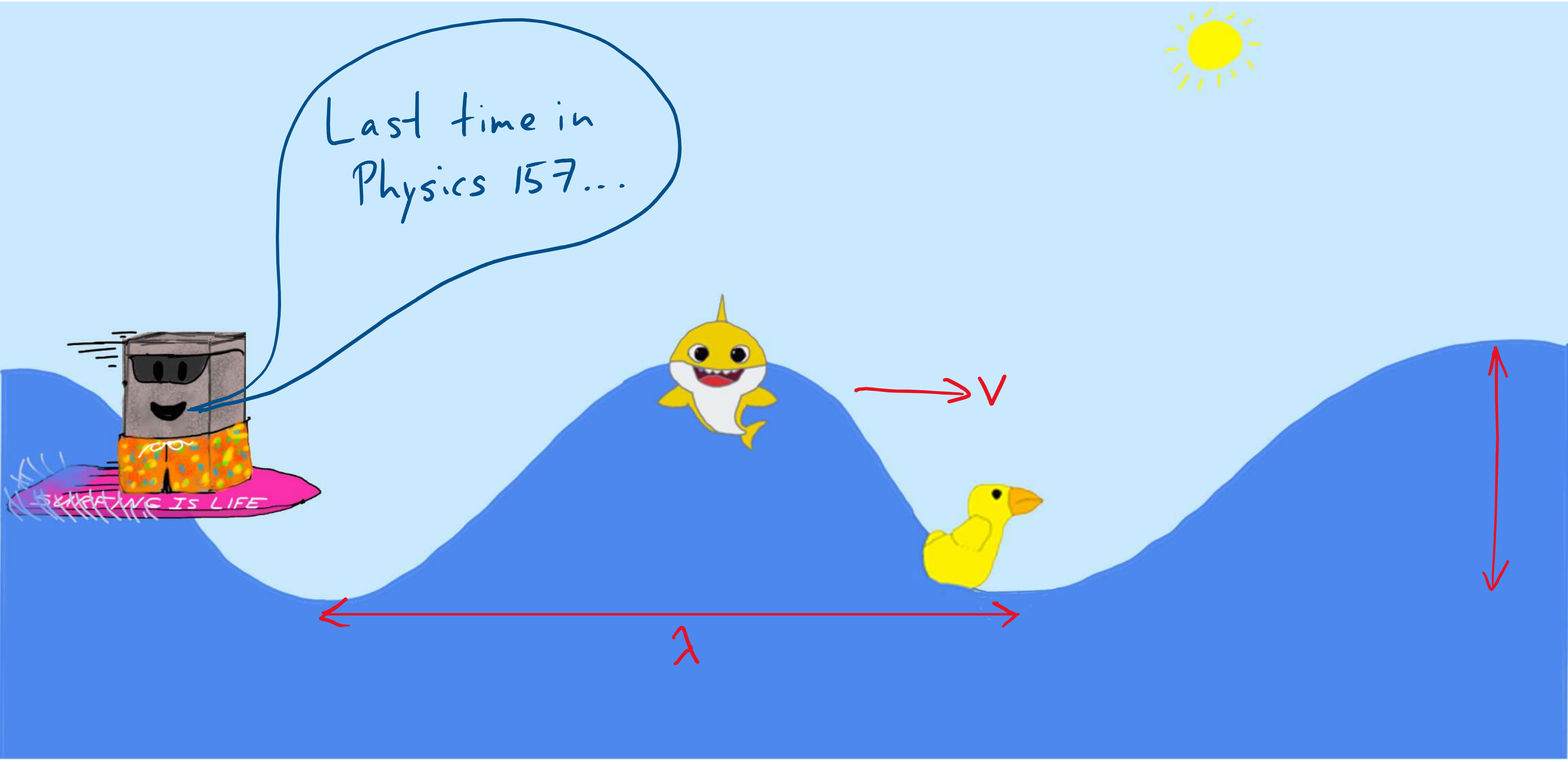
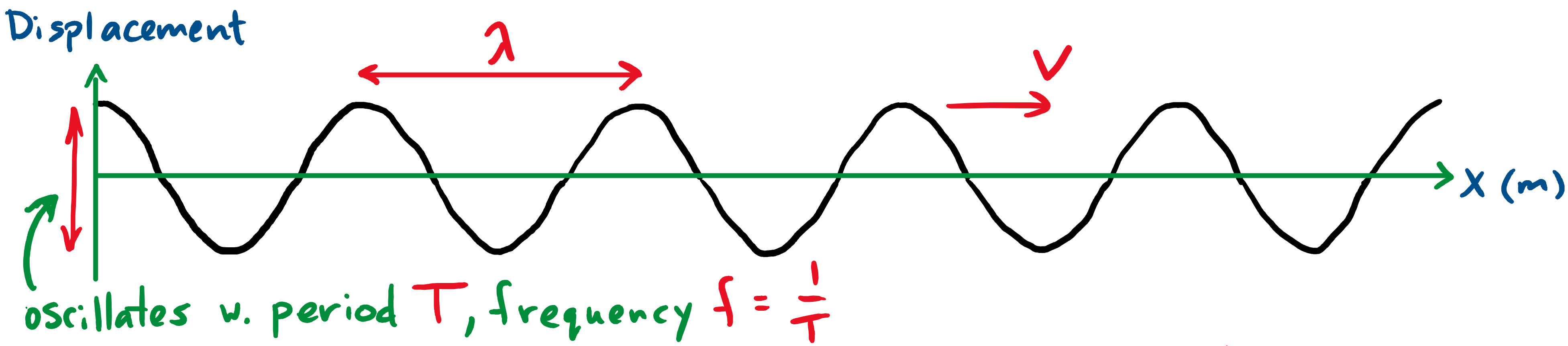

Last time in Physics 157...
SURFING IS LIFE
v
λ

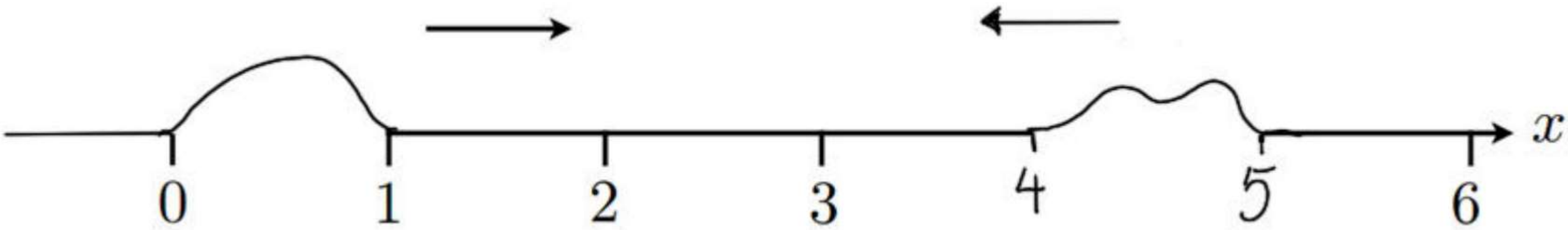
Mathematical Description of Traveling Sinusoidal Waves



Right moving wave: $D(x,t) = A\cos(kx - \omega t)$

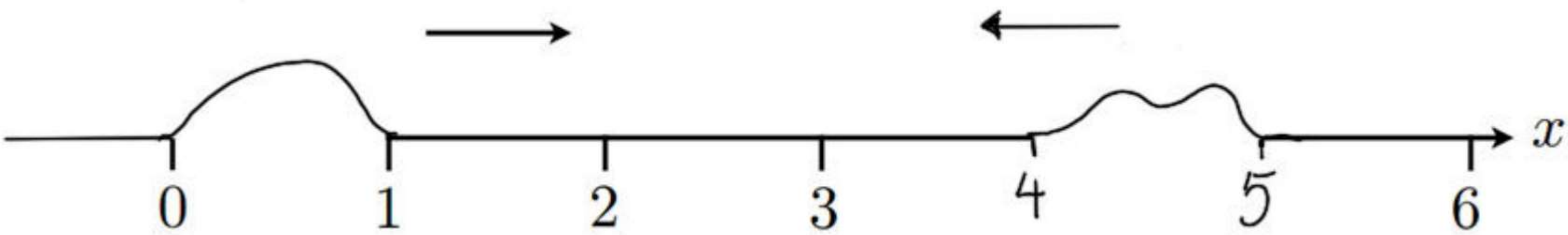
Left moving wave: $D(x,t) = A\cos(kx + \omega t)$

$k = \frac{2\pi}{\lambda}$ $\omega = \frac{2\pi}{T}$ $v = \frac{\lambda}{T}$



Two wave pulses are travelling towards each other as shown. When they meet, they will:

A) Bounce off each other and reflect backwards
B) Destroy each other, leaving a few random ripples going in either direction
C) Pass right through each other



Two wave pulses are travelling towards each other as shown. When they meet, they will:

A) Bounce off each other and reflect backwards
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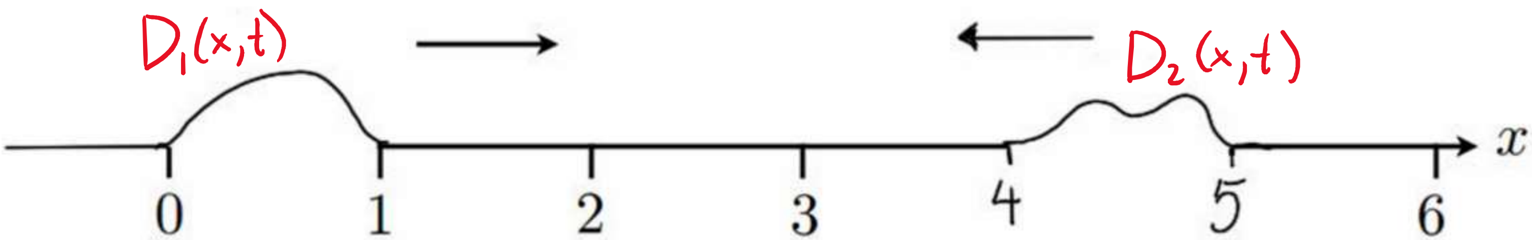
The Principle of Superposition

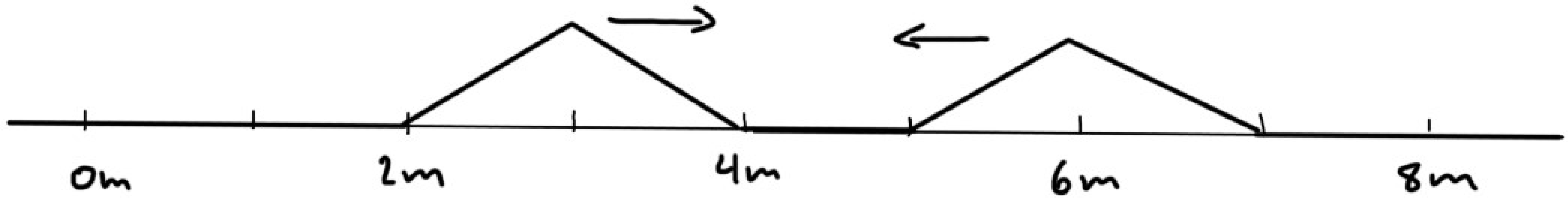
When two or more waves overlap, the net displacement D(x,t) is equal to the sum of the displacements we would have if each wave were present alone.



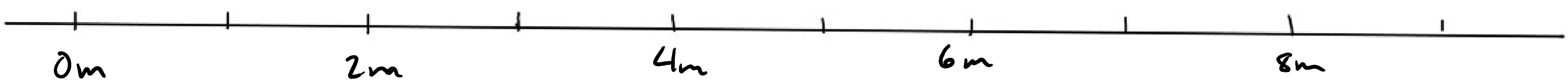
Combined wave: $D(x,t) = D_1(x,t) + D_2(x,t)$

★ waves add without disturbing each other ★

Example:

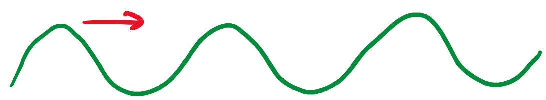


Two wave pulses, each traveling 1m/s, approach each other on a string. Sketch the displacement of the string after 1 second has passed.

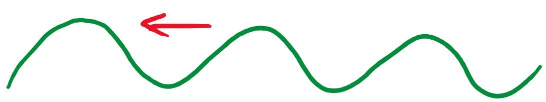


Application: standing waves

Right-moving wave: $A\cos(kx-\omega t)$



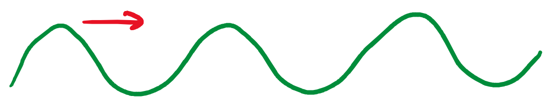
Left-moving wave: $A\cos(kx+\omega t)$



What if both are present on the same string?

Application: standing waves

Right-moving wave: $A\cos(kx-\omega t) \quad = \quad A\cos(kx)\cos(\omega t) + A\sin(kx)\sin(\omega t)$



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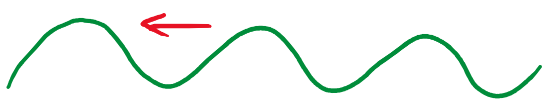
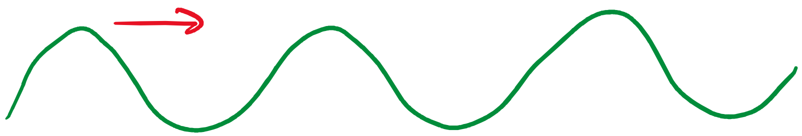


Application: standing waves

Right-moving wave: $A\cos(kx-\omega t) \quad = \quad A\cos(kx)\cos(\omega t) + A\sin(kx)\sin(\omega t)$



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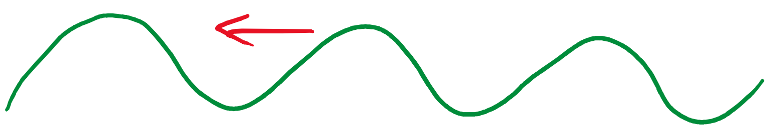
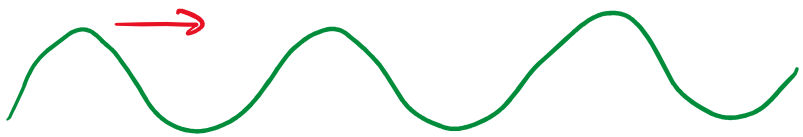
Left-moving wave: $A\cos(kx+\omega t) \quad = \quad A\cos(kx)\cos(\omega t) - A\sin(kx)\sin(\omega t)$



Application: standing waves

Right-moving wave: $A\cos(kx-\omega t) \quad = \quad A\cos(kx)\cos(\omega t) + A\sin(kx)\sin(\omega t)$



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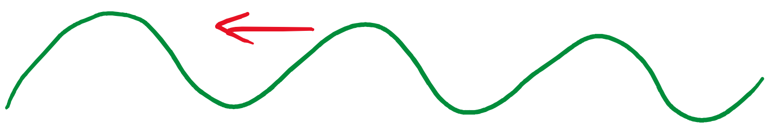
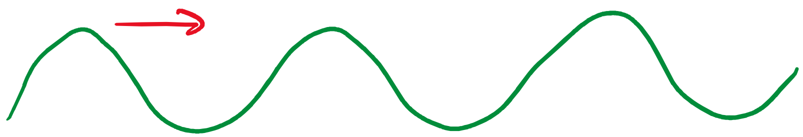
Left-moving wave: $A\cos(kx+\omega t) \quad = \quad A\cos(kx)\cos(\omega t) - A\sin(kx)\sin(\omega t)$



sum: $2A\cos(kx)\cos(\omega t)$

Application: standing waves

Right-moving wave: $A\cos(kx-\omega t) = A\cos(kx)\cos(\omega t) + A\sin(kx)\sin(\omega t)$



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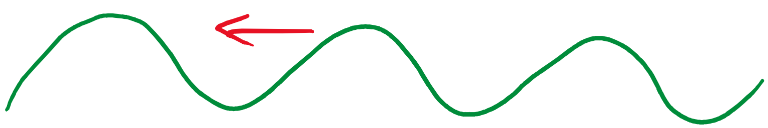

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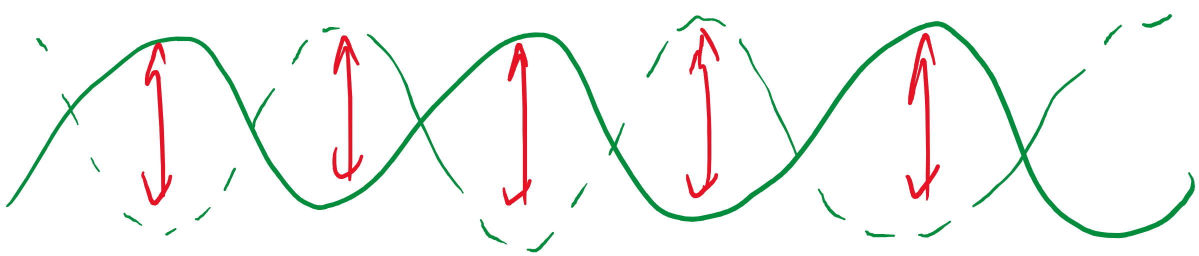
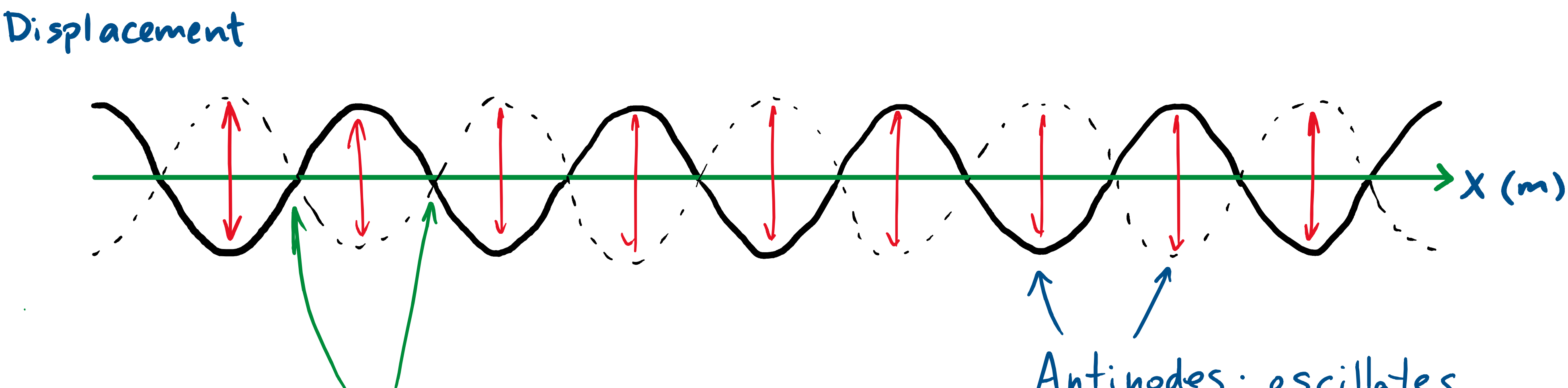


= STANDING WAVE

Standing Waves

$$D(x,t) = A\cos(kx)\cdot\cos(\omega t)$$



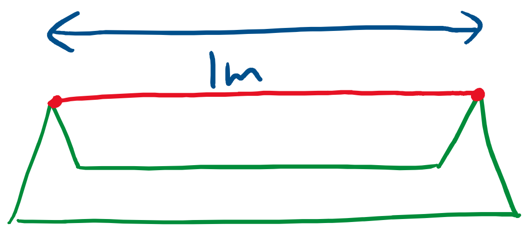
Nodes: displacement fixed at 0

$\cos(kx) = 0$

Antinodes: oscillates w. maximum displacement

$\cos(kx) = \pm 1$

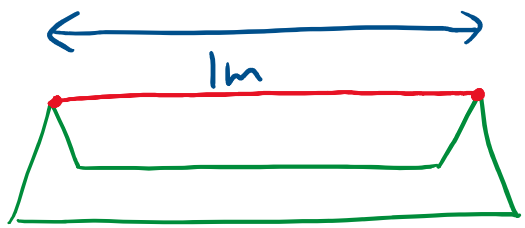
Example: guitar/violin string - displacement must be zero at ends



Q: What are the possible wavelengths for standing waves on a guitar string w. length 1 m?

(Hint: draw the shapes of the possible waves?)

Example: guitar/violin string – displacement must be zero at ends



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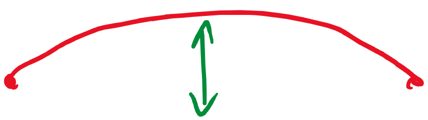
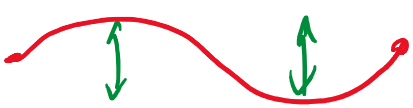
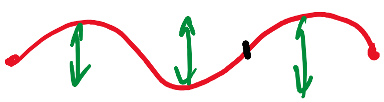


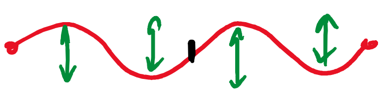
$\lambda = 2m$



$\lambda = 1m$



$\lambda = \frac{2}{3}m$

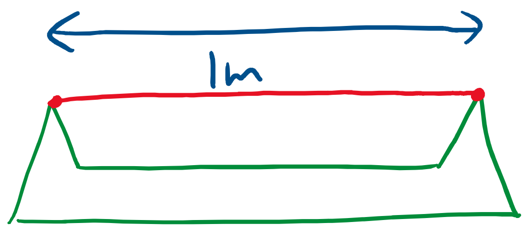


$\lambda = \frac{1}{2}m$

generally: $\lambda = \frac{2m}{n}$

Demo

Example: guitar/violin string – displacement must be zero at ends



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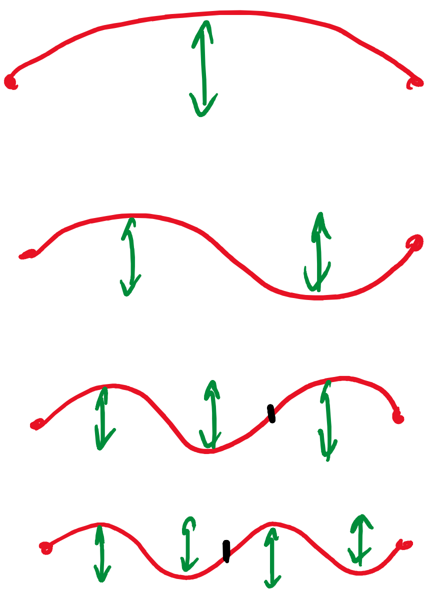


$\lambda = 2m$

$\lambda = 1m$

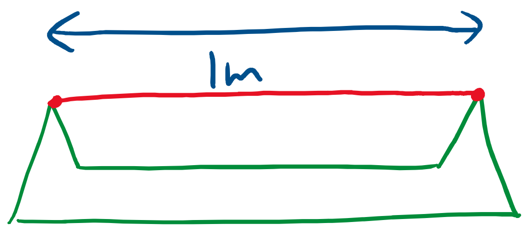
$\lambda = \frac{2}{3} m$

$\lambda = \frac{1}{2} m$

generally: $\lambda = \frac{2m}{n}$

How do we calculate the frequencies?

Example: guitar/violin string – displacement must be zero at ends



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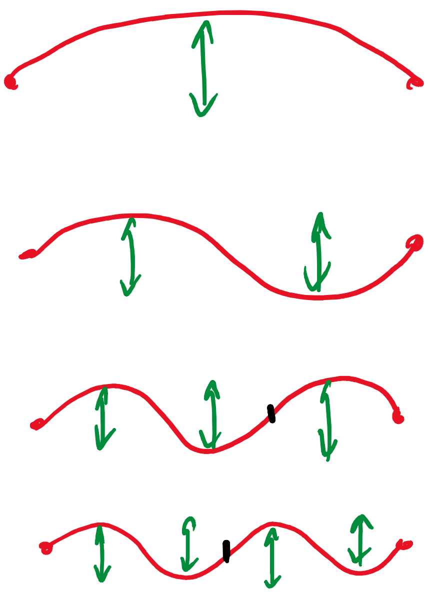


$\lambda = 2m$

$\lambda = 1m$

$\lambda = \frac{2}{3}m$

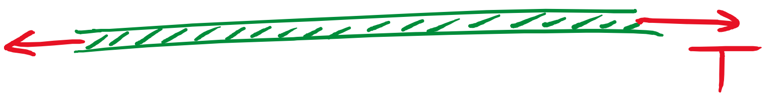
$\lambda = \frac{1}{2}m$

generally: $\lambda = \frac{2m}{n}$

How do we calculate the frequencies?

☆ use $f = \frac{v}{\lambda}$ ☆

Important: v depends only on properties of our string,
does not depend on λ, f

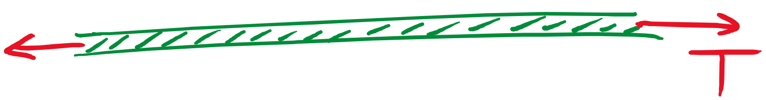


depends on tension T units $N = \frac{kg\,m}{s^2}$

depends on mass/length μ units $\frac{kg}{m}$

Which combination of these has units of velocity?

Important: v depends only on properties of our string, does not depend on λ, f



depends on tension T units $N = \frac{kg\,m}{s^2}$

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Which combination of these has units of velocity?

$v = \sqrt{\frac{T}{\mu}}$ has right units (+ is the right answer!)

"Dimensional Analysis" detailed derivation in 15.4

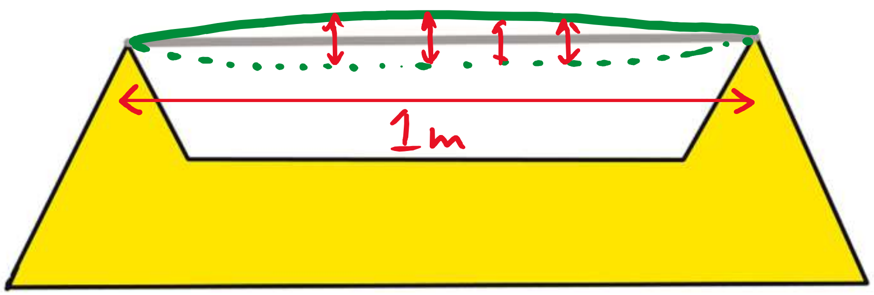
Example: Which note started the Royal Singing and Hopping Race?

$T = 800N$

wire diameter: 1mm

density of platinum: $2.14 \times 10^4 kg/m^3$

Question 1:



You are the Royal Engineer for the Kingdom of Grrrrrx (pronounced as written). Each year in the kingdom, on the last day of summer, a new Knightship of Grrrrrx is awarded to the winner of the Royal Singing-and-Hopping Race, in which participants (18 years of age and older) must hop and sing through three full laps of the castle perimeter, adhering to the strict regulations of the Royal Singing-and-Hopping Commission.

The race begins when the King of Grrrrrx plucks a single note on the Royal Plucking Instrument, which consists of a single 1mm thick platinum wire stretched between two points on a solid gold frame, as shown in the picture. To achieve the proper note, the wire must be at a tension of 800N. On the morning of the race, you notice the temperature is a chilly 5 degrees Celcius . . .