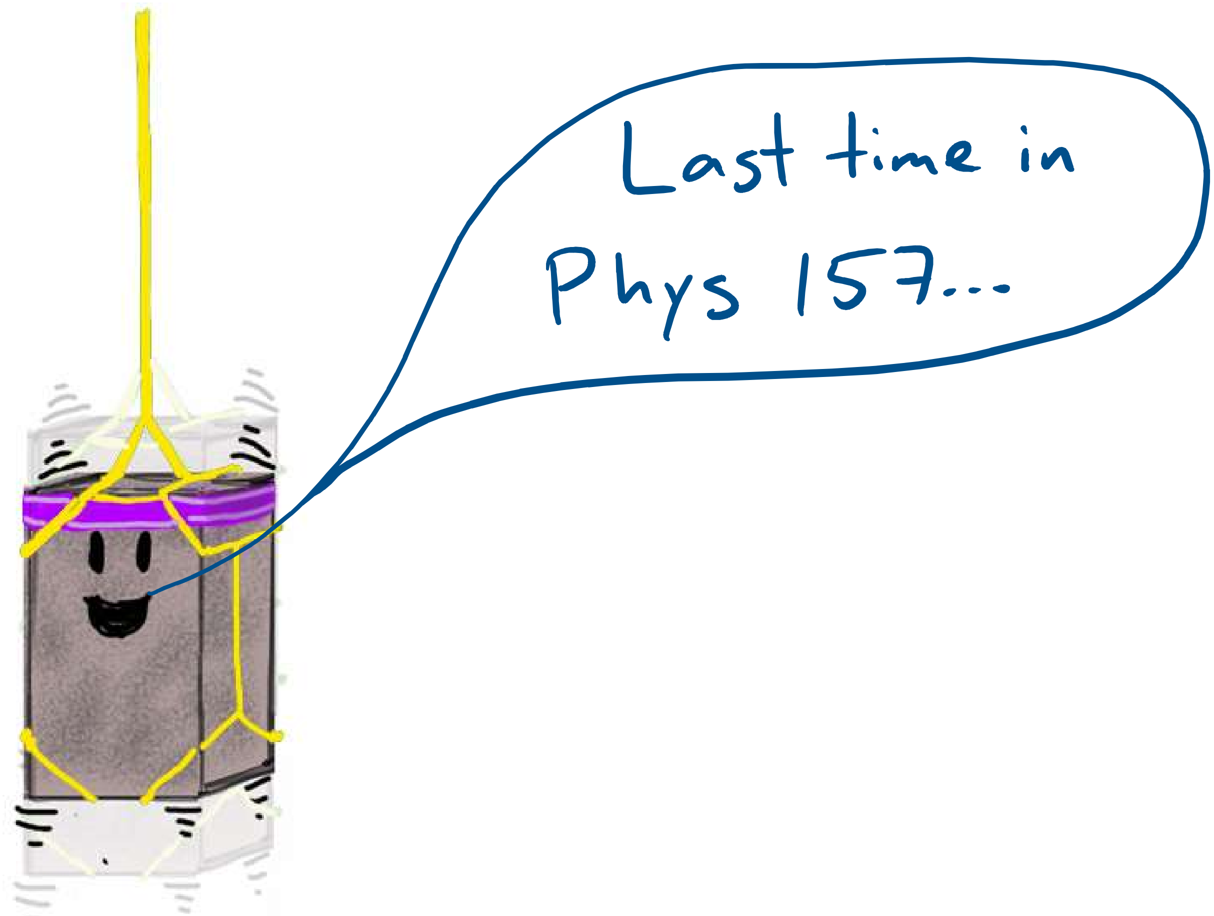
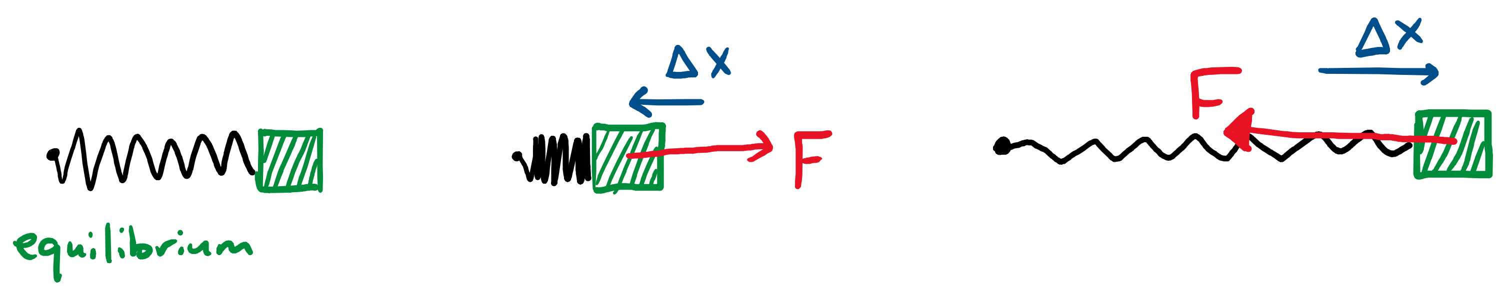

Last time in Phys 157...

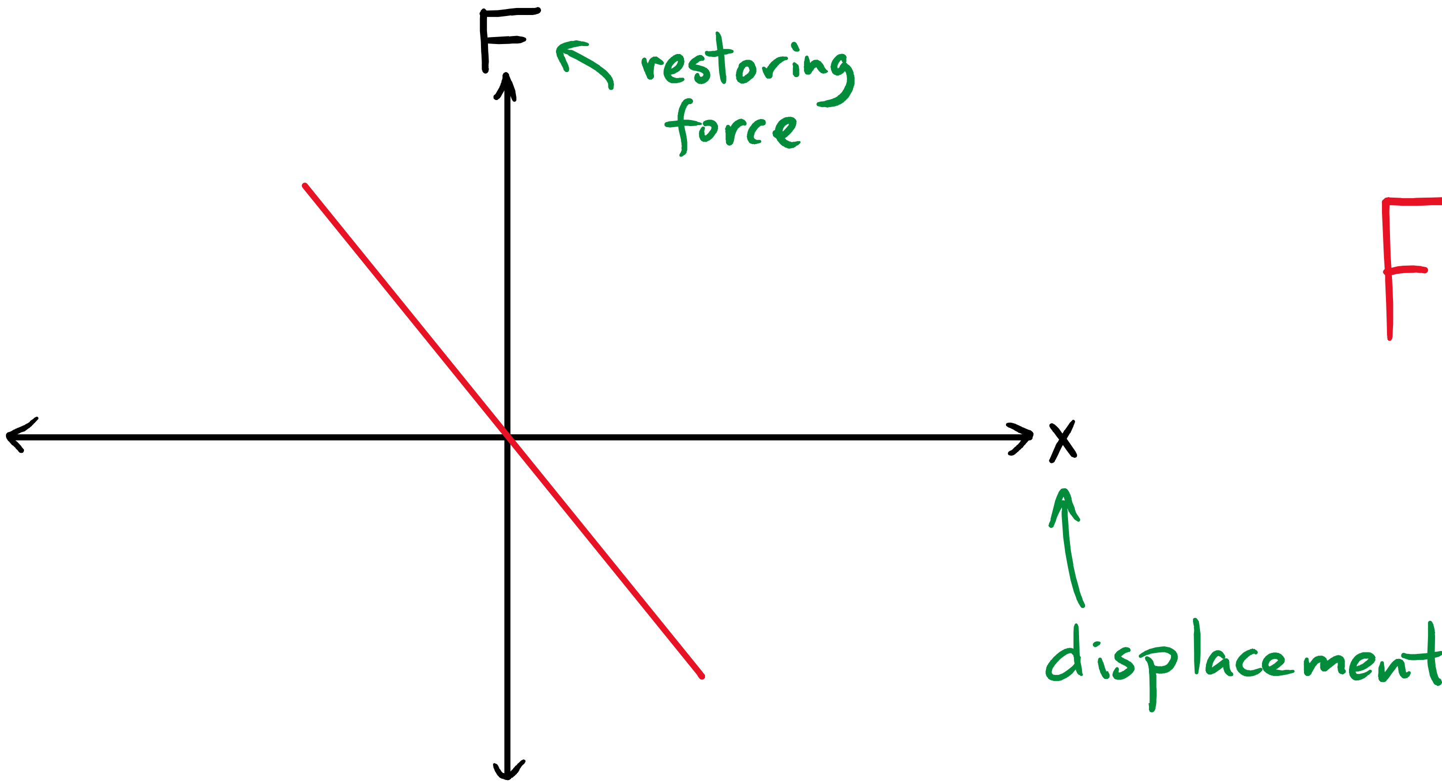
RESTORING FORCES: For an object in STABLE equilibrium, a displacement in one direction leads to a net force in the other direction.

e.g.



This leads to oscillations.

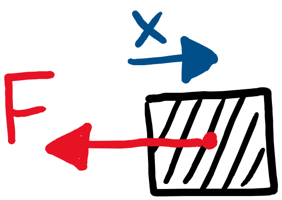
HOOKE'S LAW: Applies to almost any system perturbed a small amount from stable equilibrium



$$F = -kx$$

exact for "ideal spring"

Oscillations with Hooke's Law:



$F = -kx$

Newton: $a = \dfrac{F}{m} = -\dfrac{k}{m} \cdot x$

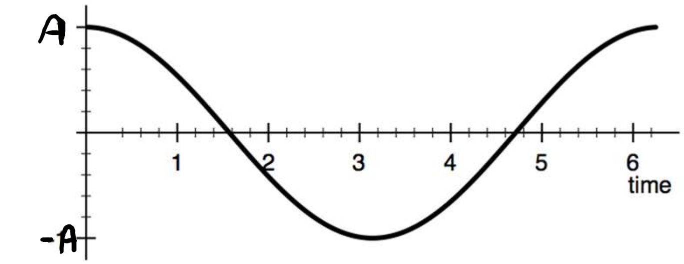
$$\frac{dv}{dt} = -\frac{k}{m} x$$

$$\frac{dx}{dt} = v$$

We can predict how velocity and position change with time.

Solution is $x(t) = A\cos(\omega t + \phi)$ with $\omega = \sqrt{\dfrac{k}{m}}$

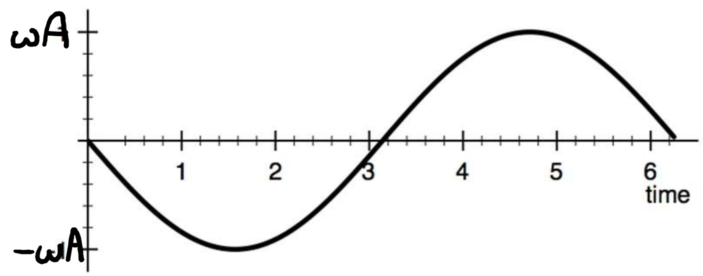
Position:



$$x(t) = A\cos(\omega t + \phi)$$

$\frac{d}{dt}$ (slope)

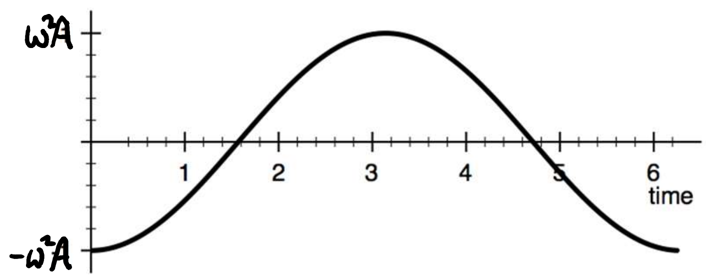
Velocity:



$$v(t) = -A\omega \sin(\omega t + \phi)$$

$\frac{d}{dt}$ (slope)

Acceleration:



$$a(t) = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 x(t)$$

Newton's Law $a = -\frac{k}{m}x$ holds if $\omega = \sqrt{\frac{k}{m}}$

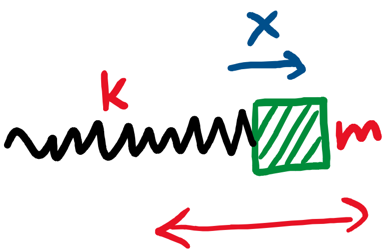
SIMPLE HARMONIC MOTION

$$x(t) = A\cos(\omega t + \phi)$$

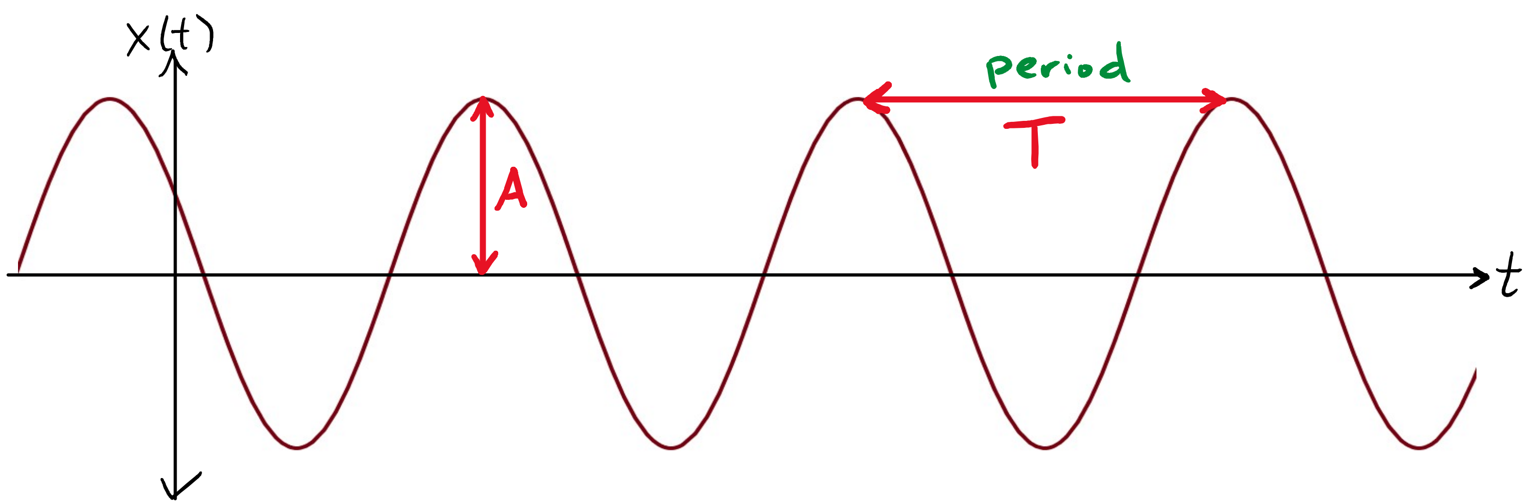
Amplitude

angular frequency

phase



$$\omega = \sqrt{\frac{k}{m}}$$



DISCUSSION:
What determines
A & ϕ?

Demo with duck

For the function $x(t) = 5\cos(3t + 5)$, what is the period?

A) 3
B) $1/3$
C) 6π
D) $2\pi/3$
E) 5

For the function x(t) = 5 cos(3t + 5), what is the period?

A) 3
B) 1/3
C) 6π
D) 2π/3
E) 5

cos repeats when 2π is added to the inside (i.e. the argument)

adding $T = \frac{2\pi}{3}$ to t adds 2π to $(3t + 5)$

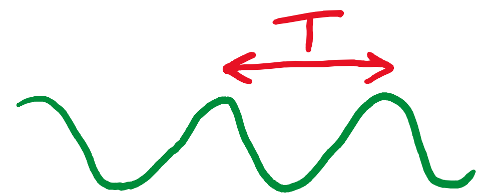
so $T = \frac{2\pi}{3}$ is the period

Frequency & Period

angular frequency

$$x(t) = A\cos(\omega t + \phi)$$

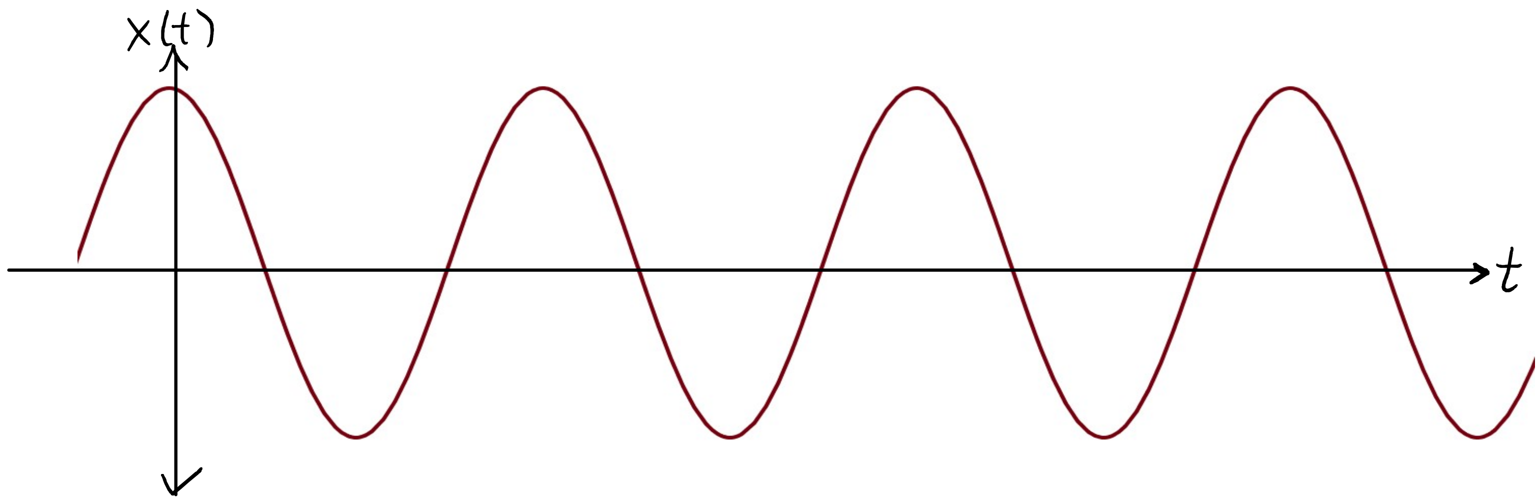
Period T: time from max → max



$$T = \frac{2\pi}{\omega}$$ since cos repeats every 2π.

Frequency f: oscillations per time $f = \frac{1}{T}$

gives: $\omega = 2\pi f$



The graph shows a displacement $x(t) = A\cos(\omega t)$. Adding a small positive phase $x(t) = A\cos(\omega t + \phi)$ will

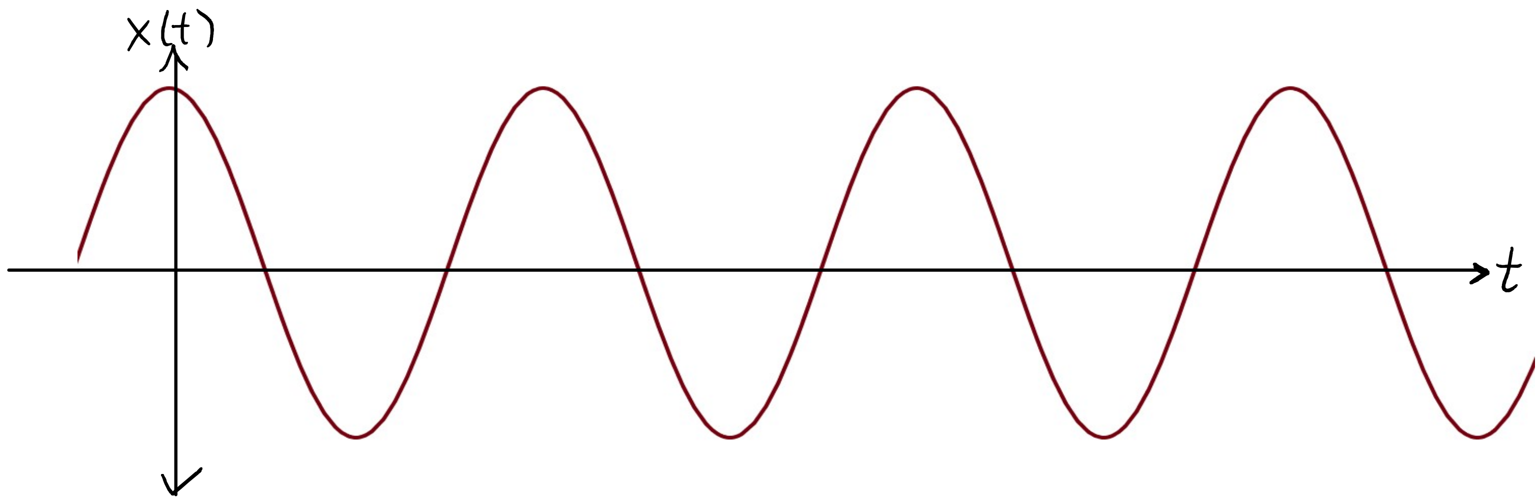
A) Shift the graph to the right

B) Shift the graph to the left

C) Squish the graph so the peaks are closer together

D) Stretch the graph so the peaks are further apart

E) Both A and C



With positive ϕ, we are seeing a later part of the cosine graph than previously, so the graph is shifted to the left

The graph shows a displacement x(t) = Acos(ωt). Adding a small positive phase x(t) = Acos(ωt + ϕ) will

A) Shift the graph to the right

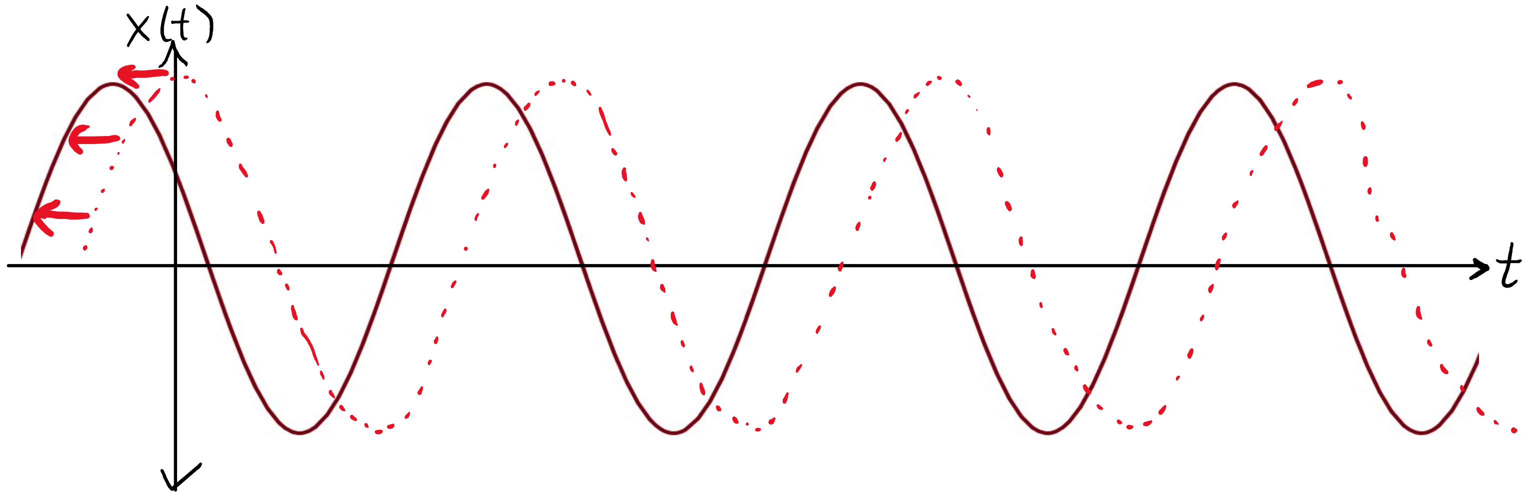
B) Shift the graph to the left

C) Squish the graph so the peaks are closer together

D) Stretch the graph so the peaks are further apart

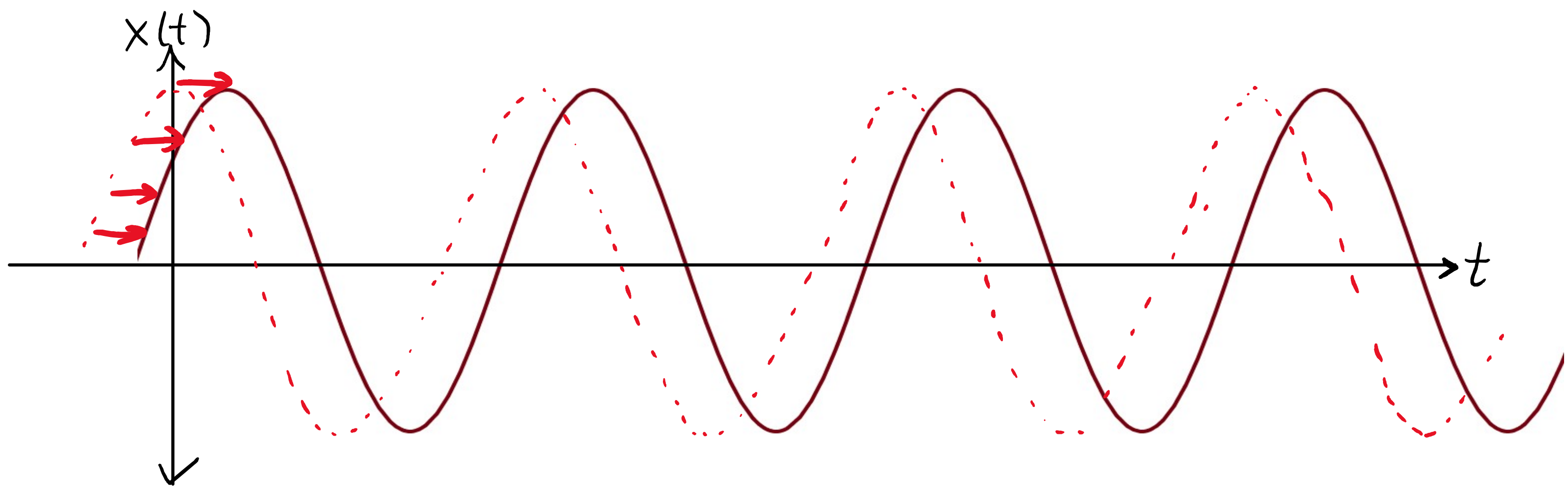
E) Both A and C

– same as adding $\frac{\phi}{\omega}$ to our time positive ϕ shifts left



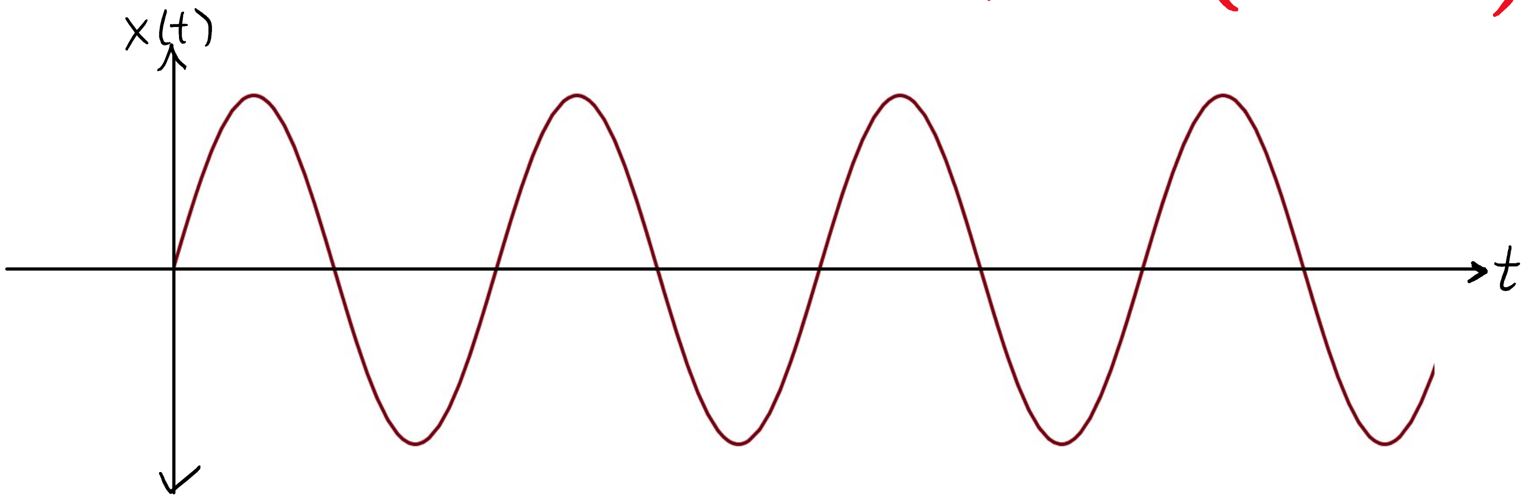
$A\cos(\omega t+\phi)$

negative ϕ shifts right



★ shift of 2π is a whole period ★

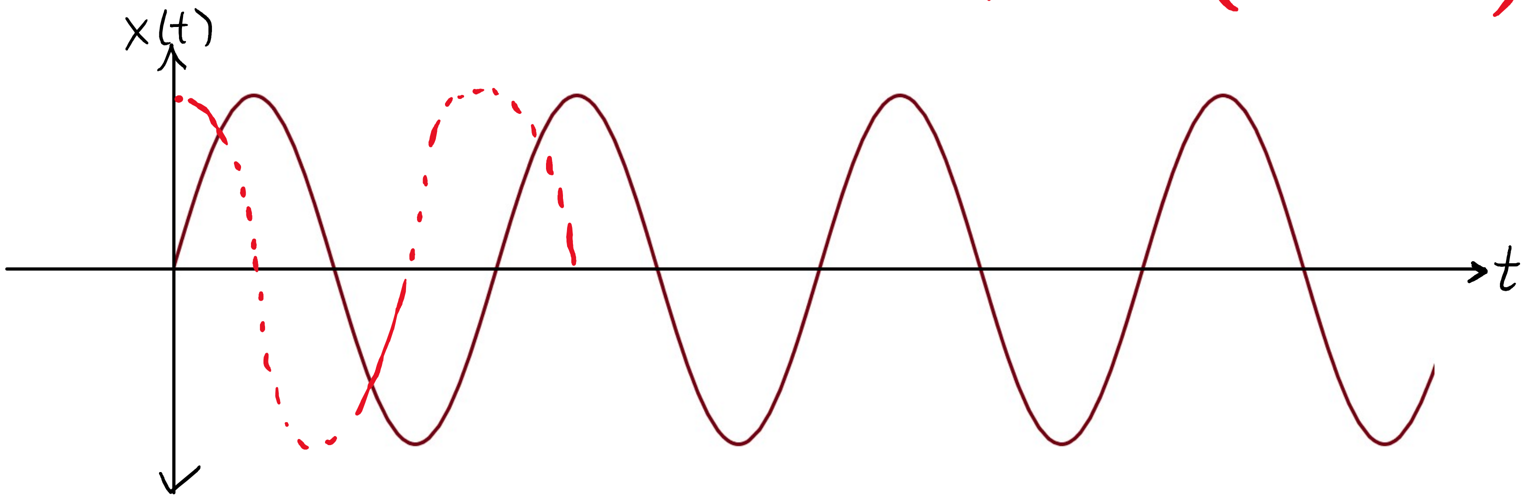
$$x(t) = A\cos(\omega t + \phi)$$



For the displacement graph shown, what is the phase ϕ?

A 0 B) π/2 C) π D) - π/2

$$x(t) = A\cos(\omega t + \phi)$$



shifts to the right by $\frac{1}{4}$ period

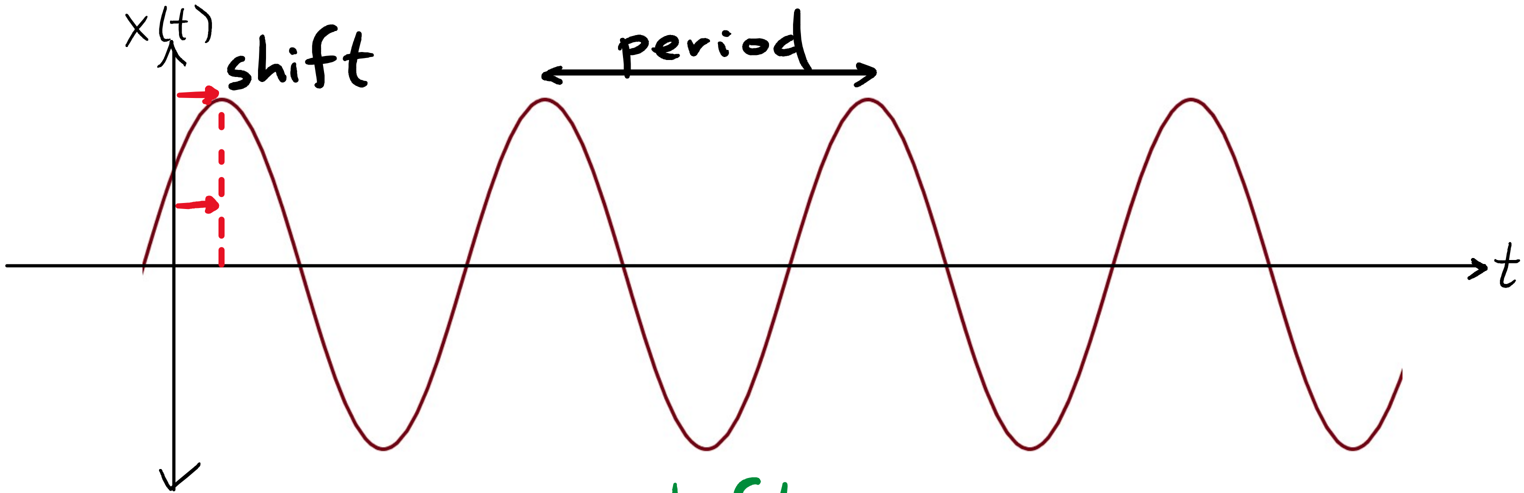
so $\phi = -\frac{1}{4} \times 2\pi$

$= -\frac{\pi}{2}$

For the displacement graph shown, what is the phase ϕ?

A 0 B) π/2 C) π D) - π/2

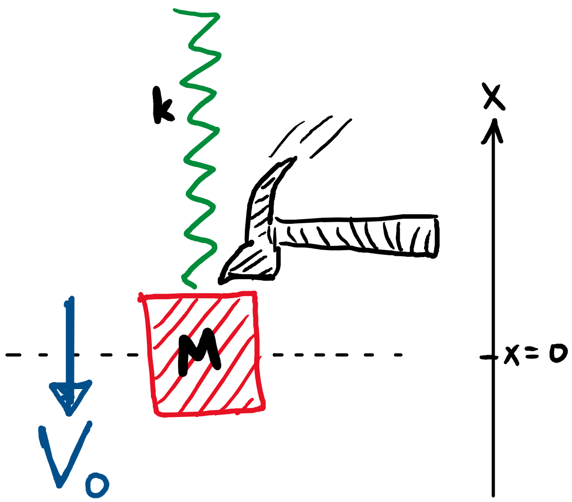
How to find ϕ



to the left

$$\phi = \pm\, 2\pi \times \frac{\text{shift}}{\text{period}}$$

to the right

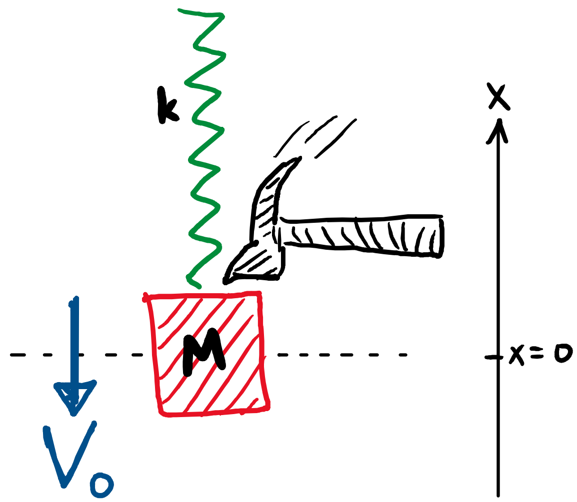


A mass on a spring is struck with a hammer, giving it an initial downward velocity when it is at its equilibrium position. Which of the following functions could describe the motion of the mass?

A) $x(t) = A\cos(\omega t - \pi/2)$
B) $x(t) = A\cos(\omega t)$
C) $x(t) = A\cos(\omega t + \pi/2)$
D) $x(t) = A\cos(\omega t + \pi)$
E) None of the above

Hint: sketch the graph of $x(t)$

EXTRA: can you determine A in terms of v_0 and ω?



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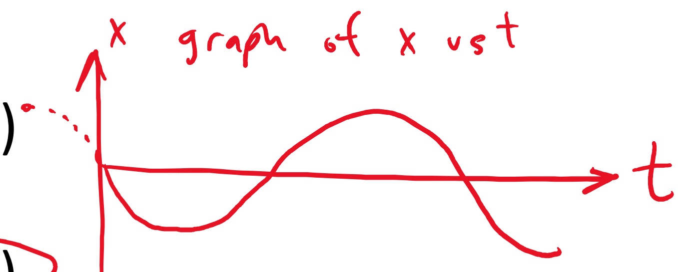


A) $x(t) = A\cos(\omega t - \pi/2)$
B) $x(t) = A\cos(\omega t)$
C) $x(t) = A\cos(\omega t + \pi/2)$
D) $x(t) = A\cos(\omega t + \pi)$
E) None of the above

initial position: $x=0$

after $t=0$, x decreases until some min. value, then comes back up to $x=0$ but with +ve velocity. So x then increases above 0, etc..

Hint: sketch the graph of $x(t)$

like cosine, but shifted to left by $\frac{1}{4}$ period.

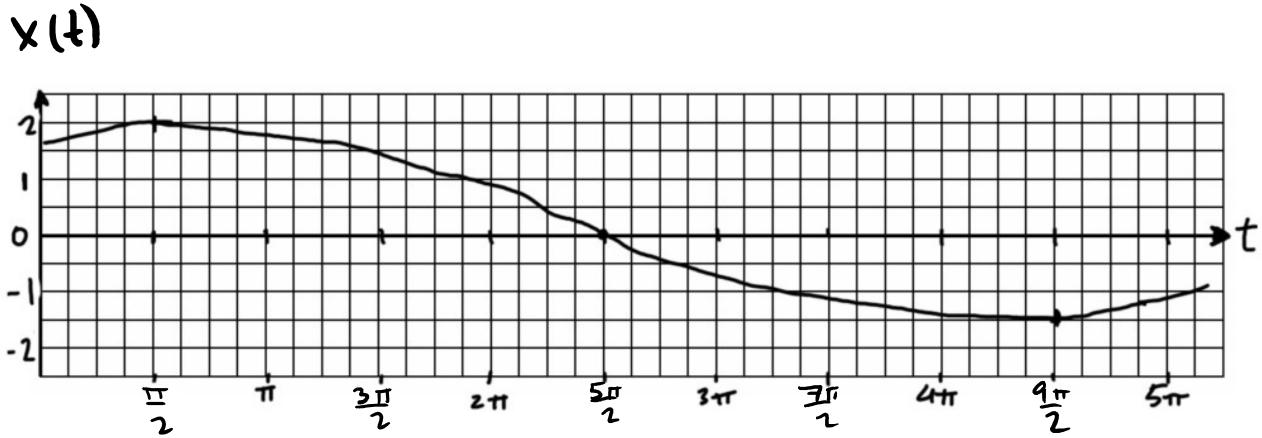
$\therefore \phi = +\frac{1}{4} \times 2\pi$

$= \frac{\pi}{2}$

EXTRA: can you determine A in terms of v_0 and ω?

$\hookrightarrow$ velocity is $\frac{dx}{dt} = -A\omega \sin(\omega t + \frac{\pi}{2})$. At $t=0$, $v = -v_0$, so: $-v_0 = -A\omega \sin(\frac{\pi}{2}) \Rightarrow A = \frac{v_0}{\omega}$

EXTRA:

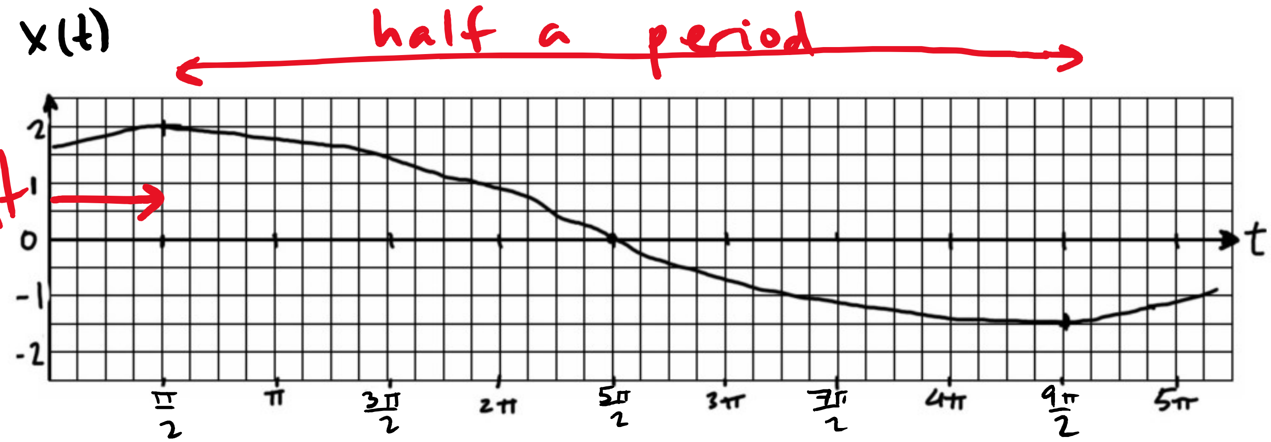


$$\phi = \pm 2\pi \cdot \frac{\text{shift}}{\text{period}}$$

$$x(t) = A\cos(\omega t + \phi)$$

For the displacement graph shown, what is the phase ϕ?

A) -π/8 B) -π/4 C) –π/2 D) π/4 E) π/8



$\phi = \pm 2\pi \cdot \frac{\text{shift}}{\text{period}}$

$$x(t) = A\cos(\omega t + \phi)$$

period is

$T = 2 \times \left(\frac{9\pi}{2} - \frac{\pi}{2}\right)$

$\Rightarrow T = 8\pi$

shift is by $\frac{\pi}{2}$

For the displacement graph shown, what is the phase ϕ?

A) -π/8 B) -π/4 C) –π/2 D) π/4 E) π/8

phase is : $\phi = -2\pi \times \frac{\text{shift}}{\text{period}} = -2\pi \times \frac{\pi/2}{8\pi} = -\frac{\pi}{8}$