Last time in Physics 157...

OFFICE HOURS TODAY:

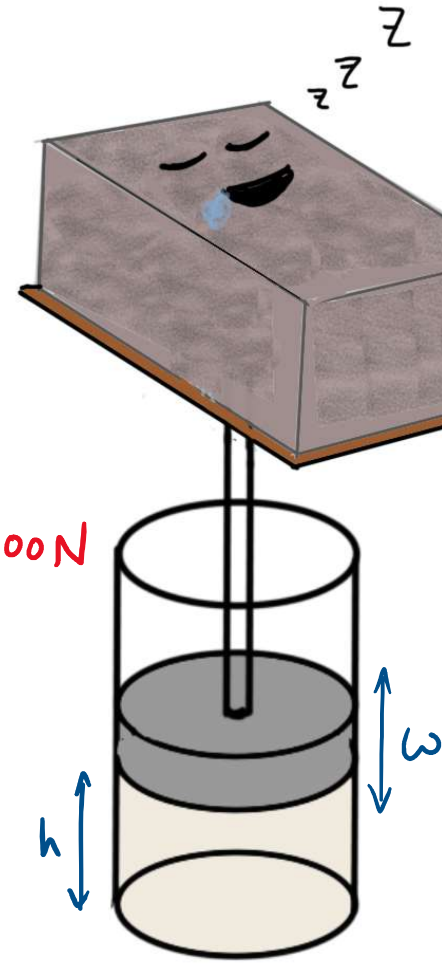
3:30 - 5:00pm

Hennings 420

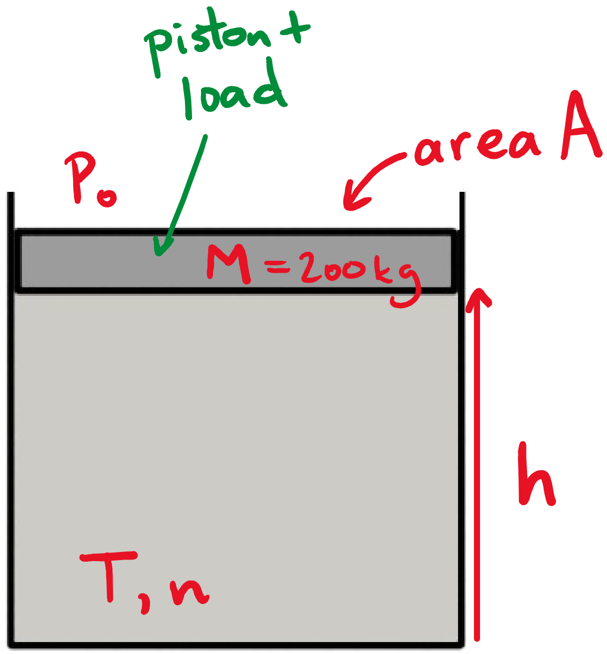
$$F_{NET} = \frac{1500\,J}{h} - 5000\,N$$

What is $\omega$?

($M = 200\,kg$)




# Example: air leg



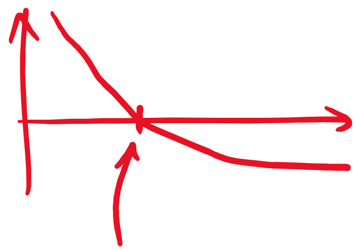

$$F_{NET} = \frac{1500\,J}{h} - 5000\,N$$

d) What is the equilibrium height of the piston?

equilibrium height: $F_{NET} = 0$

$$h_{eq} = 0.3\,m$$

e) What is the oscillation frequency f?



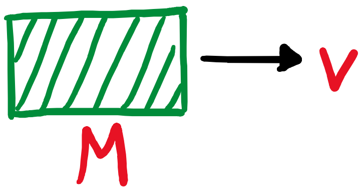
$$k = -\frac{dF}{dh} \text{ at } h_{eq} = \frac{1500\,J}{h_{eq}^2} = 1.67 \times 10^4\,N/m$$

$$\longrightarrow \quad \omega = \sqrt{\frac{k}{m}} = 9.1\,s^{-1}$$

$$f = \frac{\omega}{2\pi} = 1.45\,s^{-1}$$

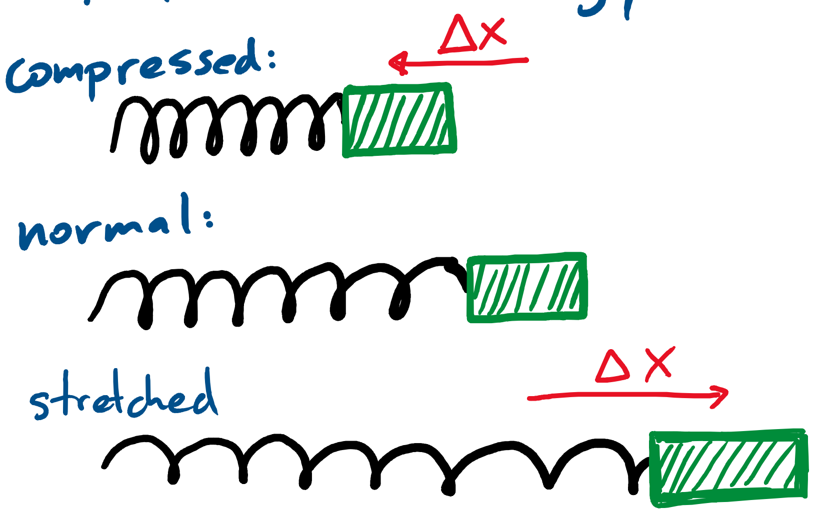
# Energy in S.H.M.

## Kinetic energy:



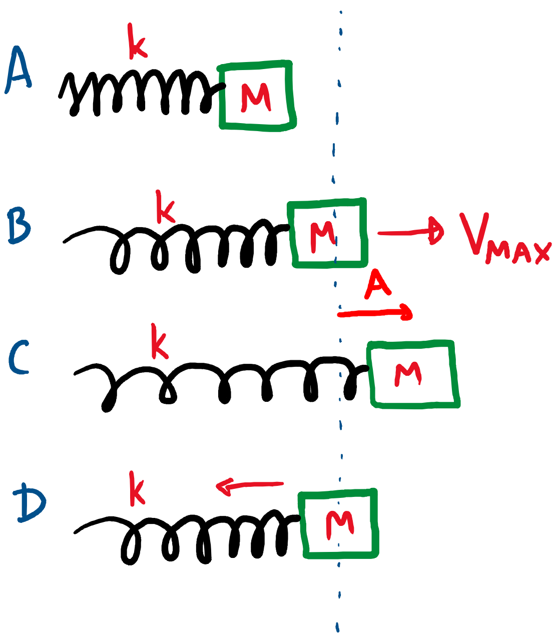

$$\boxed{K.E. = \frac{1}{2} M v^2}$$

## Potential Energy relative to equilibrium:




$$\boxed{P.E. = \frac{1}{2} k (\Delta x)^2}$$

# Energy in simple harmonic motion:




Energy at B relative to mass at equilibrium:

$$\frac{1}{2} M v_{MAX}^2$$

Energy at C: same: $\frac{1}{2} M v_{MAX}^2$

Rewrite in terms of $k$ and $A$:

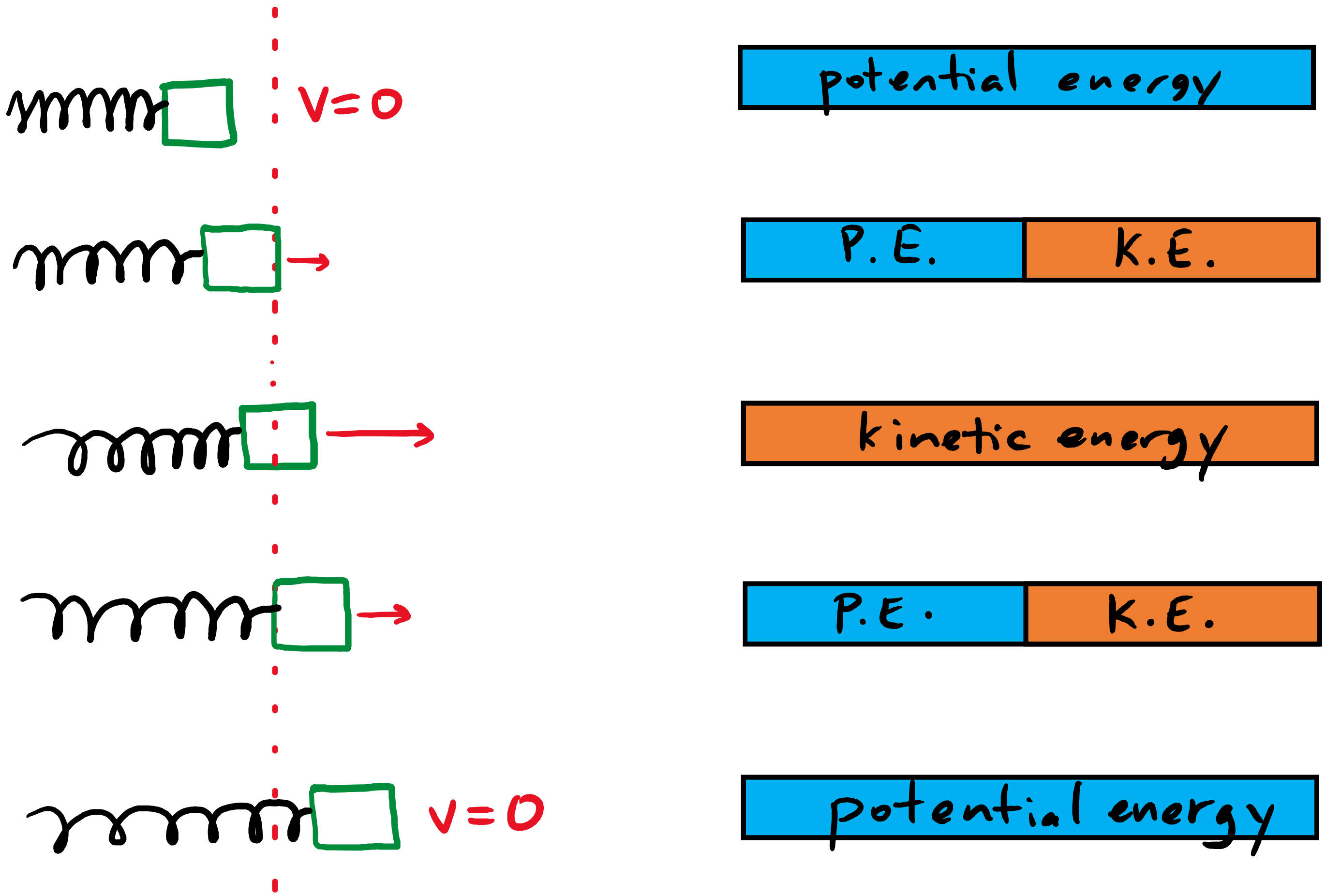
$$V_{MAX} = A\omega = A\sqrt{\frac{k}{M}}$$

$$\frac{1}{2} M v_{MAX}^2 = \frac{1}{2} M \times \left(A \times \sqrt{\frac{k}{m}}\right)^2 = \frac{1}{2} k A^2$$

this must be the formula for potential energy when $\Delta x = A$

Total energy is conserved $\frac{1}{2}Mv^2 + \frac{1}{2}kx^2 = E$ constant

equal to initial energy



A 0.5 kg mass is attached to a horizontal spring of spring constant 200 N/m. If the spring is initially compressed by 0.1m, and the mass is then released, what is the speed of the block when the spring is at its equilibrium length?

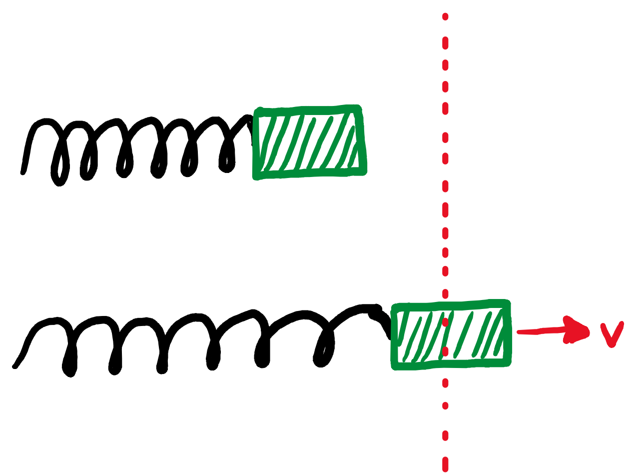
A. 1 m/s

B. 2 m/s

C. 3 m/s

D. 4 m/s

E. 5 m/s



M k

A 0.5 kg mass is attached to a horizontal spring of spring constant 200 N/m. If the spring is initially compressed by 0.1m, and the mass is then released, what is the speed of the block when the spring is at its equilibrium length?

Δx

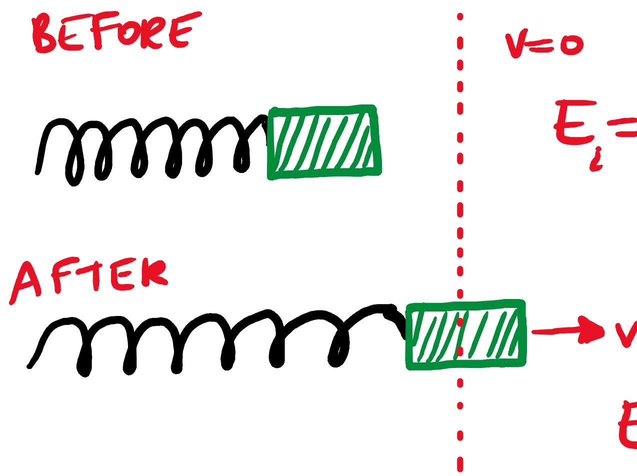
A. 1 m/s

B. 2 m/s

C. 3 m/s

D. 4 m/s

E. 5 m/s



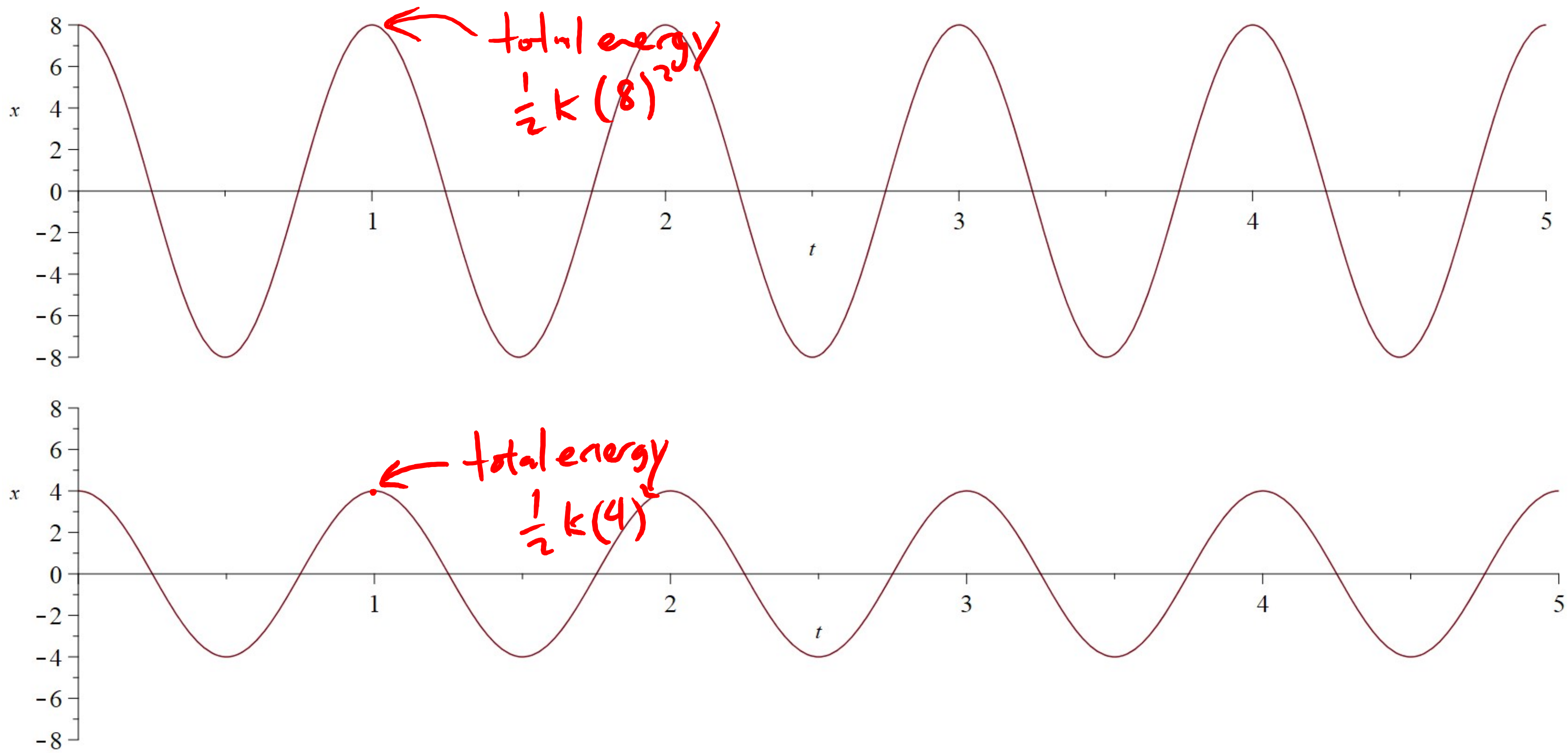

$v=0$

$E_i = \frac{1}{2} k(\Delta x)^2$

$\Delta x = 0$

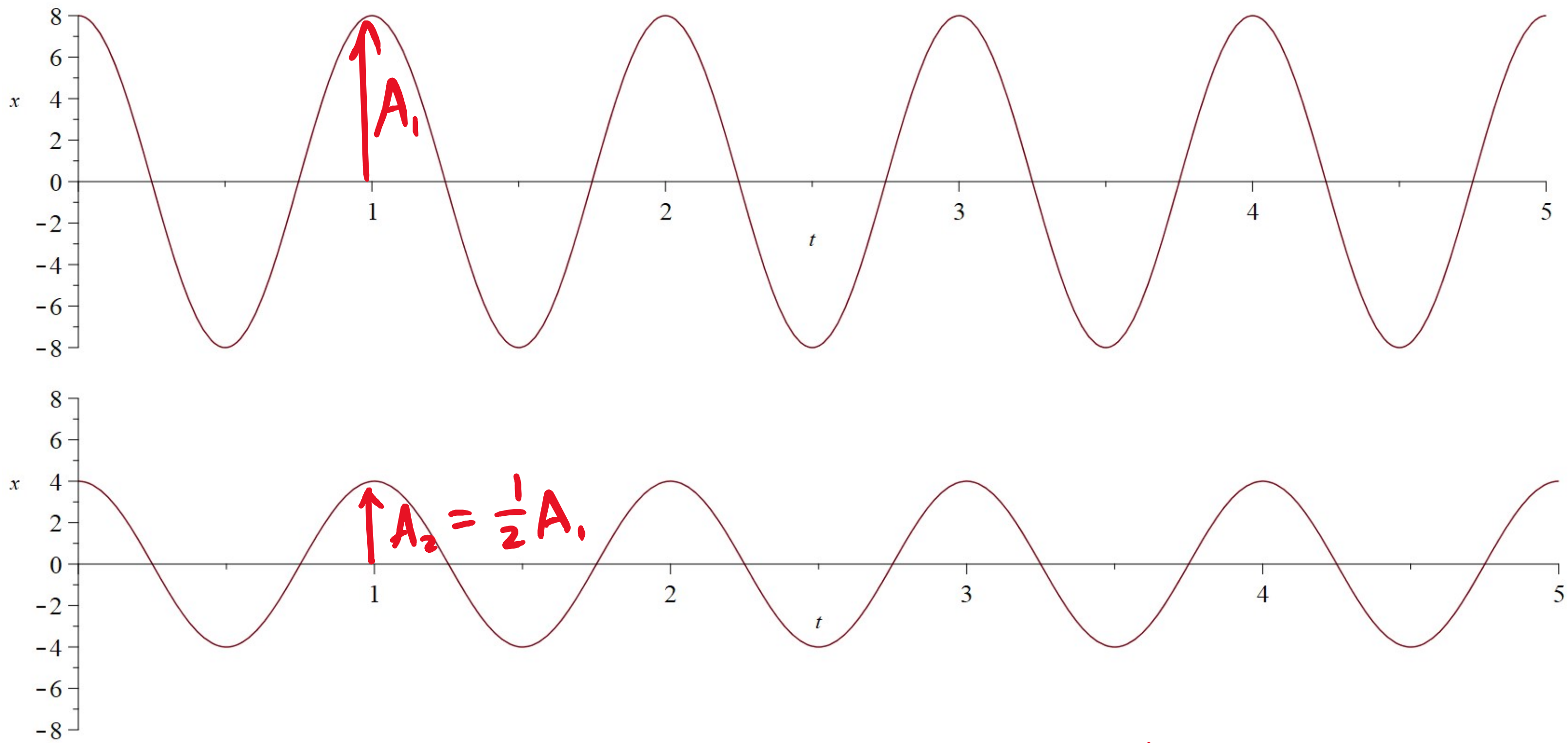
$E_f = \frac{1}{2} M v^2$

Energy conserved: $\frac{1}{2} M v^2 = \frac{1}{2} k(\Delta x)^2 \Rightarrow v = \sqrt{\frac{k}{m}} \cdot \Delta x = 2 \text{m/s}$




The two graphs show different oscillations for the same system. Compared with the first case, the total potential plus kinetic energy in the second case is

A) The same B) Twice as big C) Half as big D) One quarter as big E) One 16th as big




for each: energy same at all times. Look at time when v=0.

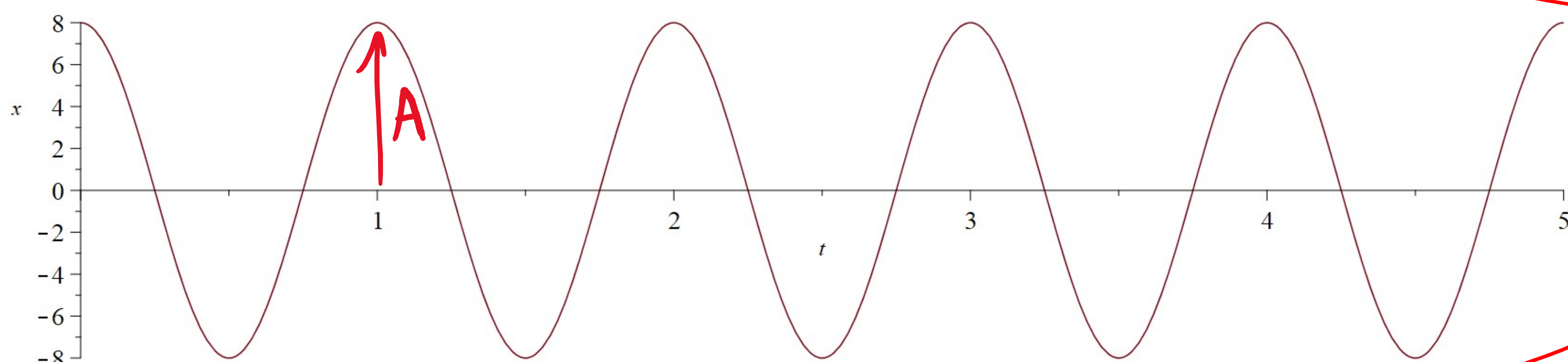
The two graphs show different oscillations for the same system. Compared with the first case, the total potential plus kinetic energy in the second case is

Then $E = \frac{1}{2} k A^2$ so $\frac{E_2}{E_1} = \frac{A_2^2}{A_1^2} = \frac{1}{4}$

A) The same B) Twice as big C) Half as big D) One quarter as big E) One 16th as big

Key fact about oscillating systems:

Energy is proportional to amplitude squared



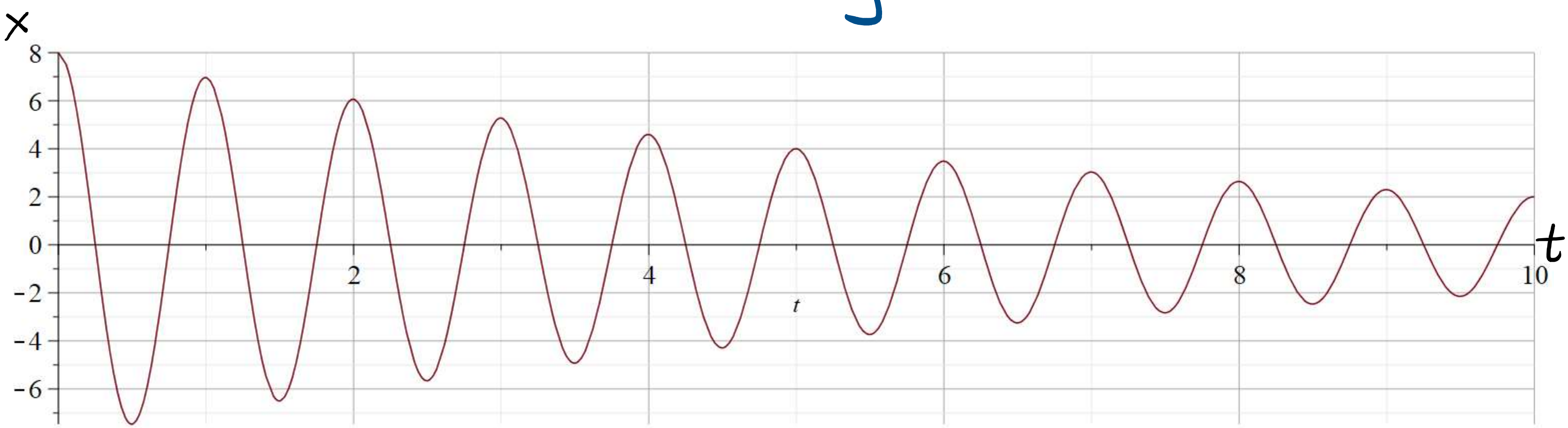

$$E_{TOT} \propto A^2$$

Real oscillators: energy is lost

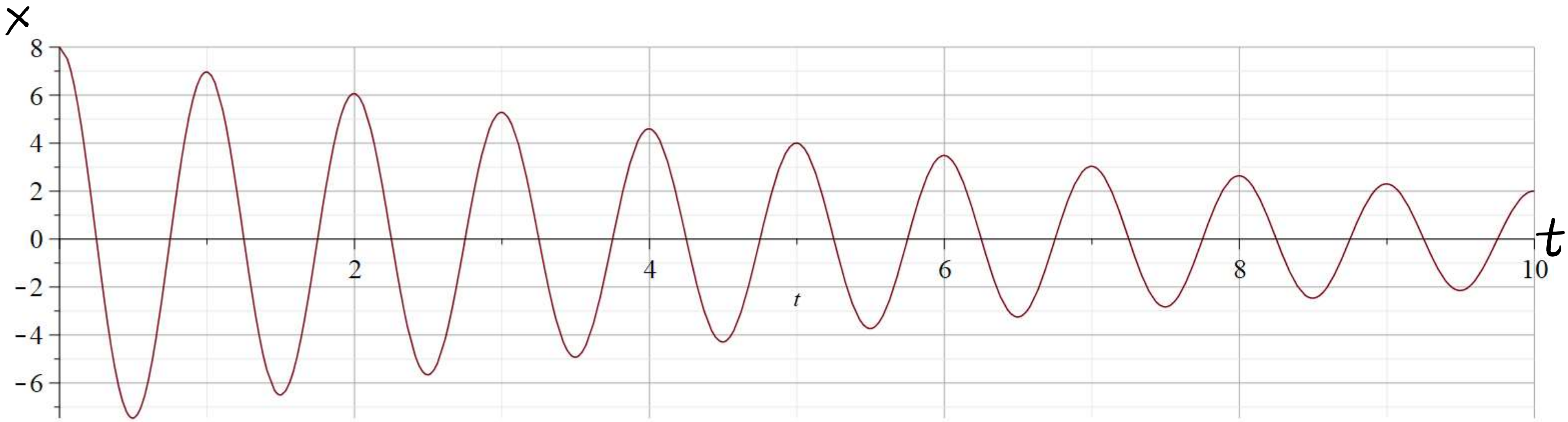
friction

air/fluid drag

heating of system



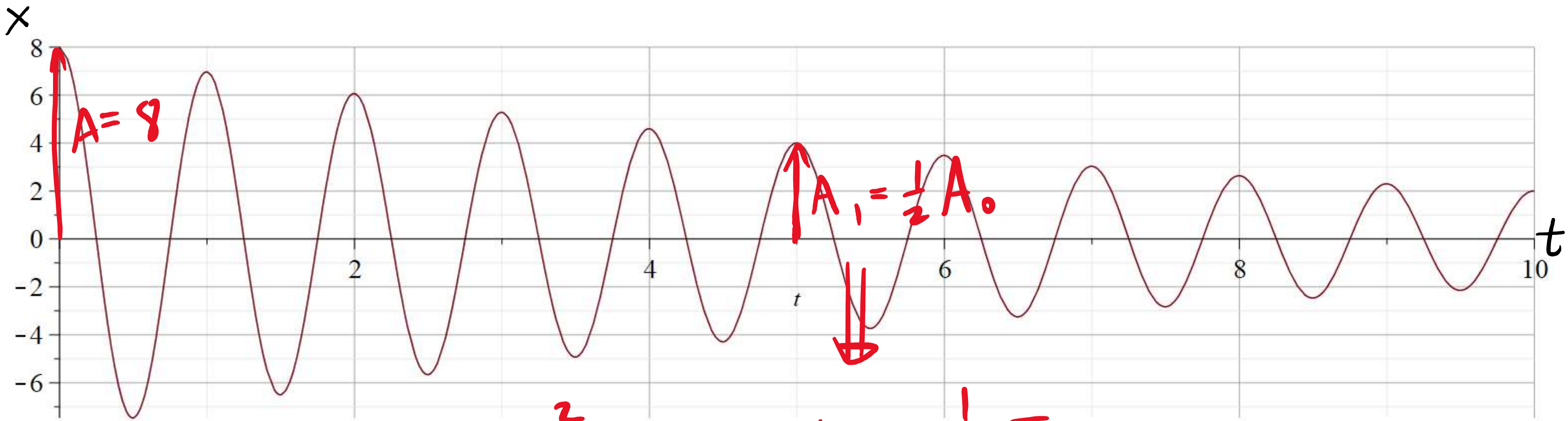

amplitude decreases with time




What fraction of the original kinetic + potential energy remains in the oscillator at t=5s?

A) All of it.
B) Half of it.
C) One quarter of it.
D) $1/\sqrt{2}$ of it.

**EXTRA:** what fraction of the energy at t=5s remains at t=10s?




$E \propto \text{amplitude}^2$ $\qquad E_1 = \frac{1}{4} E_0$

What fraction of the original kinetic + potential energy remains in the oscillator at t=5s?

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**EXTRA:** what fraction of the energy at t=5s remains at t=10s? $\frac{1}{4}$

Common situation: amplitude decreases by ~ same fraction each full oscillation.

example: $t=0 \longrightarrow A = A_0$

$t=T \longrightarrow A = A_0 \cdot r$ fraction of previous amplitude

$t=2T \longrightarrow A = A_0 \cdot r^2$

$t=3T \longrightarrow A = A_0 \cdot r^3$

number of periods

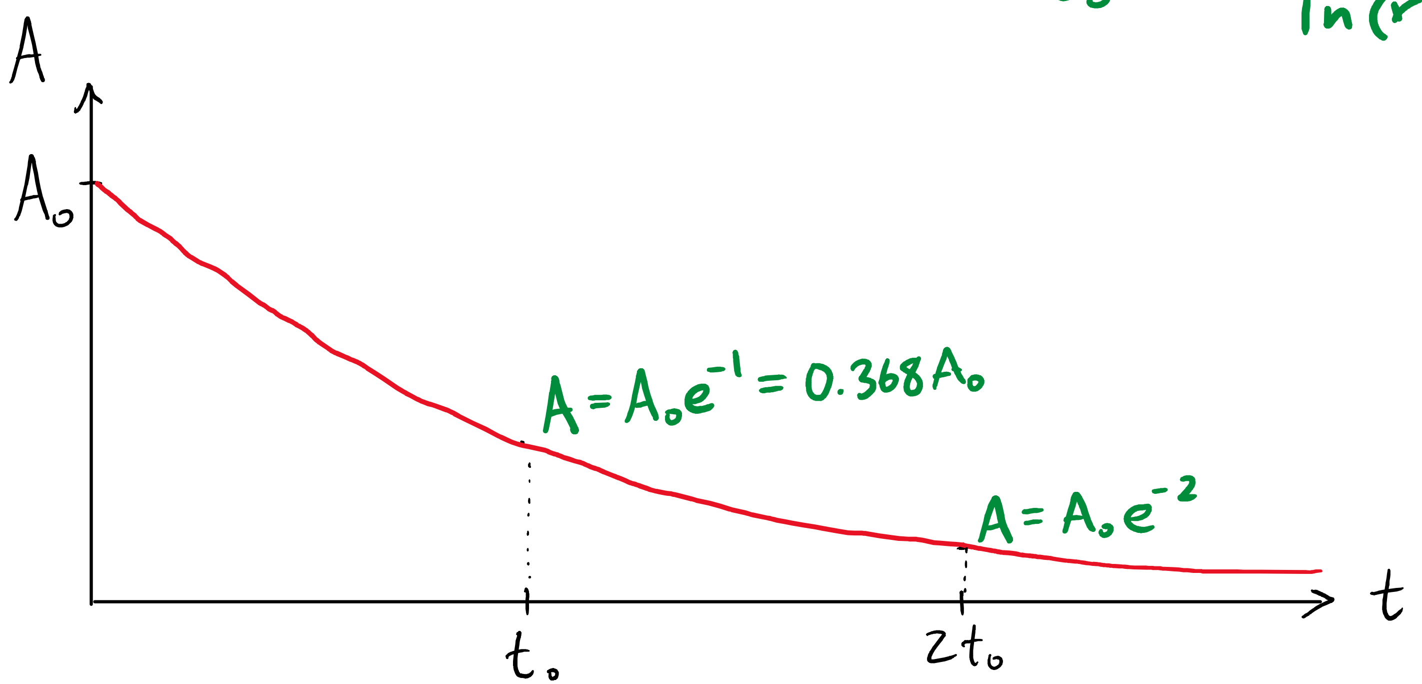
general $T \longrightarrow A = A_0 \cdot r^{t/T}$

# Exponential decay

We can rewrite this as:

$$\boxed{A = A_0 e^{-t/t_0}}$$

$$t_0 = -\frac{T}{\ln(r)}$$



# Exponential decay

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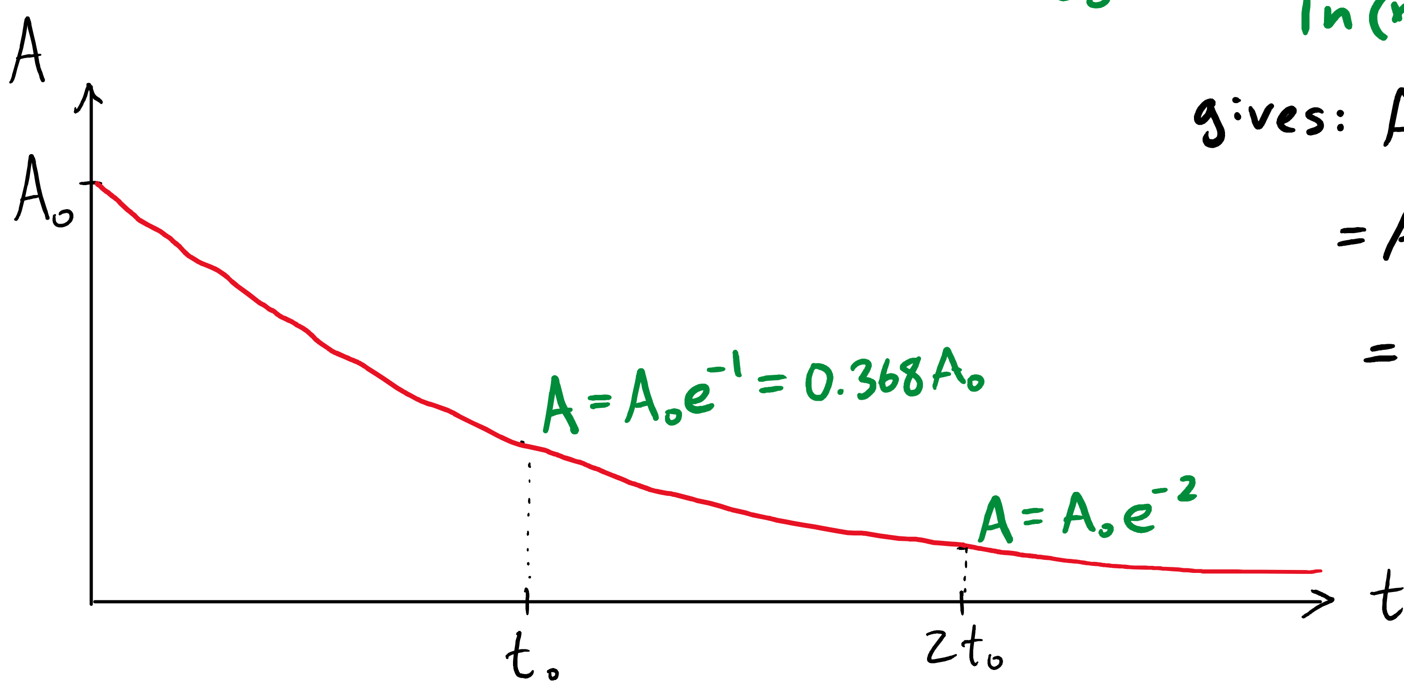
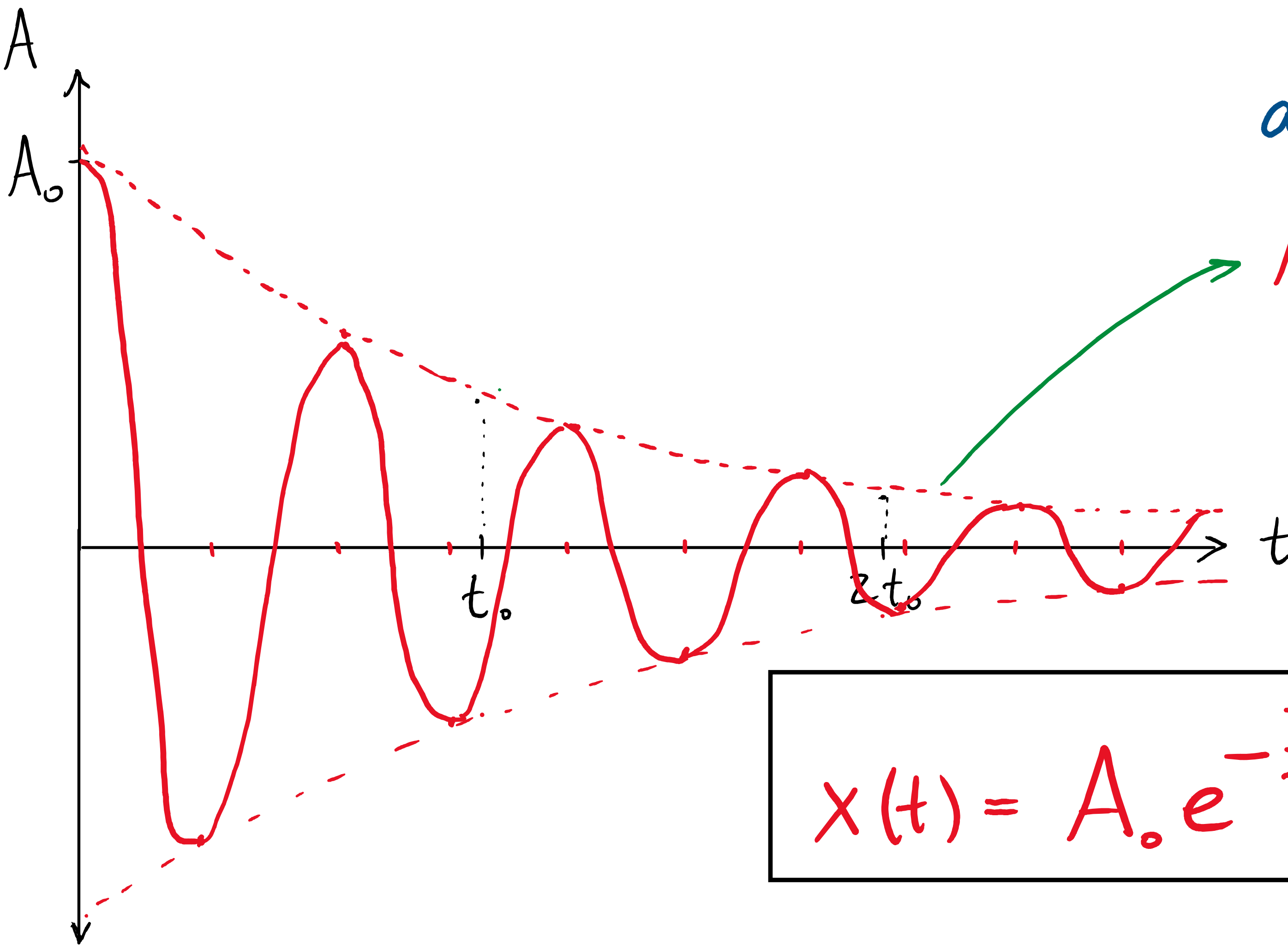



gives: $A_0 e^{\ln(r) \cdot \frac{t}{T}}$

$= A_0 e^{\ln(r^{t/T})}$

$= A_0 r^{t/T}$

# Damped oscillations

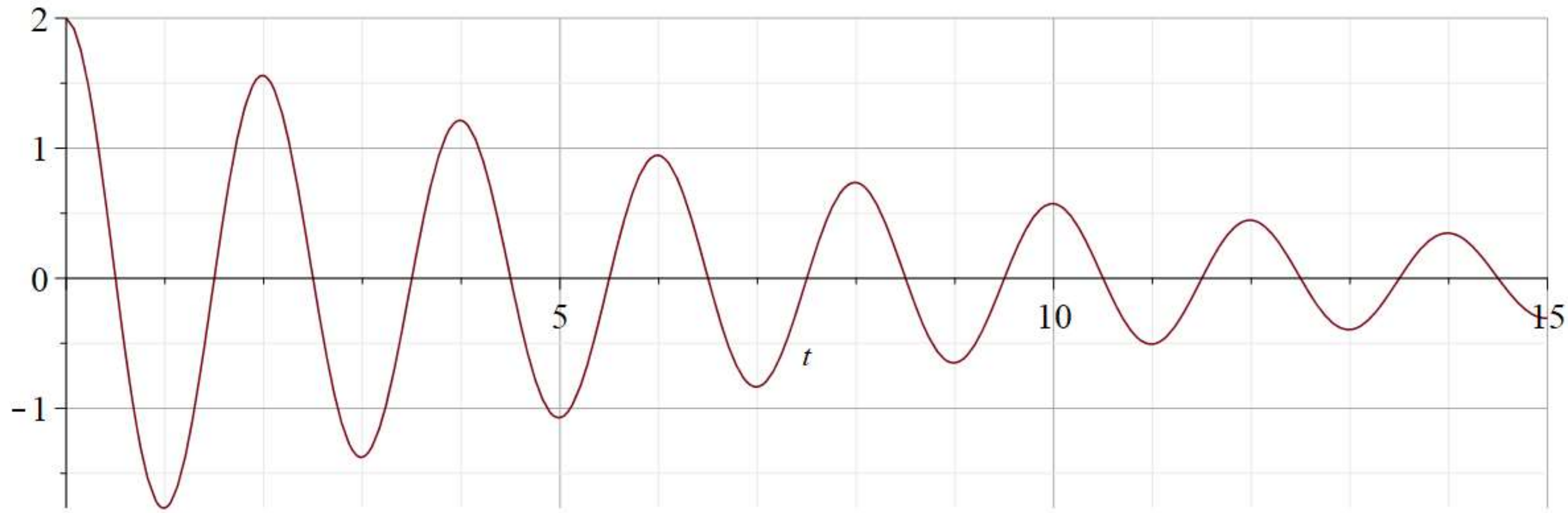



amplitude:

$$A = A_0 e^{-\frac{t}{t_0}}$$

multiplies cosine

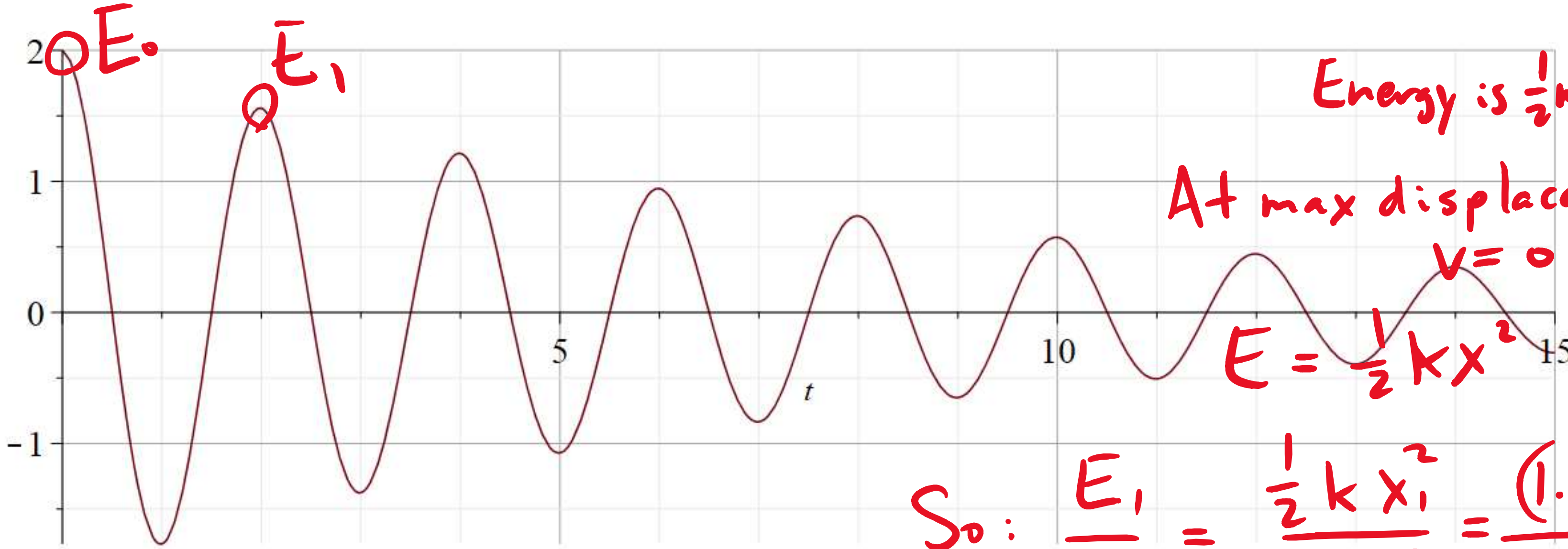
$$\boxed{x(t) = A_0 e^{-\frac{t}{t_0}} \cos(\omega t + \phi)}$$




EXTRA: An object with mass 2kg oscillates on a spring with a small amount of damping.

Roughly what fraction of the energy is lost in one complete oscillation?

A) 6%  B) 12%  C) 23%  D) 40%  E) 72%




Energy is $\frac{1}{2}mv^2 + \frac{1}{2}kx^2$

At max displacement, $v = 0$ so:

$$E = \frac{1}{2}kx^2$$

So: $$\frac{E_1}{E_0} = \frac{\frac{1}{2}kx_1^2}{\frac{1}{2}kx_0^2} = \frac{(1.55)^2}{2^2}$$

An object with mass 2kg oscillates on a spring with a small amount of damping. $\approx 0.6$

Roughly what fraction of the energy is lost in one complete oscillation?

so 40% has been lost

A) 6% B) 12% C) 23% D) 40% E) 72%