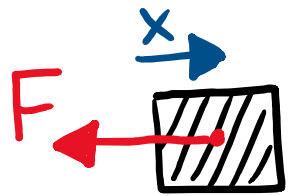


Last time in  
Phys 157...

Linear restoring force:  $F_{\text{NET}} = -kx$



$$F = -kx$$



Newton's 2nd Law

$$a = -\frac{k}{m}x$$



Simple harmonic motion

# SIMPLE HARMONIC MOTION

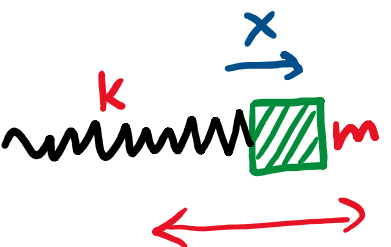
satisfies  $\frac{d^2x}{dt^2} = -\omega^2 x$   
↑  $\frac{k}{m}$

$$x(t) = A \cos(\omega t + \phi)$$

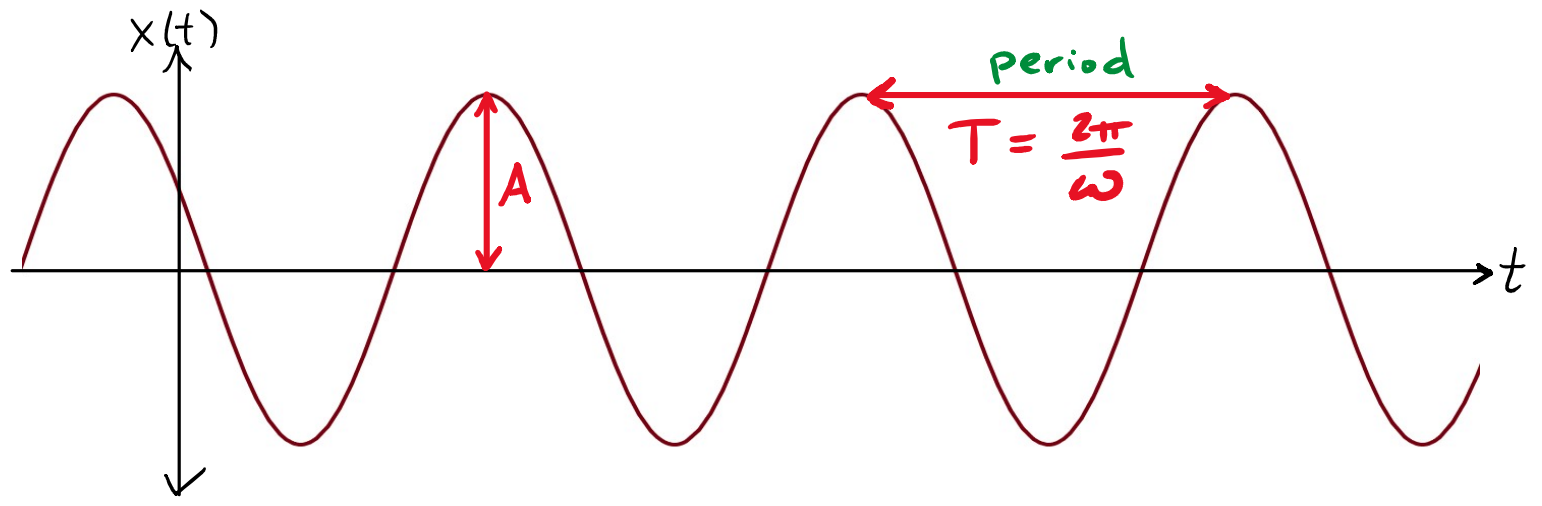
Amplitude

angular frequency

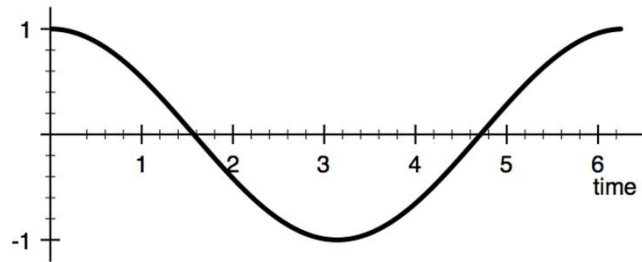
phase



$$\omega = \sqrt{\frac{k}{m}}$$



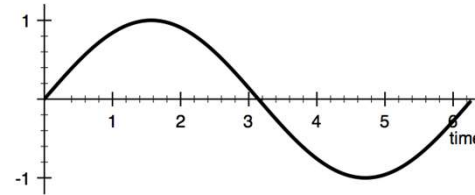
## Velocity vs displacement



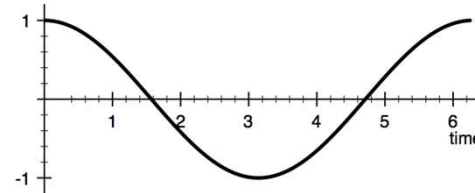
A plot of displacement as a function of time for an oscillator is shown above. Which of the diagrams to the right describes the **velocity** as a function of time for the same motion?

Phys157

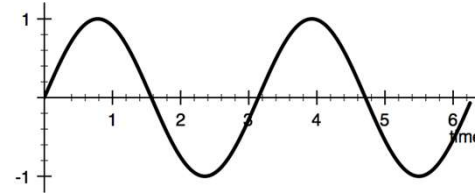
A.



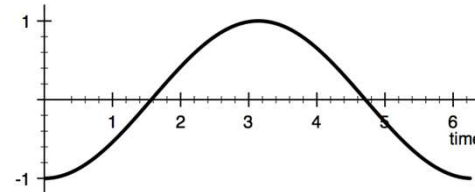
B.



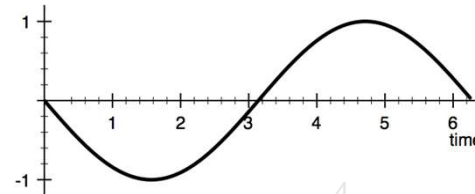
C.



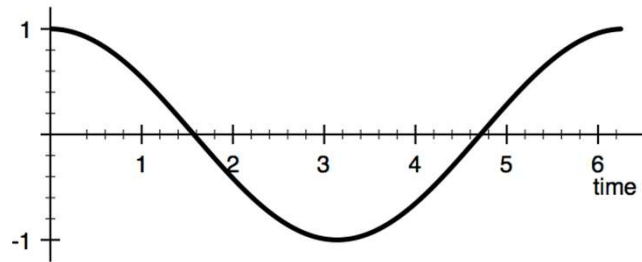
D.



E.



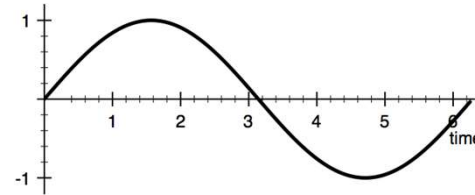
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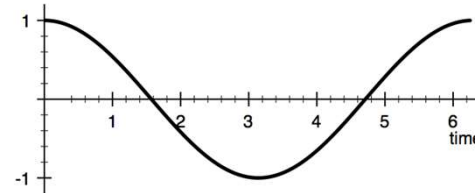
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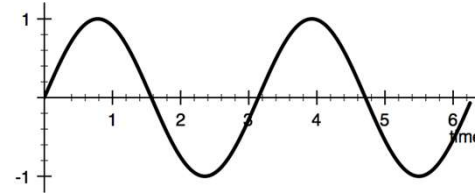
A.



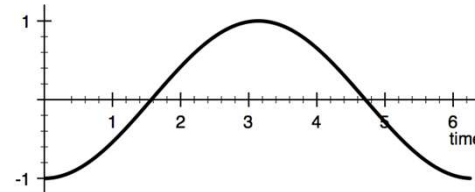
B.



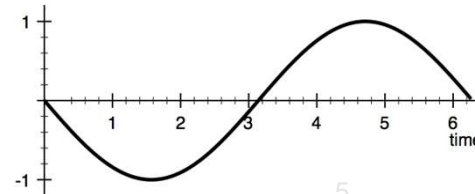
C.



D.



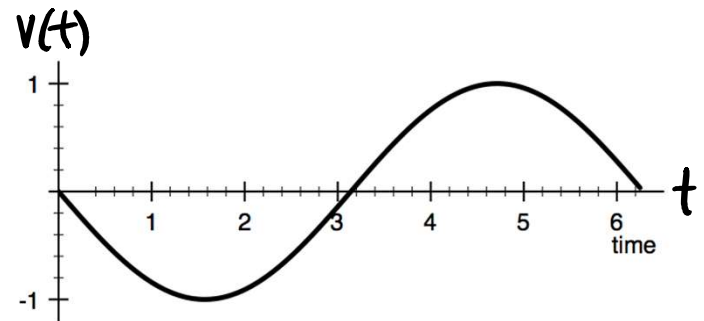
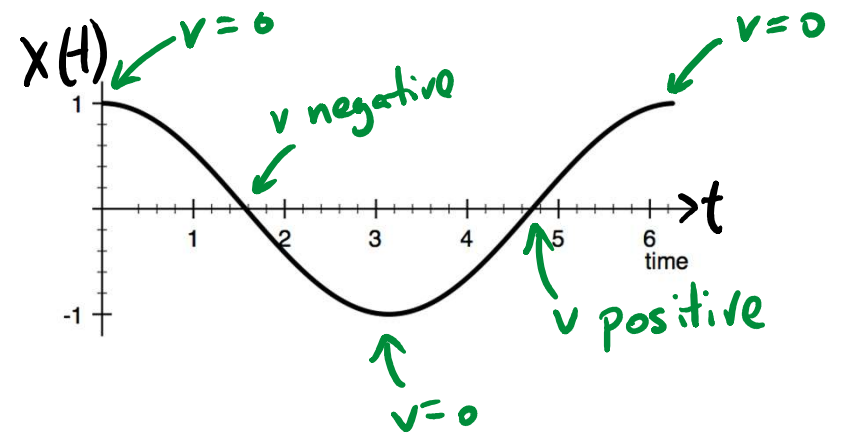
E.



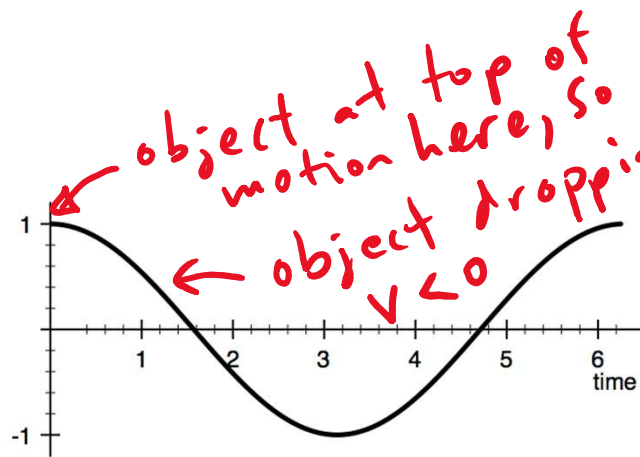
Velocity from displacement:

$$V = \frac{dx}{dt}$$

$v(t)$  = slope of  $x(t)$   
at time  $t$



# Velocity vs displacement

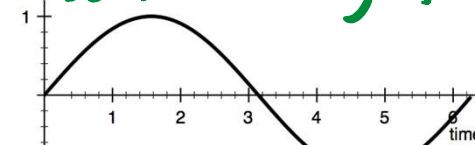


A plot of displacement as a function of time is shown above. Which of the diagrams to the right describes the **velocity** as a function of time for the same motion?

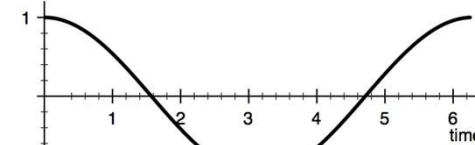
Phys157

because switching from  $v > 0$  to  $v < 0$

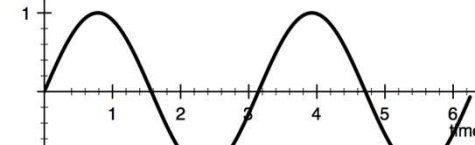
A.



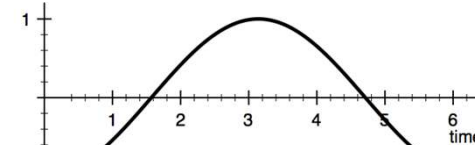
B.



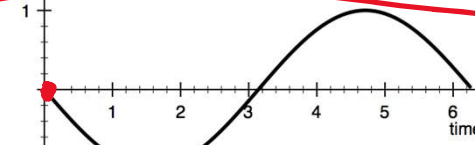
C.



D.



E.



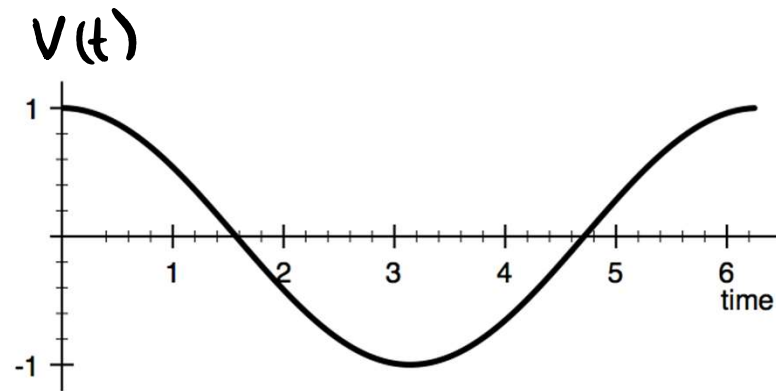
$v$  starts at 0 and goes negative

Other methods:

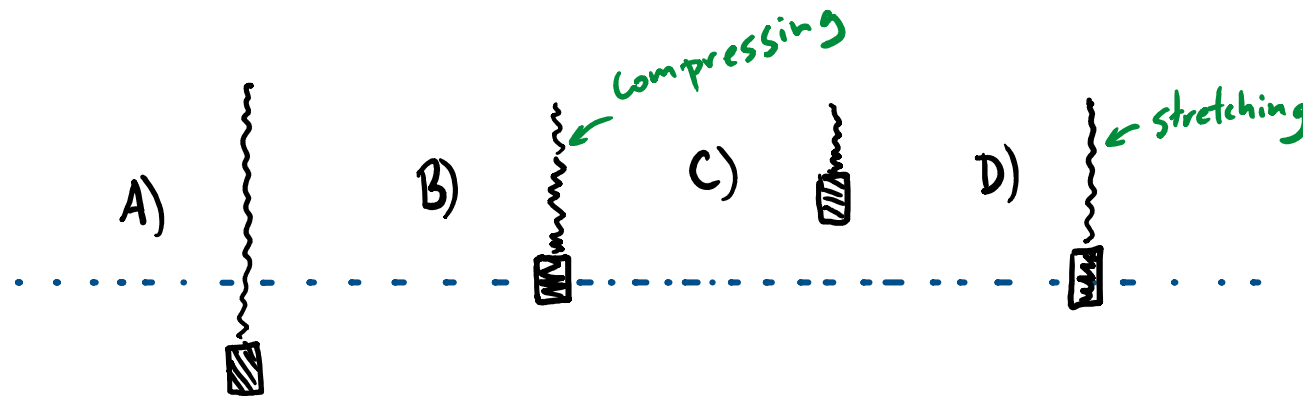
$$v = \frac{dx}{dt}$$

= slope of graph

# Simple Harmonic Motion:



A plot of upward **velocity** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures best represents the situation at  $t=1.6\text{s}$ ?

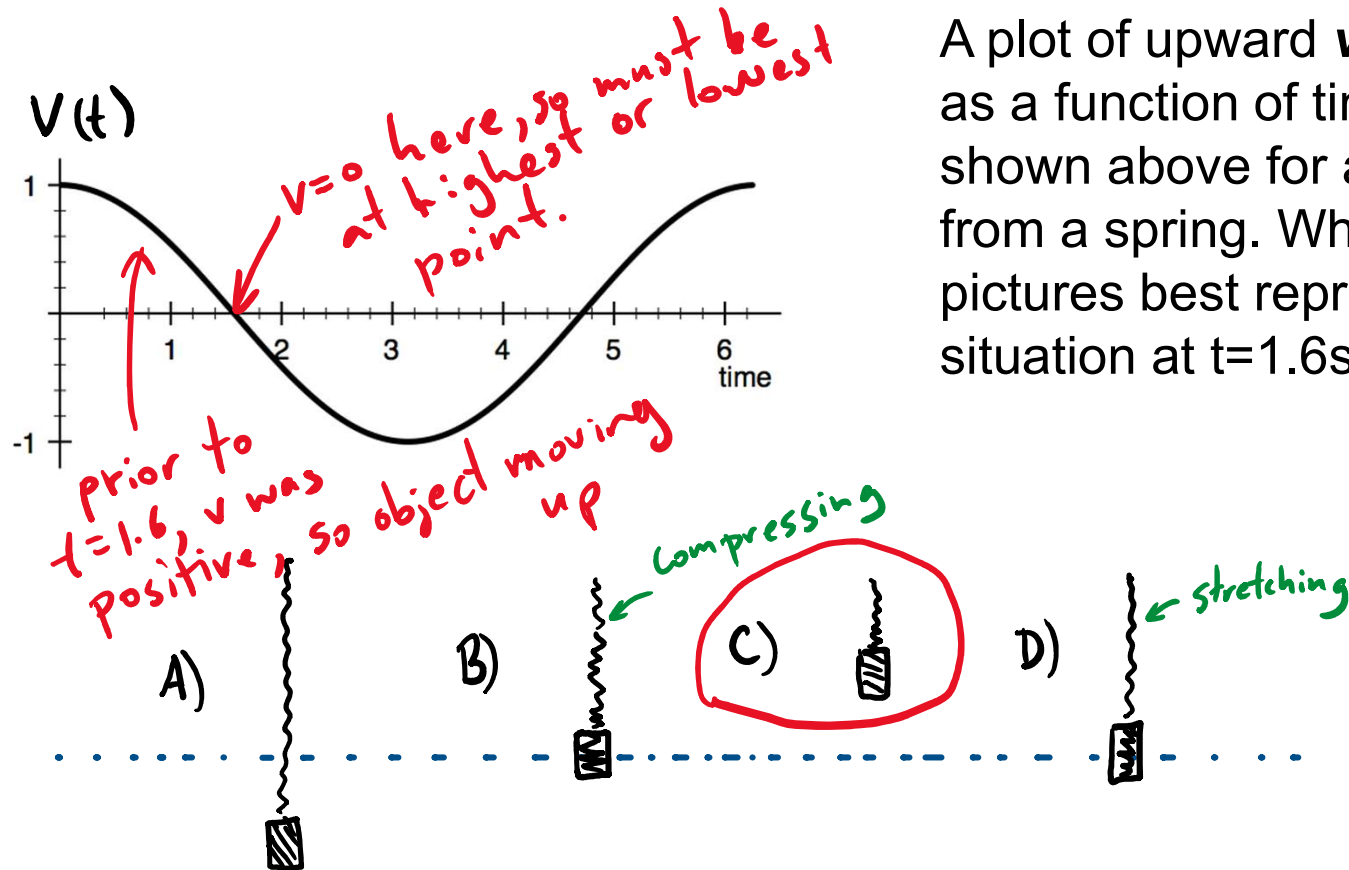


Phys157

EXTRA: what does  $x(t)$  look like?



# Simple Harmonic Motion:

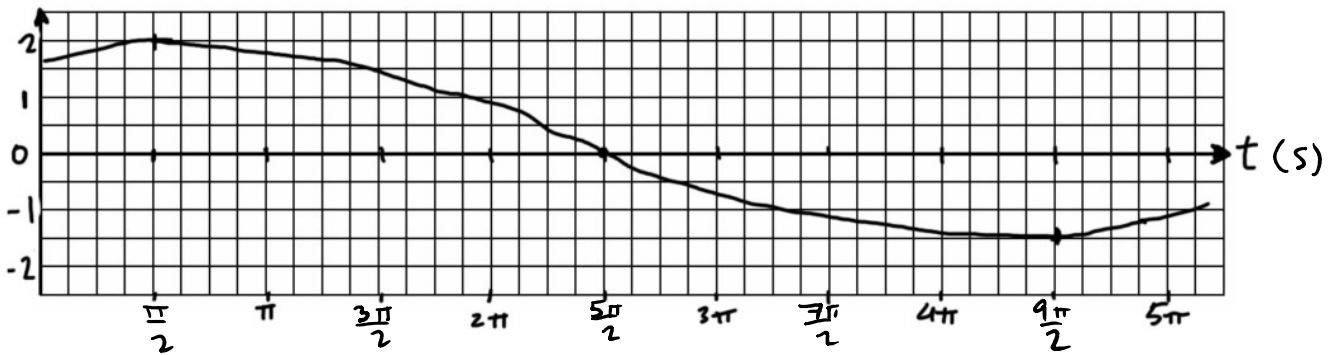


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Phys157

**EXTRA:** what does  $x(t)$  look like?

$x(t)$  (cm)



$$\omega = \frac{2\pi}{T}$$

$$x(t) = A \cos(\omega t + \phi)$$

For the displacement graph shown, what is the maximum magnitude of velocity, in cm/s?

A) 4

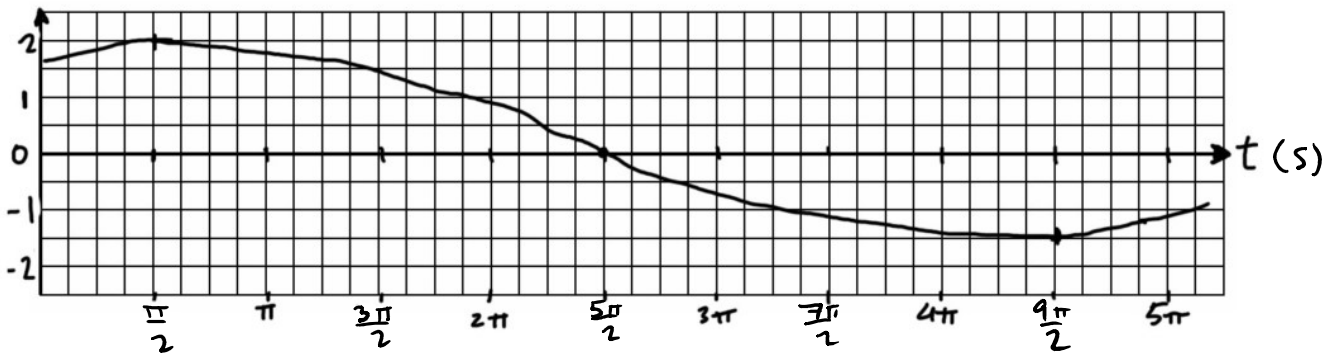
B) 2

C) 1

D) 1/2

E) 1/4

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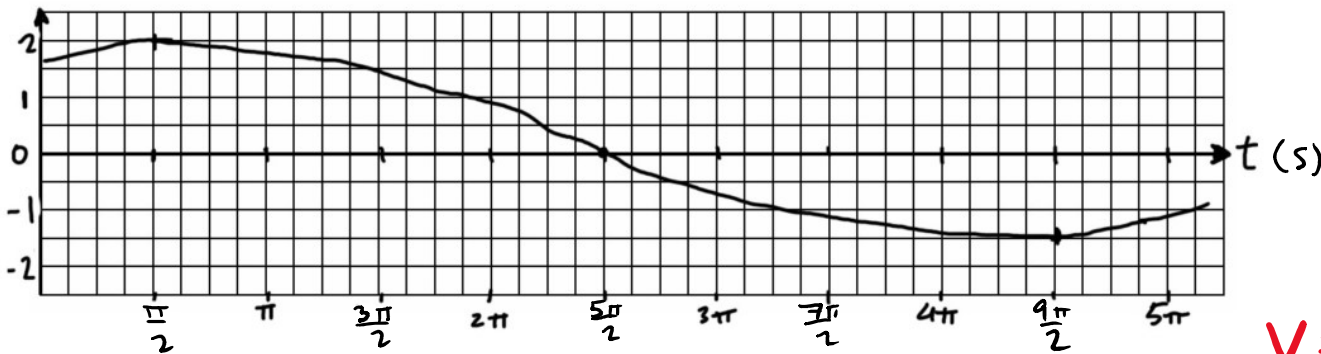
B) 2

C) 1

D) 1/2

E) 1/4

$x(t)$  (cm)



$$\omega = \frac{2\pi}{T}$$

$$v = \frac{dx}{dt}$$

$$x(t) = A \cos(\omega t + \phi)$$

$$= -A\omega \sin(\omega t + \phi)$$

For the displacement graph shown, what is the maximum magnitude of velocity, in cm/s?

A) 4

B) 2

C) 1

D) 1/2

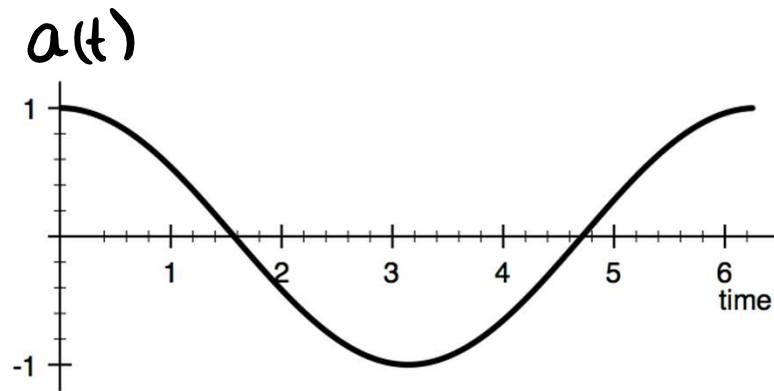
E) 1/4

sin goes from -1 to 1, so max value of  $v$  is  $A\omega$

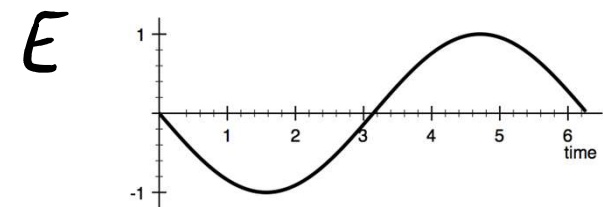
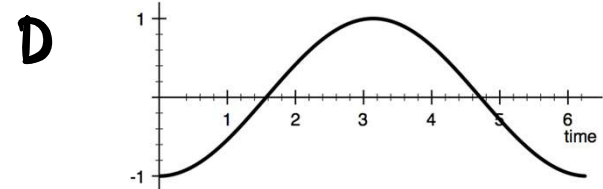
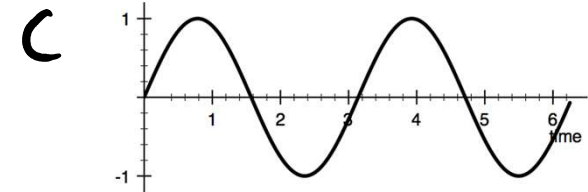
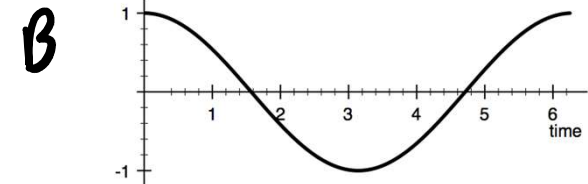
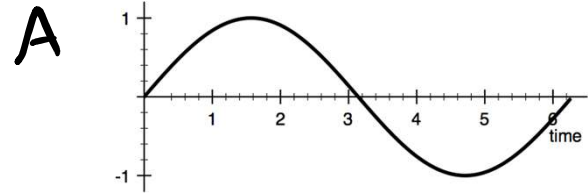
$$A = 2 \text{ cm}, \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{8\pi} = \frac{1}{4}$$

$$A\omega = \frac{1}{2}$$

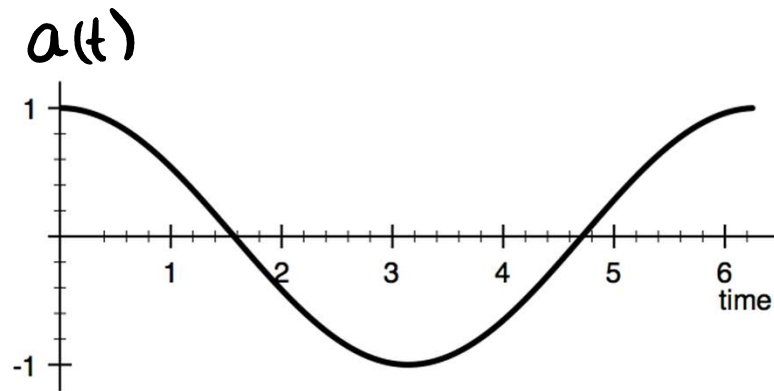
Acceleration vs displacement:



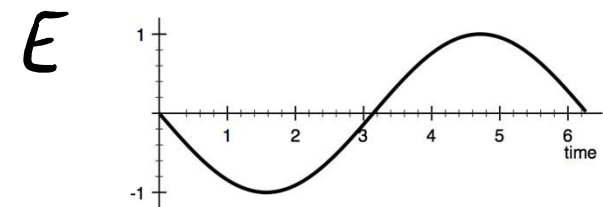
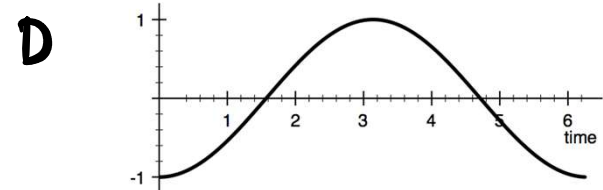
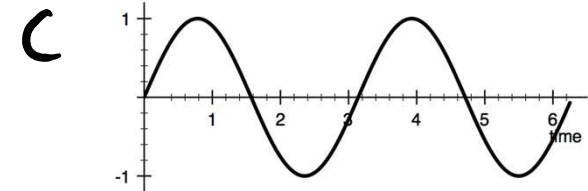
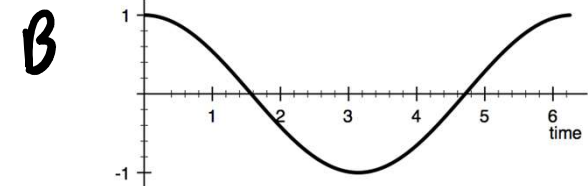
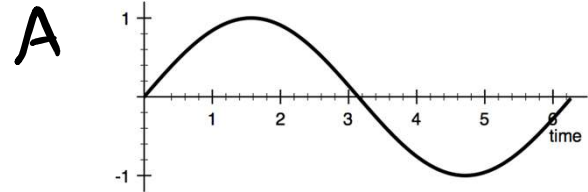
A plot of upward **acceleration** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures to the right could represent  $x(t)$ ?



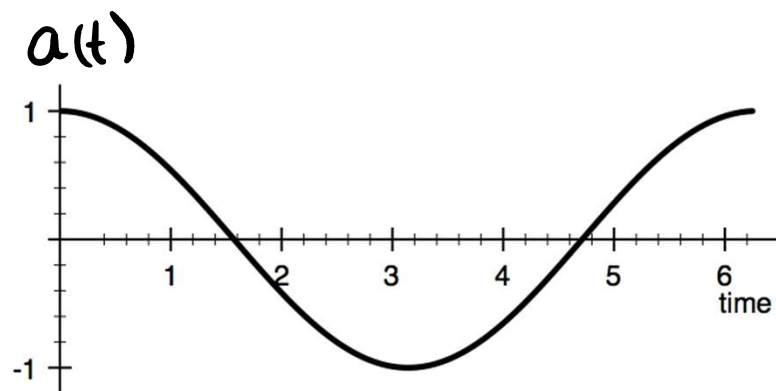
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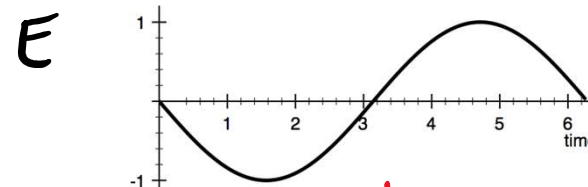
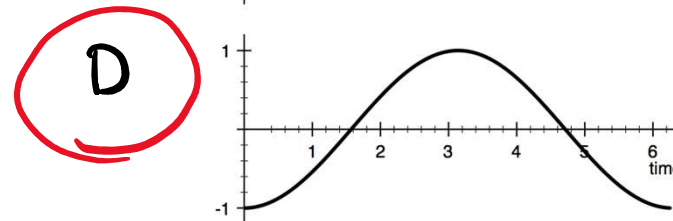
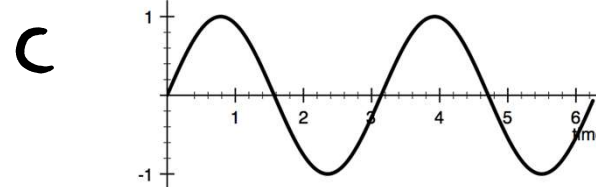
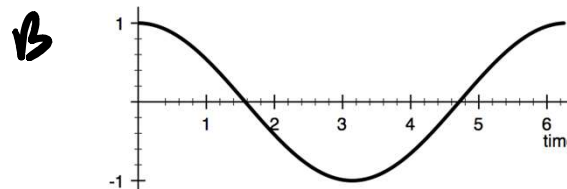
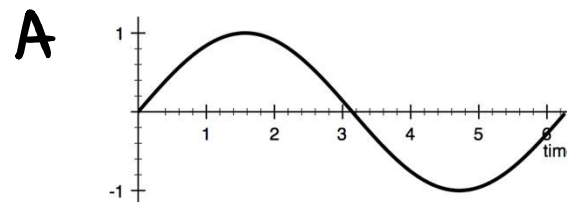


# Acceleration vs displacement:

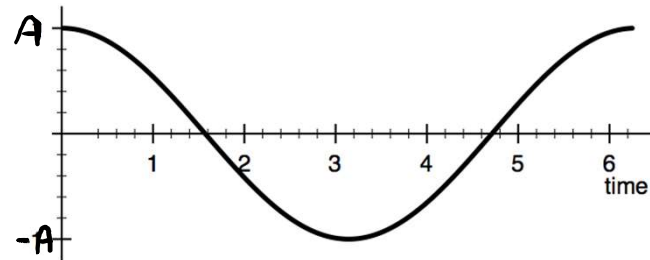


A plot of upward **acceleration** (in cm/s) as a function of time (in s) is shown above for a mass hanging from a spring. Which of the pictures to the right could represent  $x(t)$ ?

Have  $a = -\omega^2 x$  for SHM, so  $x$  is maximum when  $a$  is minimum



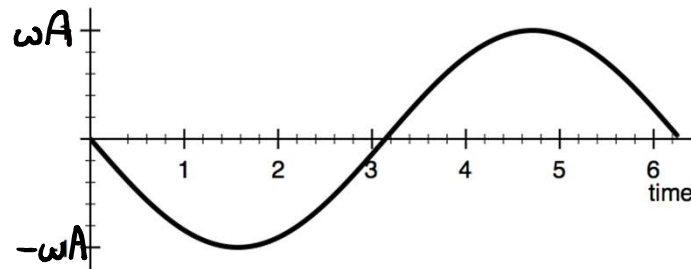
Position:



$$x(t) = A \cos(\omega t + \phi)$$

$$\downarrow \frac{d}{dt} \quad (\text{slope})$$

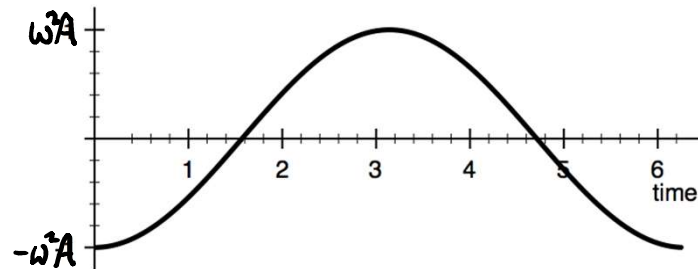
Velocity:



$$v(t) = -A\omega \sin(\omega t + \phi)$$

$$\downarrow \frac{d}{dt} \quad (\text{slope})$$

Acceleration:

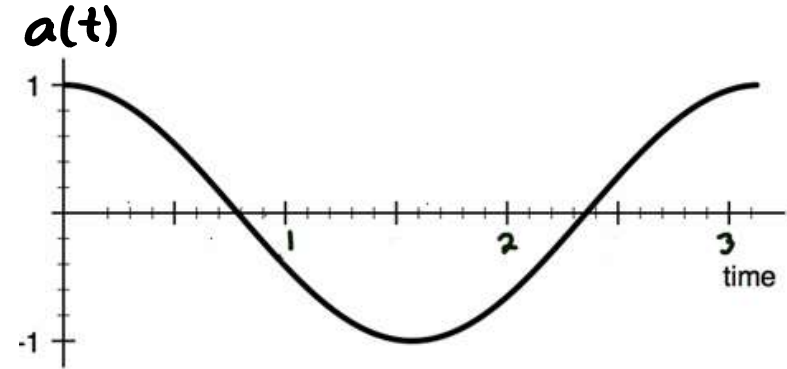
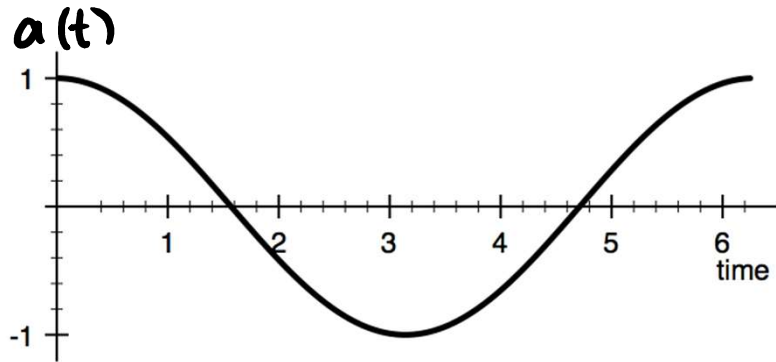


$$a(t) = -A\omega^2 \cos(\omega t + \phi)$$

||

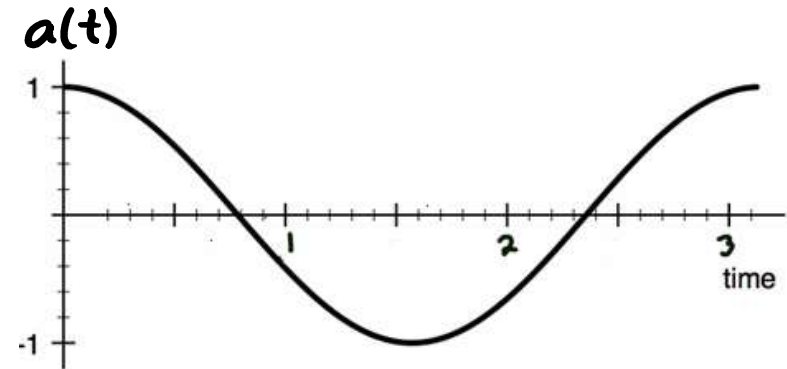
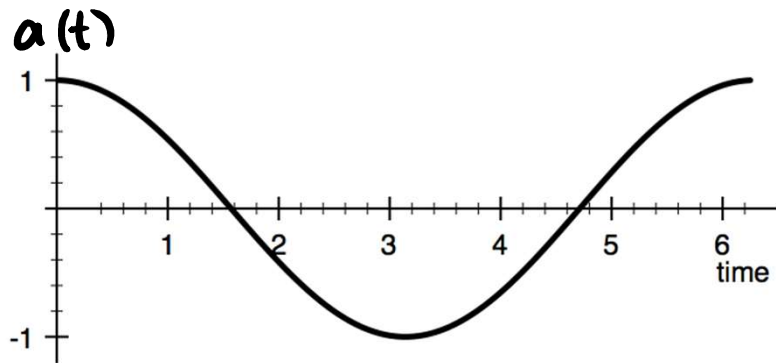
$$- \omega^2 x(t)$$





The graphs show **acceleration** as a function of time for two different harmonic oscillators. The amplitude of the **displacement** in the first case is 1cm. For the second oscillator, the amplitude of the **displacement** is

- A) 4cm      B) 2cm      C) 1cm      D) 0.5 cm      E) 0.25 cm



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- A) 4cm      B) 2cm      C) 1cm      D) 0.5 cm      E) 0.25 cm

Have  $a = -\omega^2 x$ , so  $x = -\frac{a}{\omega^2}$ .  $T$  is half in 2nd case so  $\omega$  is double, so amplitude of  $x$  is  $\frac{1}{4}$

$$\phi = \pm 2\pi \frac{t_{\max}}{T}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A \cos(\omega t + \phi) \quad \omega = \frac{2\pi}{T}$$

Approximately what is the spring constant of the spring in the simulation?

A) 1 N/m

B) 2 N/m

C) 4 N/m

D) 8 N/m

E) 16 N/m

$$\phi = \pm 2\pi \frac{t_{\max}}{T}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A \cos(\omega t + \phi) \quad \omega = \frac{2\pi}{T}$$

\* in our actual example it was 1.3 s.

Approximately what is the spring constant of the spring in the simulation?

A) 1 N/m

B) 2 N/m

C) 4 N/m

D) 8 N/m

E) 16 N/m

have:  $T = 1\text{ s}$  so  $\omega = \frac{2\pi}{T} \approx 6.28\text{ s}^{-1}$

Using  $\omega = \sqrt{\frac{k}{m}}$  have:  $k = m\omega^2 = 0.1 \times (6.28)^2 \frac{\text{N}}{\text{m}} \approx 4\text{ N/m}$