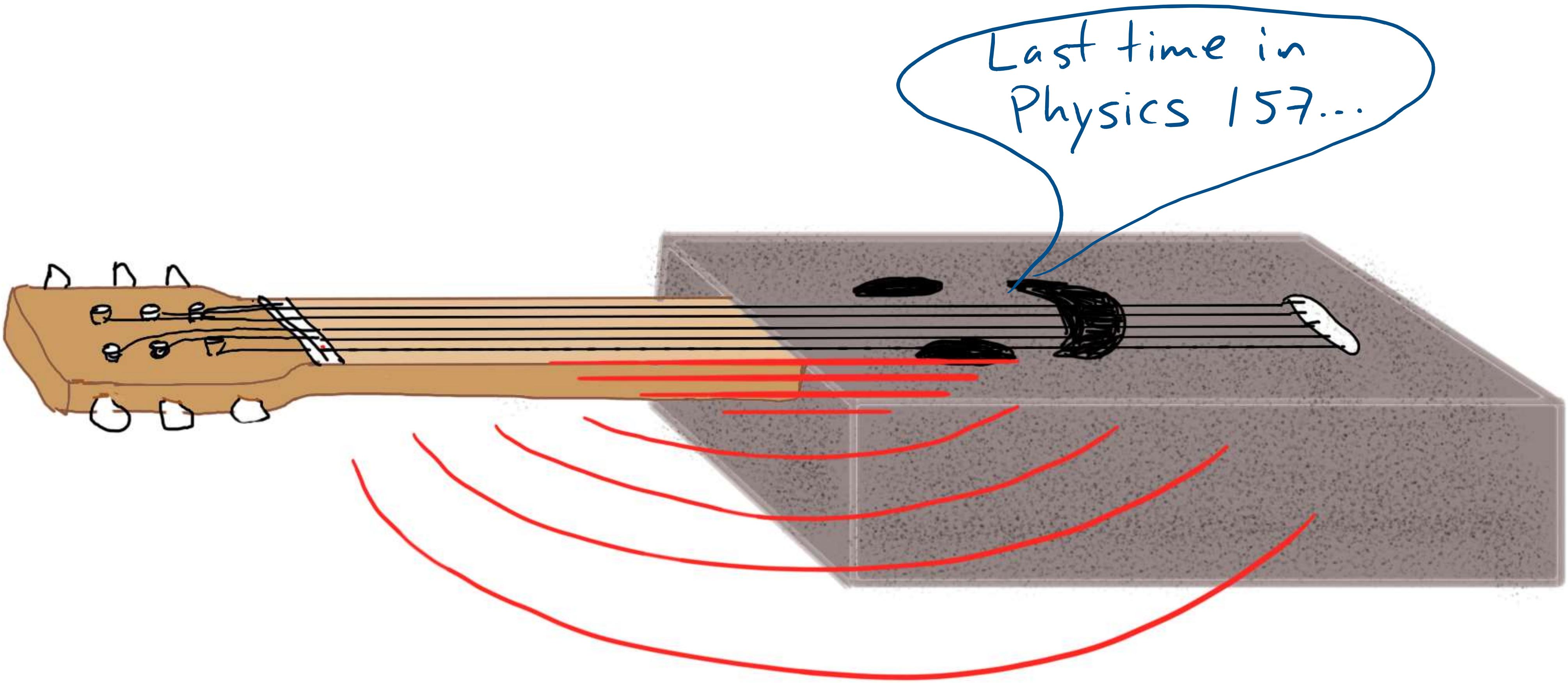
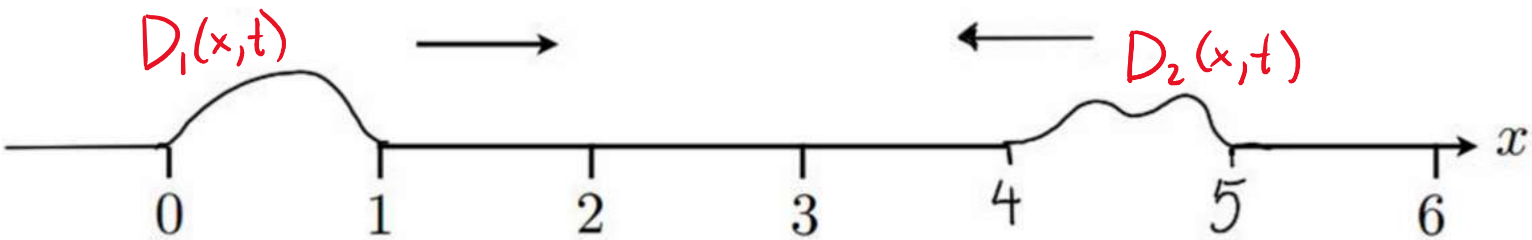

Last time in
Physics 157...

The Principle of Superposition

When two or more waves overlap, the net displacement D(x,t) is equal to the sum of the displacements we would have if each wave were present alone.

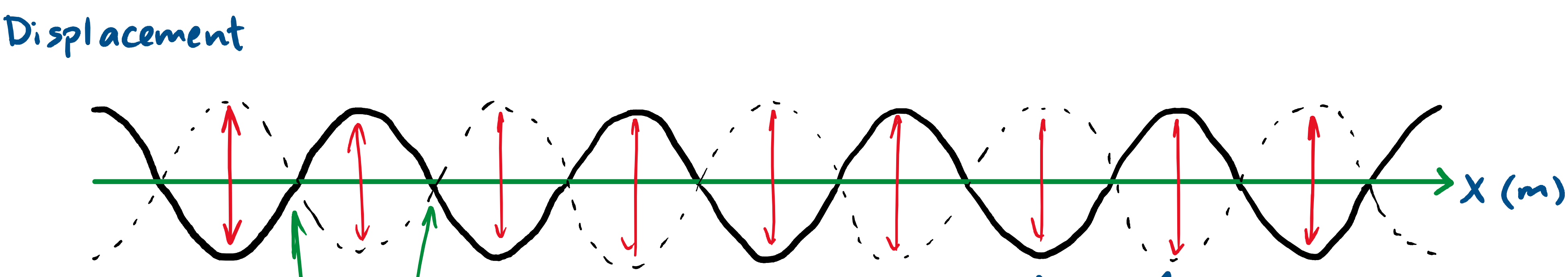


combined wave: $D(x,t) = D_1(x,t) + D_2(x,t)$

★ waves add without disturbing each other ★

Standing Waves

$$D(x,t) = A\cos(kx)\cdot\cos(\omega t)$$



Nodes: displacement fixed at 0

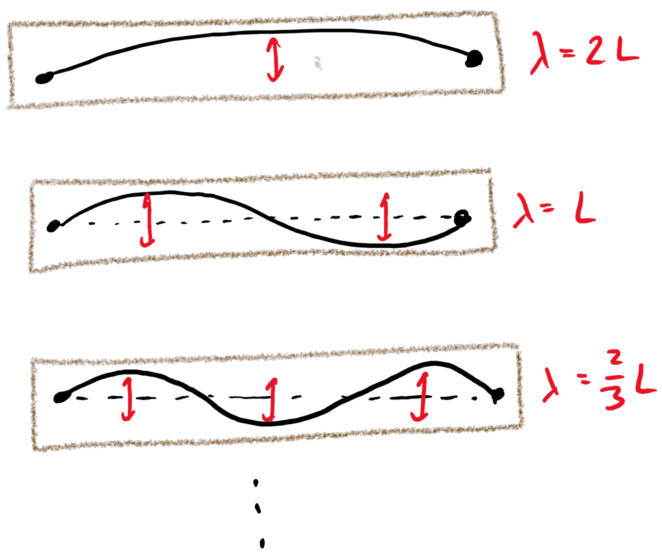
$\cos(kx) = 0$

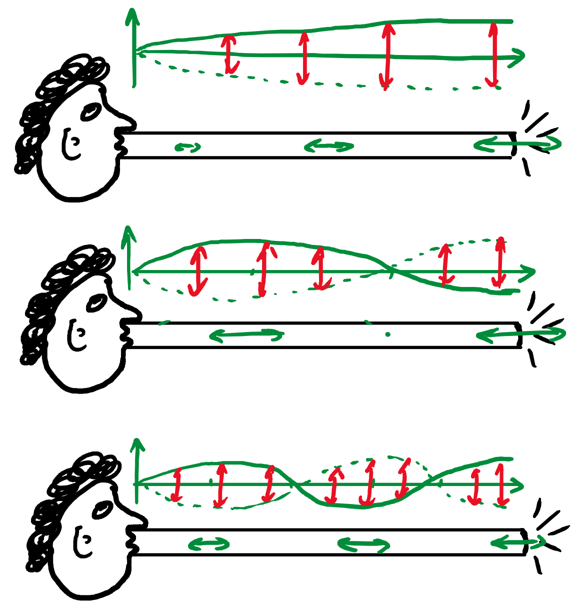
Antinodes: oscillates w. maximum displacement

$\cos(kx) = \pm 1$

= sum of left-moving wave + right-moving wave

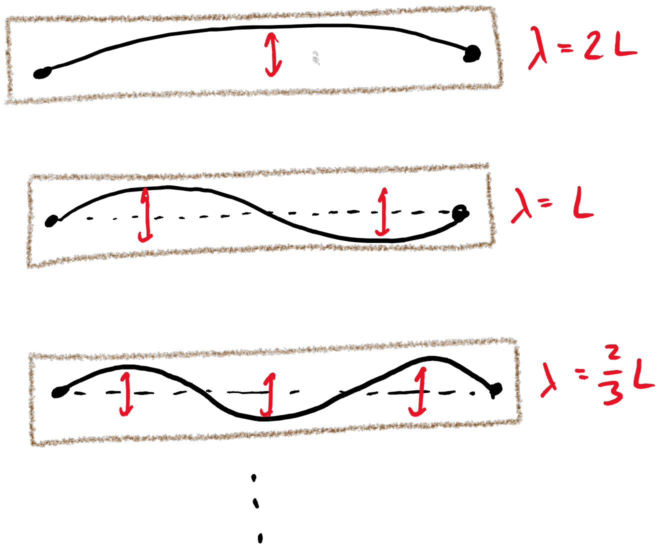
Musical Instruments:

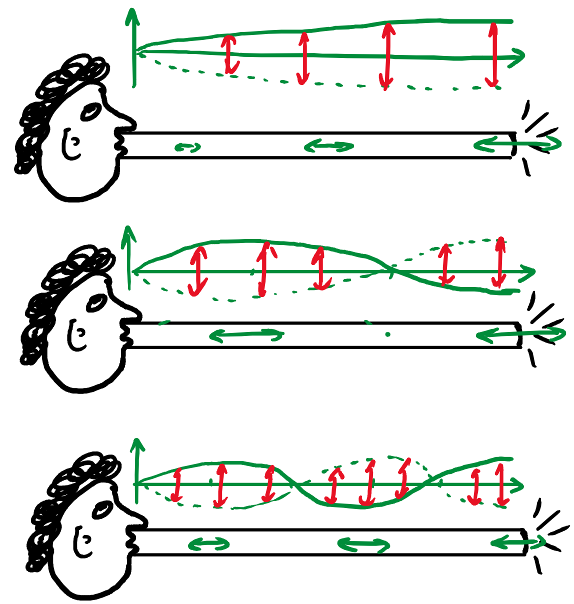




$$f = \frac{v}{\lambda}$$

Musical Instruments:





$$f = \frac{v}{\lambda}$$ → determined by properties of the medium

THE WAVE EQUATION:

Q: What determines v and $D(x,t)$?

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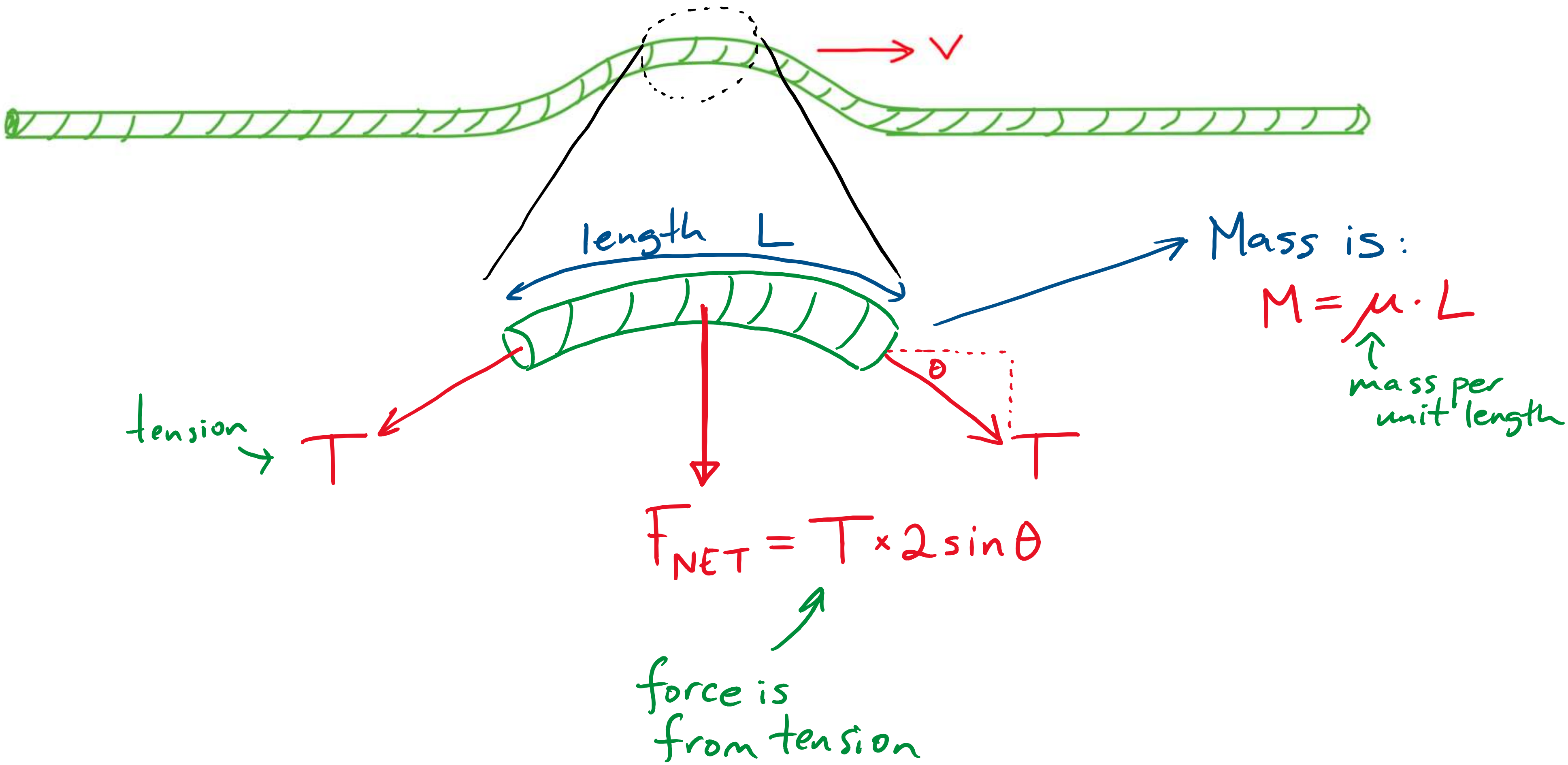
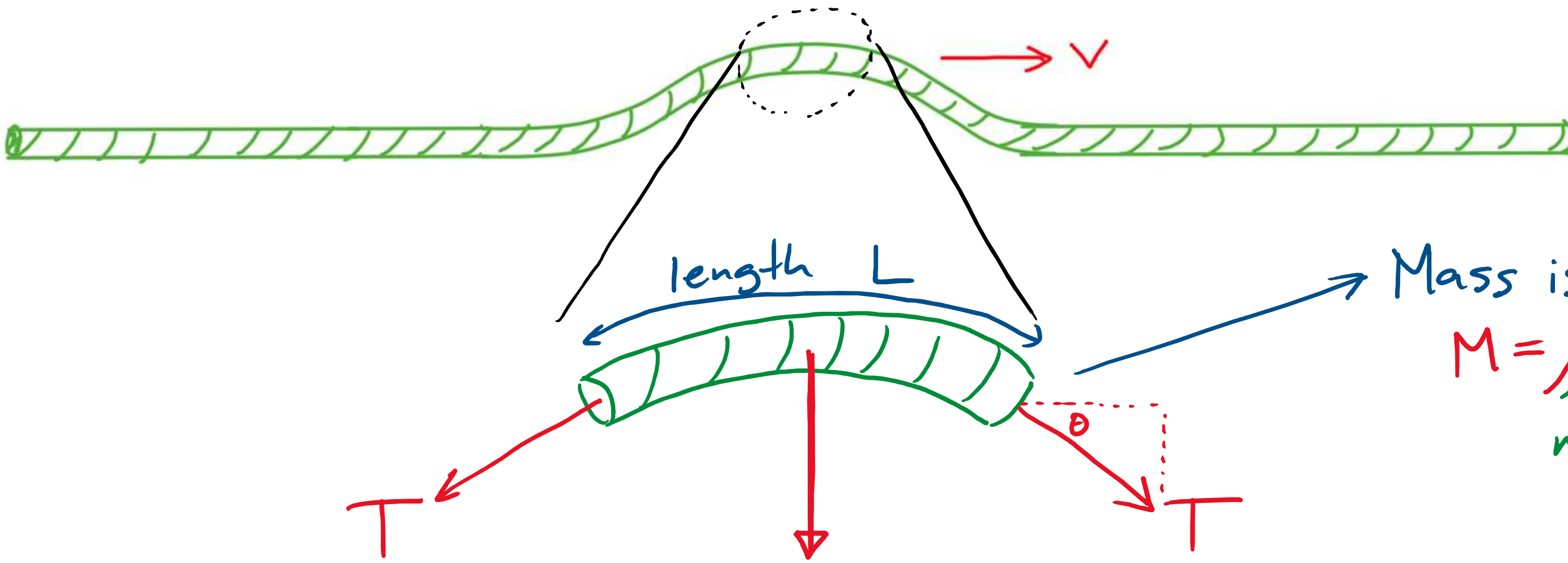
A: Newton's 2nd Law!

THE WAVE EQUATION:



THE WAVE EQUATION:



Mass is:

$M = \mu \cdot L$

mass per unit length

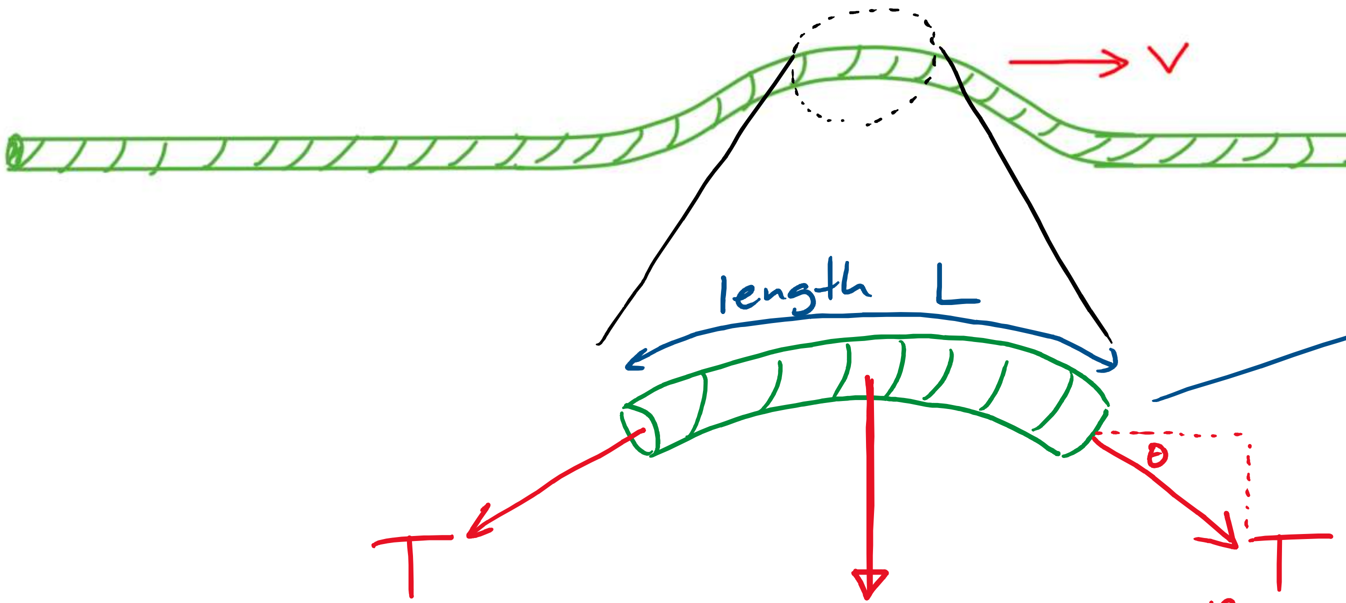
$$F_{NET} = T \times 2 \sin\theta = T \times L \frac{d^2 D}{dx^2}$$

force is from tension

angle is smaller if we look at smaller part

no net force unless rope is curved

THE WAVE EQUATION:



Mass is:

$$M = \mu \cdot L$$

mass per unit length

$$F_{NET} = T L \frac{d^2D}{dx^2}$$

Newton's 2nd Law:

$$a = \frac{F_{NET}}{m} \Longrightarrow \boxed{\frac{d^2D}{dt^2} = \frac{T}{\mu}\frac{d^2D}{dx^2}}$$

THE WAVE EQUATION:

$$\boxed{\frac{d^2D}{dt^2} = \frac{T}{\mu}\frac{d^2D}{dx^2}}$$

determines motion of wave once we specify $D(x, t=0)$ and $\frac{d}{dt}D(x, t=0)$

general solution is sum of $F_1(x-vt)$ → right-moving wave

and $F_2(x+vt)$ → left-moving wave

with $v = \sqrt{\frac{T}{\mu}}$

Example : Which note started the Royal Singing and Hopping Race?

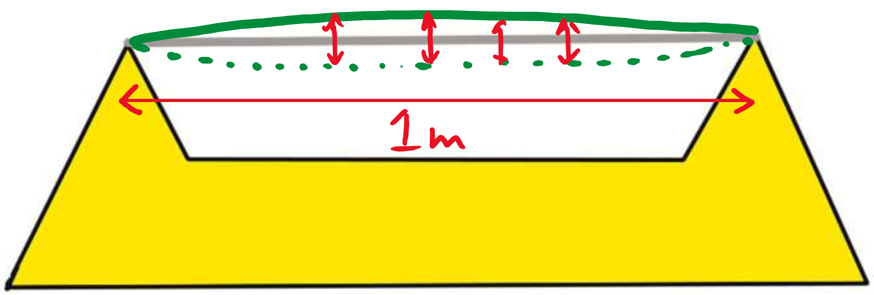
$T = 800N$

wire diameter : 1mm

density of platinum : $2.14 \times 10^4 \, kg/m^3$

gives $\mu = 0.0168 \, kg/m$

Question 1:

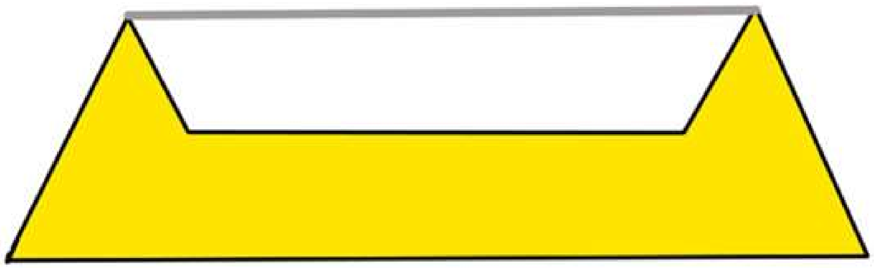


You are the Royal Engineer for the Kingdom of Grrrrrx (pronounced as written). Each year in the kingdom, on the last day of summer, a new Knightship of Grrrrrx is awarded to the winner of the Royal Singing-and-Hopping Race, in which participants (18 years of age and older) must hop and sing through three full laps of the castle perimeter, adhering to the strict regulations of the Royal Singing-and-Hopping Commission.

$\star \; v = \sqrt{\frac{T}{\mu}} \; \star$

$\star \; \lambda \cdot f = v \; \star$

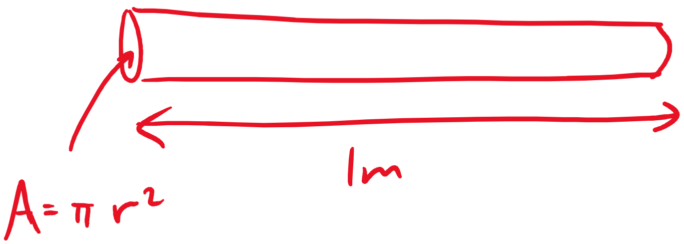
The race begins when the King of Grrrrrx plucks a single note on the Royal Plucking Instrument, which consists of a single 1mm thick platinum wire stretched between two points on a solid gold frame, as shown in the picture. To achieve the proper note, the wire must be at a tension of 800N. On the morning of the race, you notice the temperature is a chilly 5 degrees Celcius . . .



1 m of wire has volume $V = L \cdot A = 1m \cdot \pi \, (0.0005m)^2$

mass: $m = \rho \times V$

$= 2.145 \times 10^4 \, kg/m^3 \times$



So $\mu = 0.0168 \, kg/m$

Then: $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{800N}{0.0168 \, kg/m}} = 218 \, m/s$

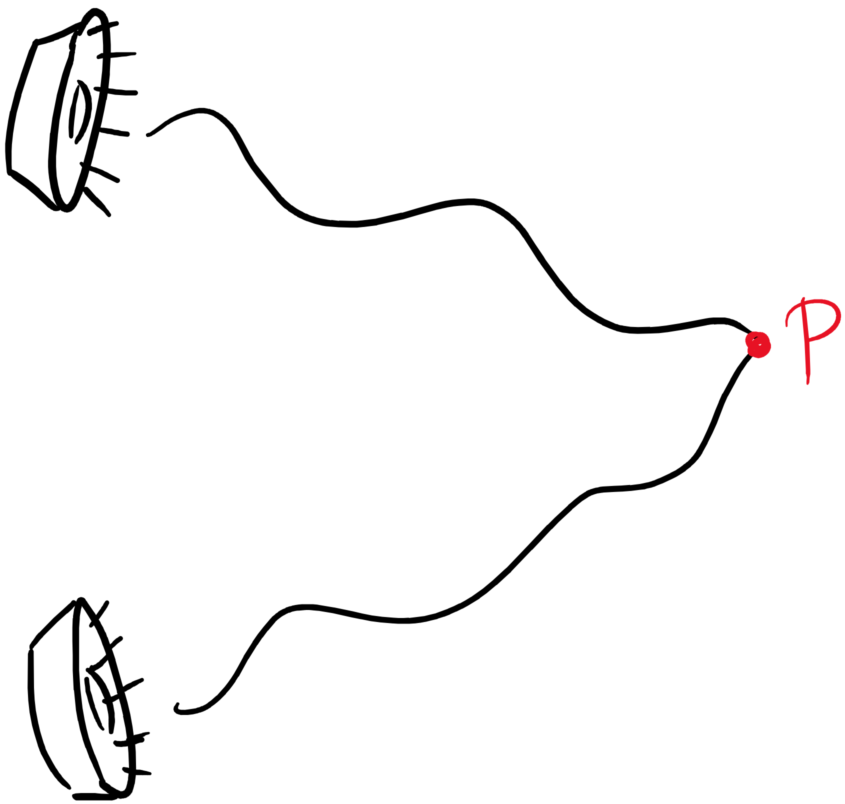
For $\lambda = 2m$, we get $f = \frac{v}{\lambda} = 109 \, Hz$

Another consequence of the superposition principle:

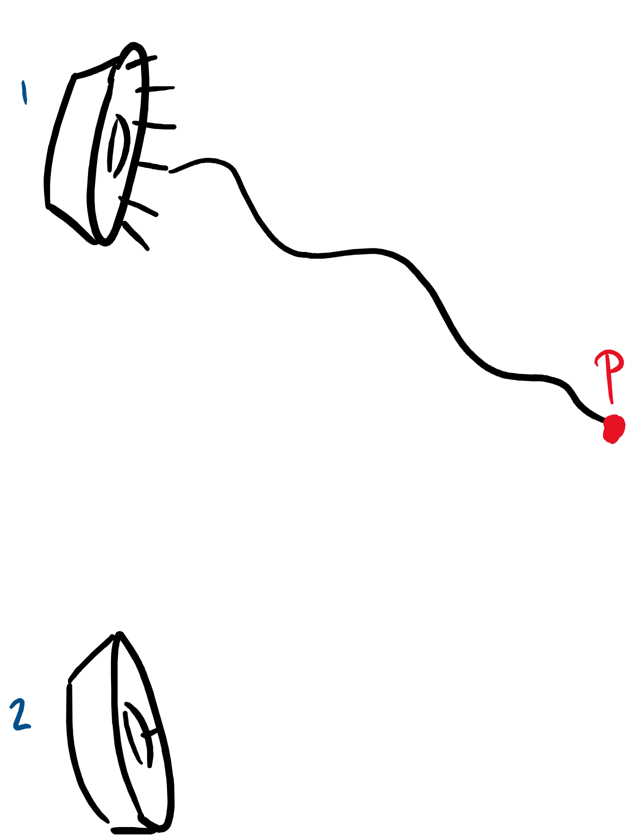
INTERFERENCE of waves

kind of a bad name...

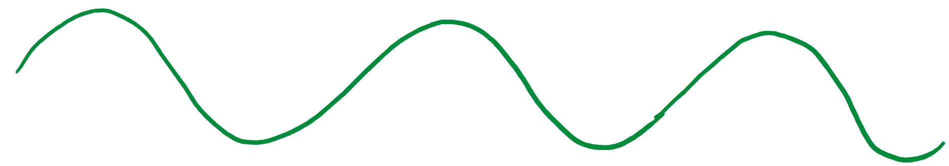
Waves from 2 sources:

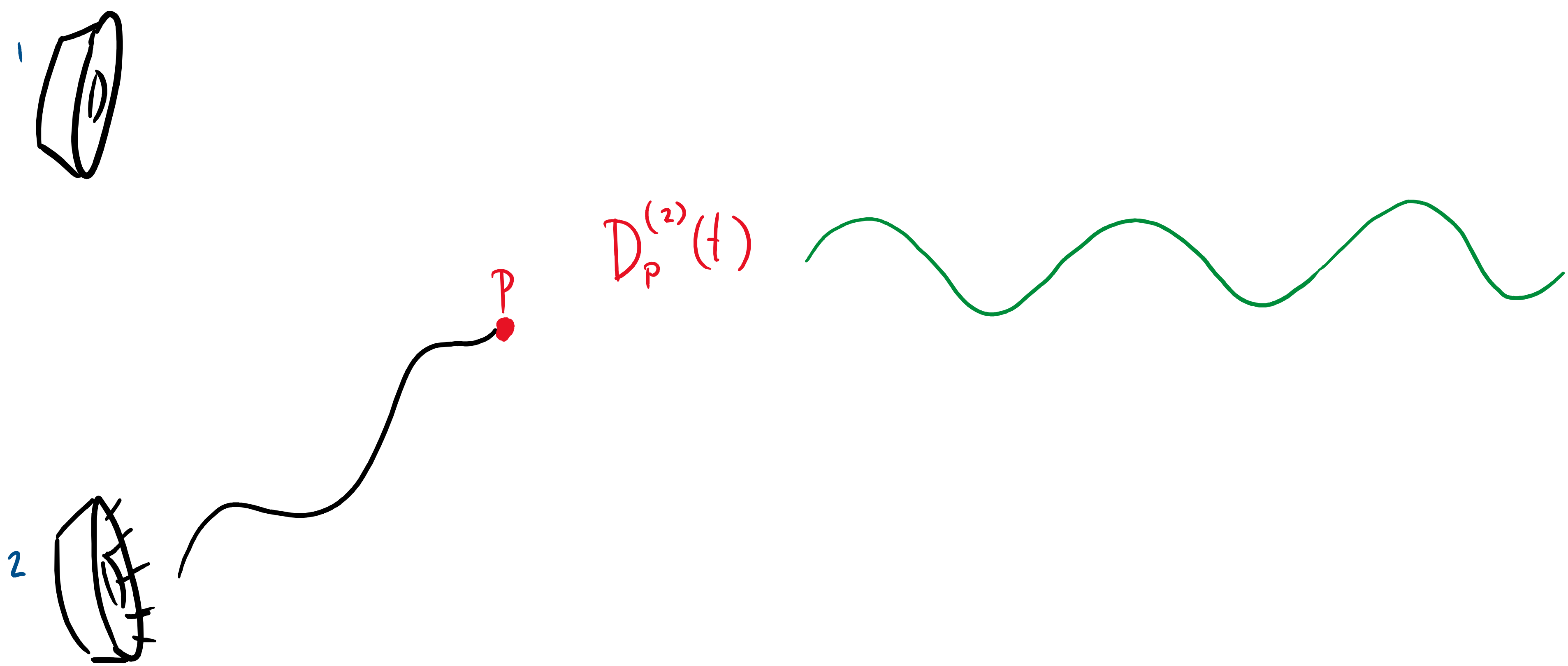


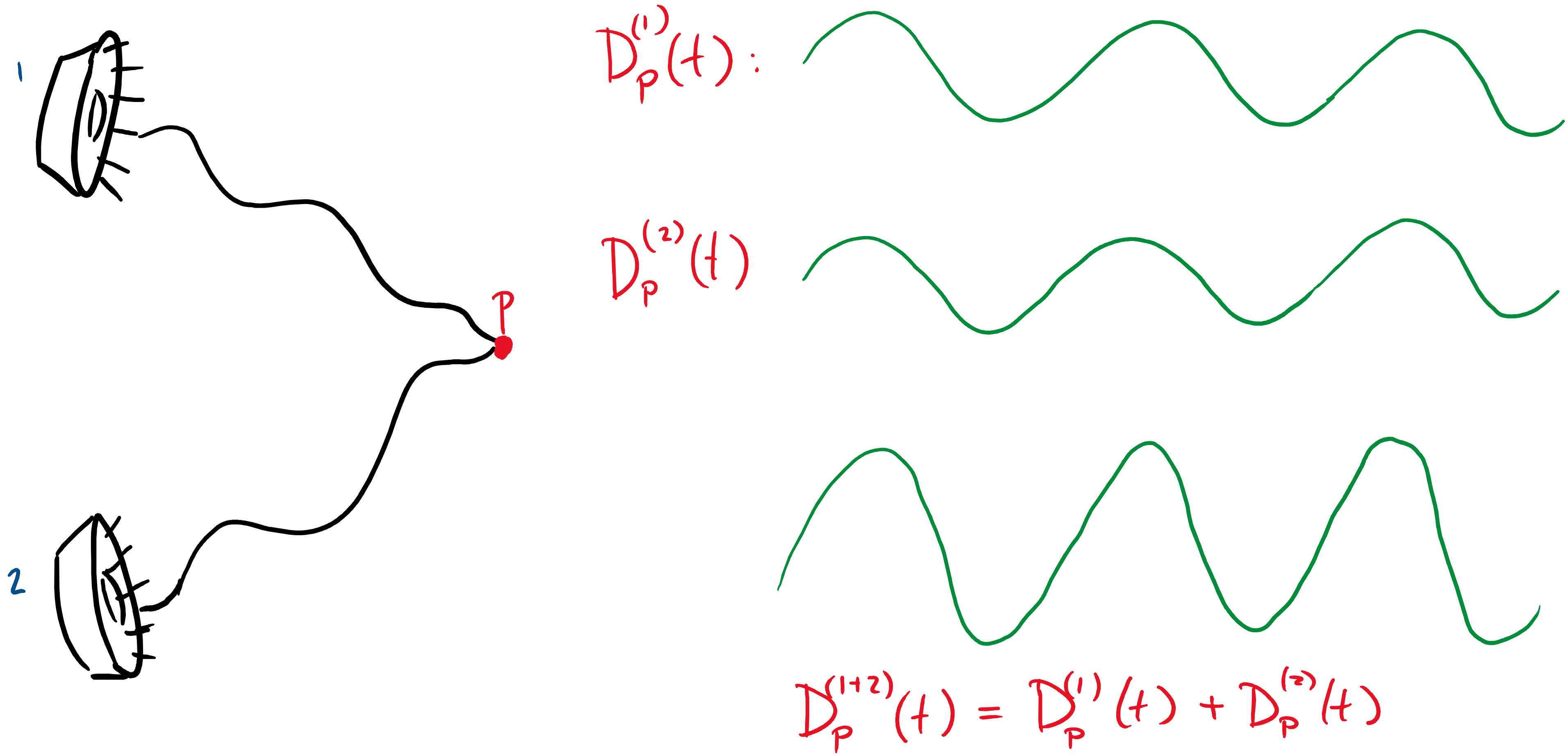
Displacement at point P is the sum of the displacements from the two individual waves.



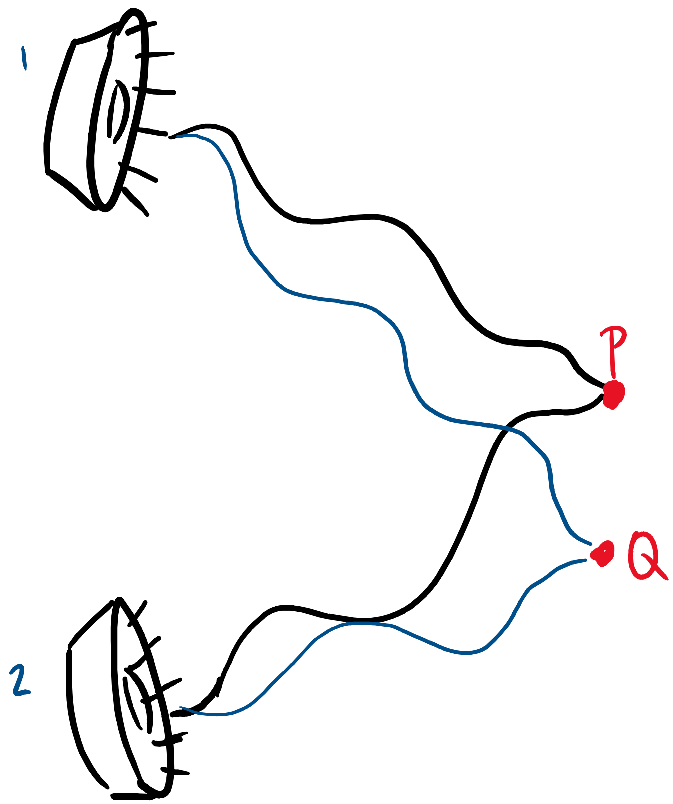
$D_P^{(1)}(t):$




1
2
P
$D_P^{(2)}(t)$

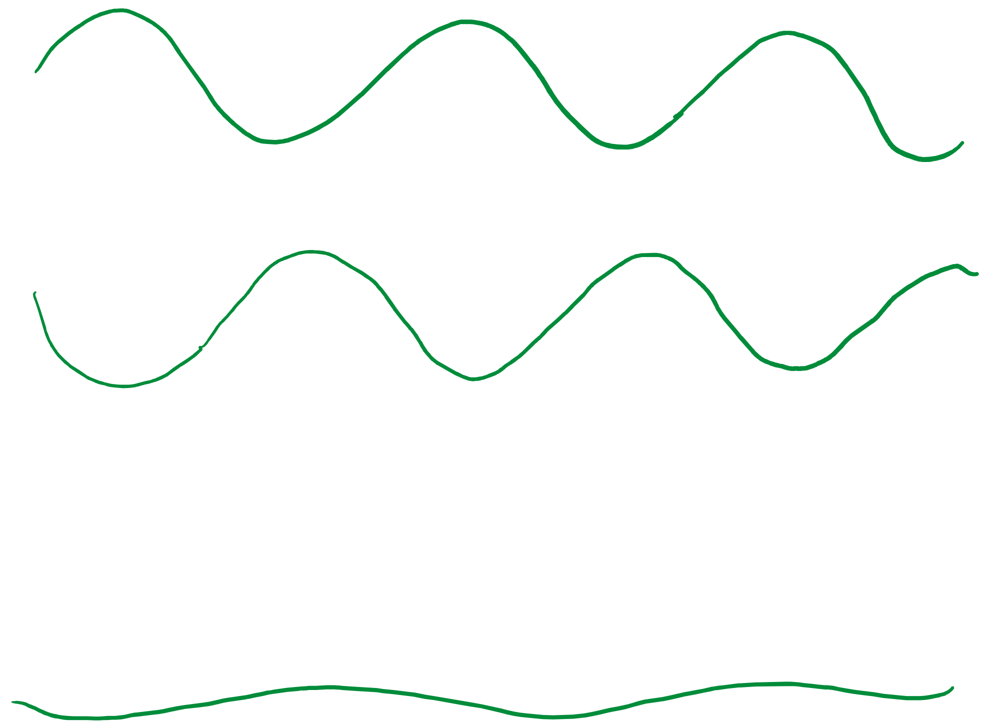


★ CONSTRUCTIVE INTERFERENCE ★



$D_Q^{(1)}(t)$:

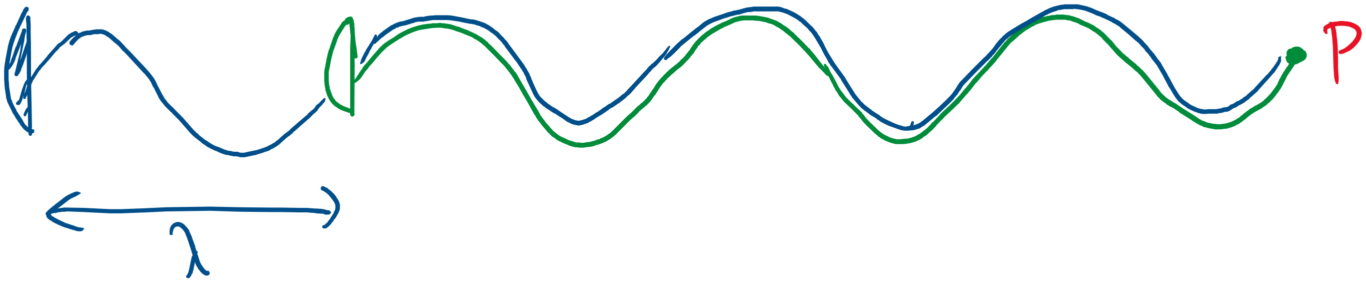
$D_Q^{(2)}(t)$



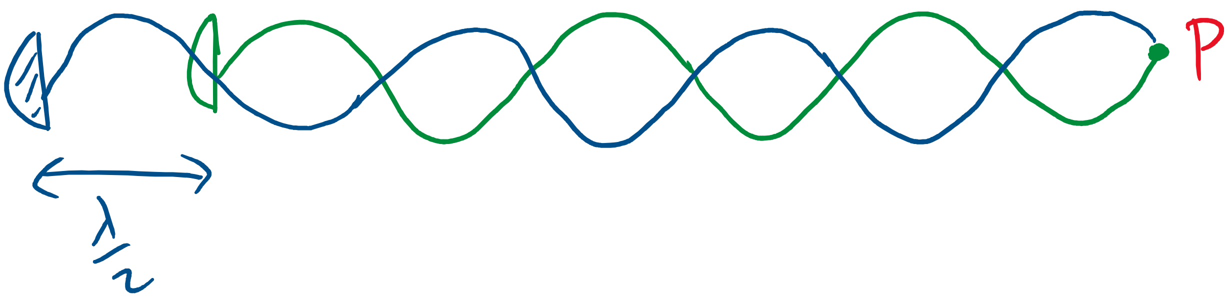
$$D_Q^{(1+2)}(t) = D_Q^{(1)}(t) + D_Q^{(2)}(t)$$

★ DESTRUCTIVE INTERFERENCE ★

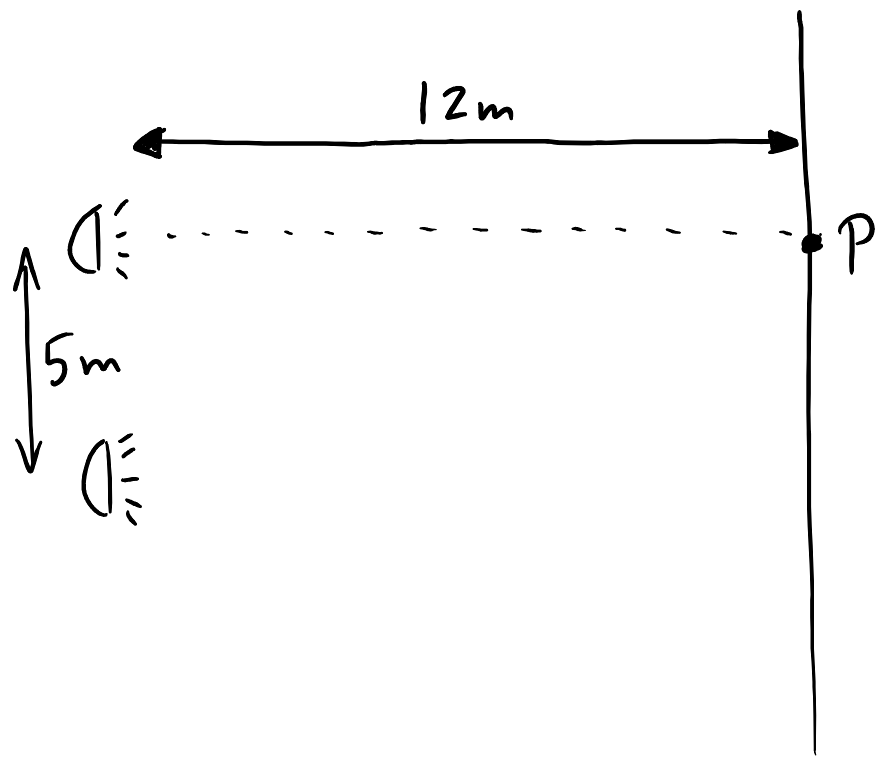
For in phase sources: have constructive interference when distances to 2 sources differ by $\lambda, 2\lambda, 3\lambda, \ldots$



Destructive interference when distances to 2 sources differ by $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$

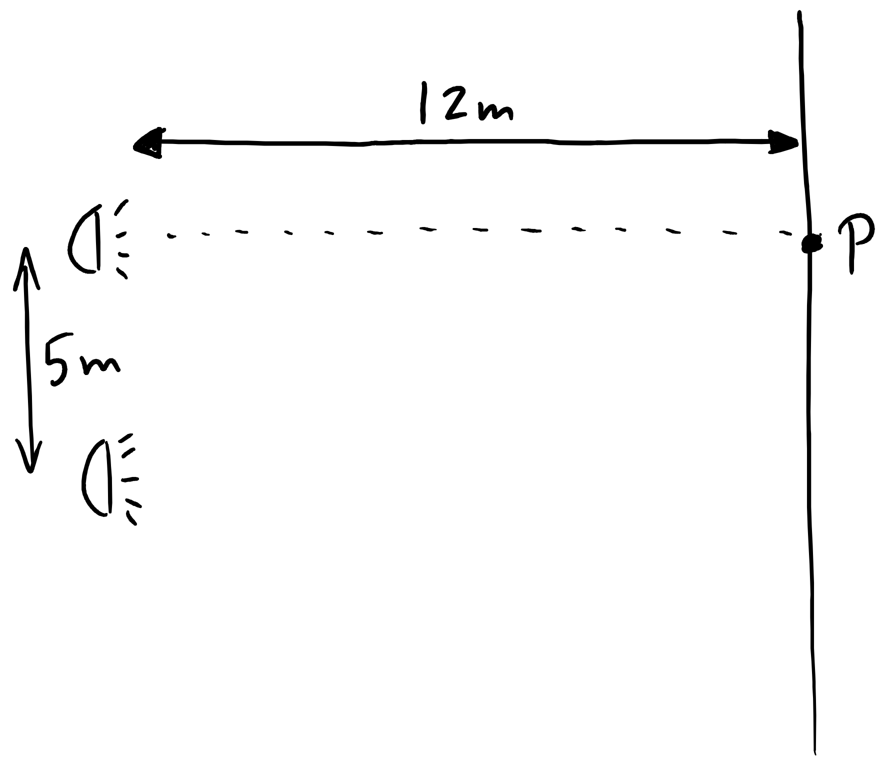


DEMO



Q : For what frequencies will there be constructive interference at P?

($v_{sound} = 340\,m/s$)



Q : For what frequencies will there be constructive interference at P?

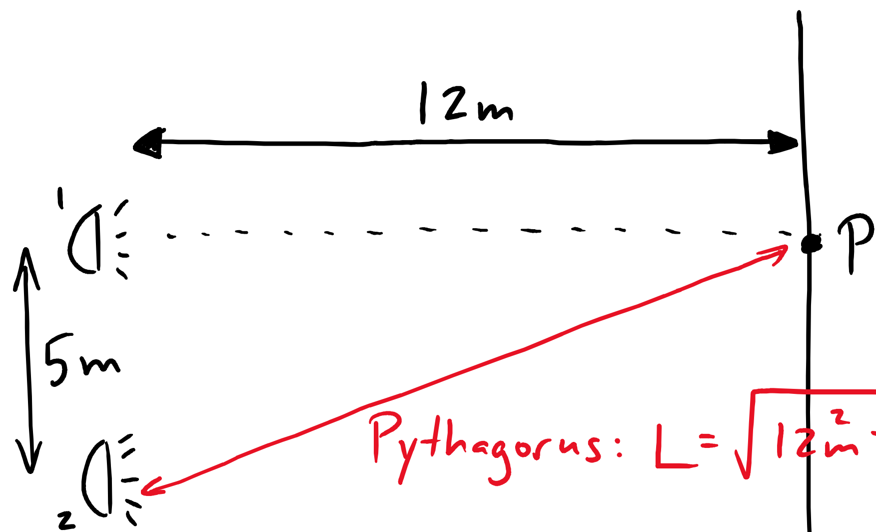
($v_{sound} = 340 \text{ m/s}$)

What is the LOWEST freq. for constructive interference?

A) 170 Hz B) 340 Hz

C) 680 Hz D) 510 Hz

E) 68 Hz



Q : For what frequencies will there be constructive interference at P?

Pythagorus: $L = \sqrt{12m^2 + 5m^2} = 13m$

Path length difference: 1m

Constructive interference if $1m = \lambda, 2\lambda$, etc...

So: $\lambda = \frac{1m}{n}$

$$f = \frac{v}{\lambda} = 340\,Hz \times n$$