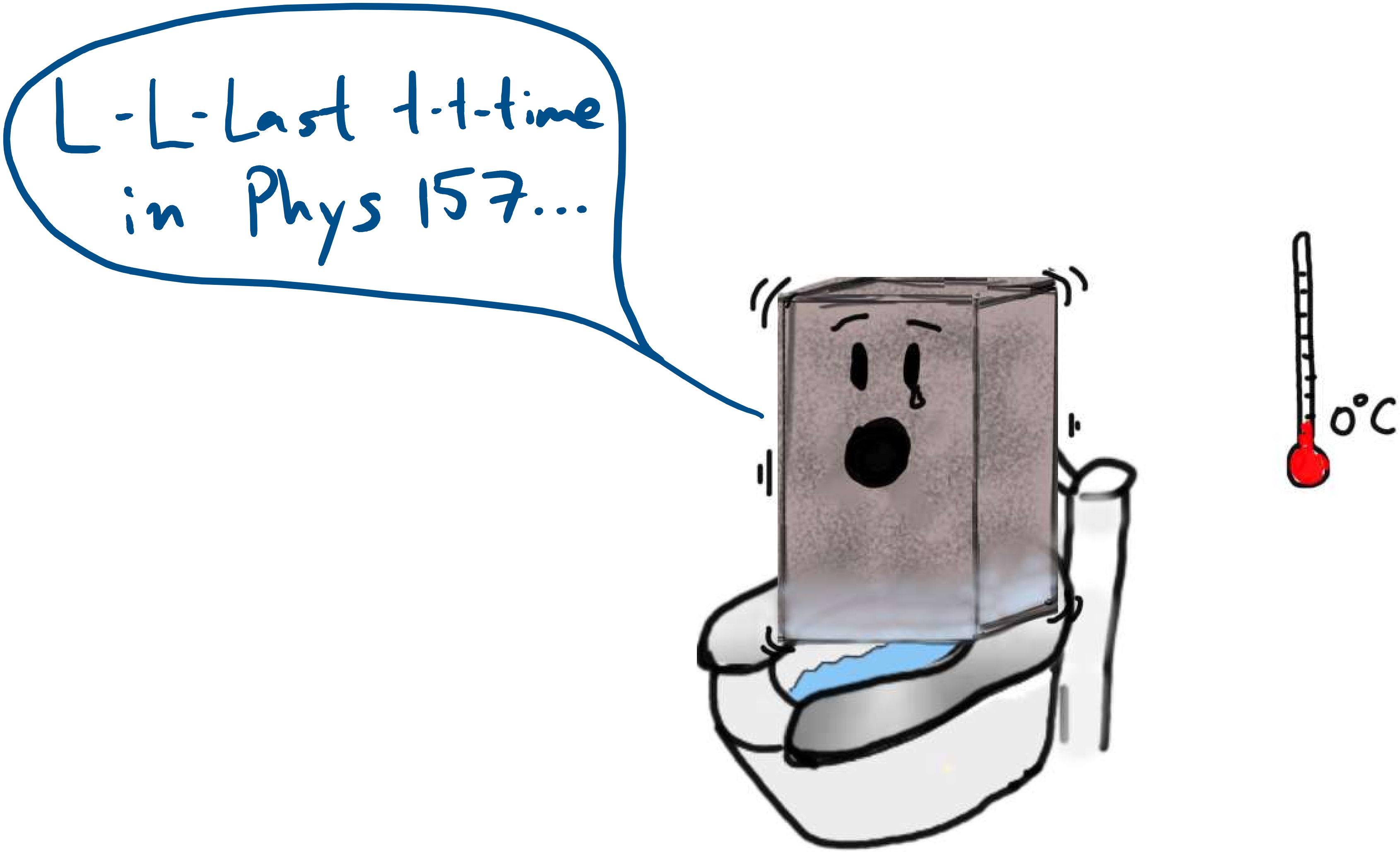
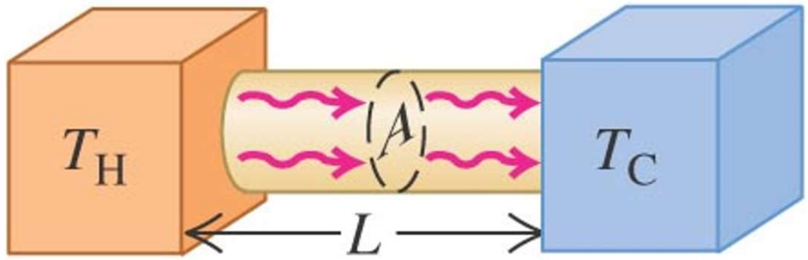
Learning goals:

- When heat is flowing steadily from an object with a higher thermal conductivity to an object with a lower thermal conductivity, explain how the heat currents can be the same
- Calculate heat flow or interface temperatures in systems with materials of various thermal conductivities
- Given the heat current into or out of an object, calculate the heat transferred in a given amount of time, or the temperature change of that object in a given amount of time


L-L-Last t-t-time in Phys 157...
0°C

THERMAL CONDUCTIVITY: Determines heat current from temperature gradient.

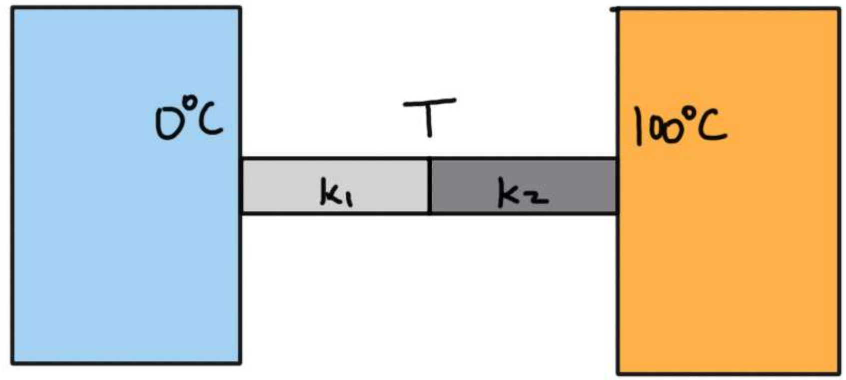


$$H = kA\underbrace{\frac{T_H - T_C}{L}}_{\text{temperature gradient}}$$

Heat current
"
Heat per time

Thermal conductivity

Two materials of equal dimensions but different thermal conductivities are placed side to side between objects kept at 0ºC and 100ºC, and a steady heat flow is established. If $k_1 > k_2$, we can say that the temperature T in the middle is:

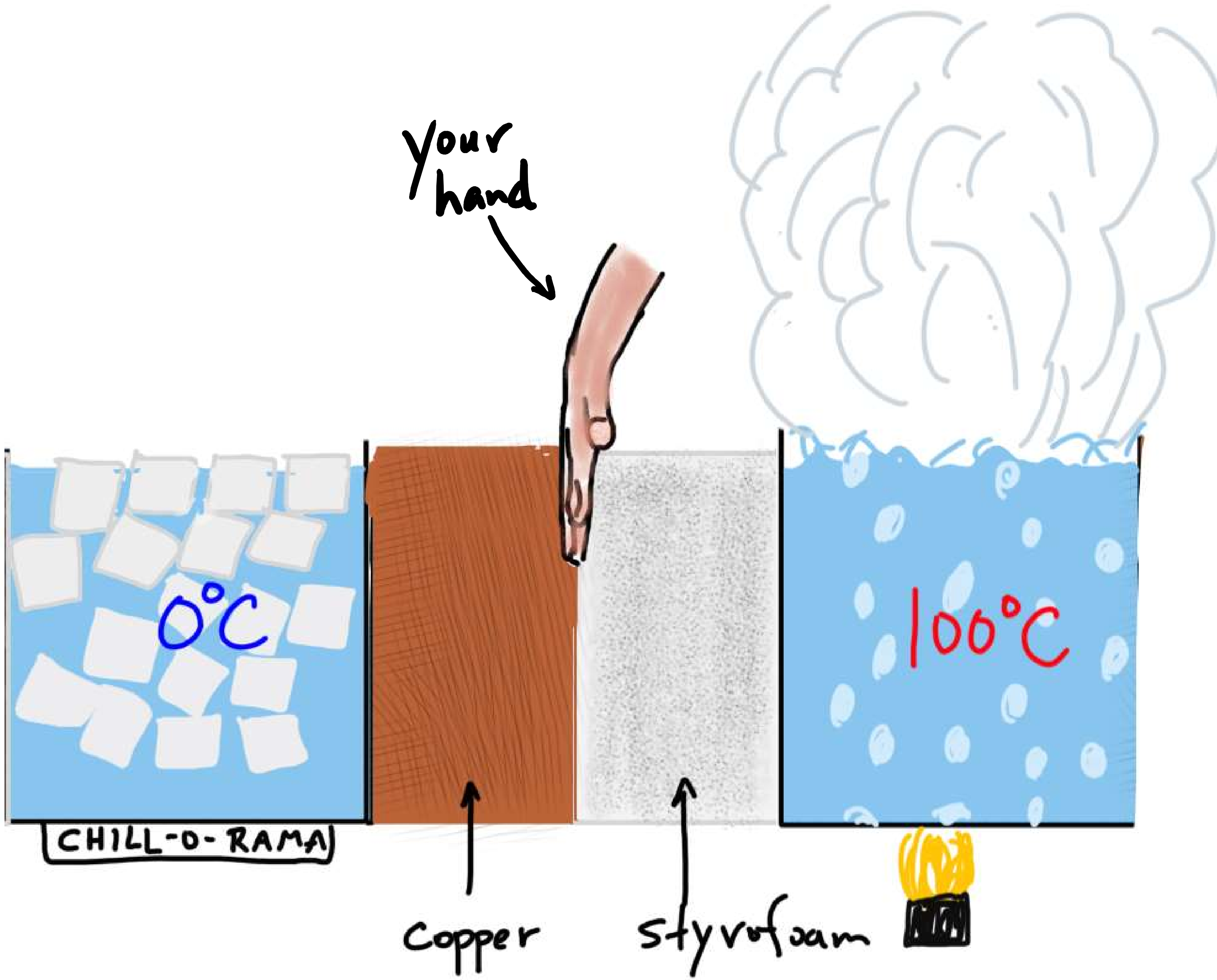


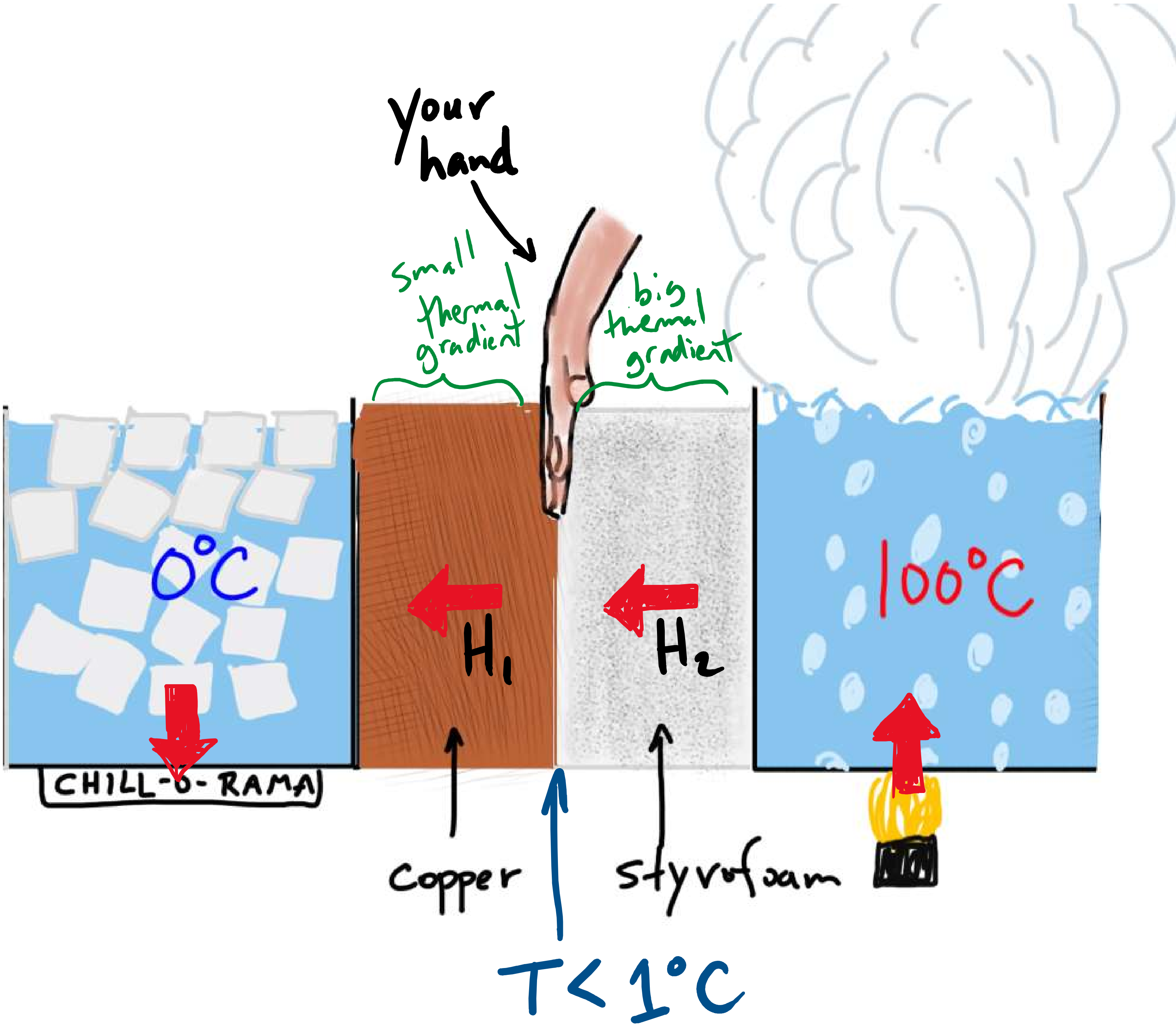
A) Equal to 50ºC

B) Greater than 50ºC

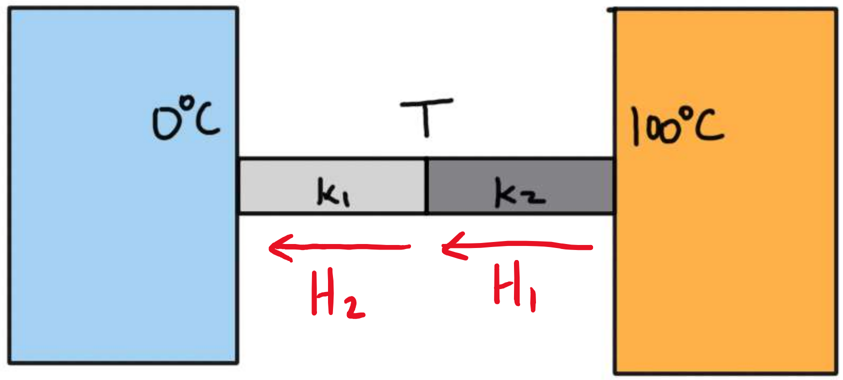
C) Less than 50ºC

EXTRA: How would you calculate the temperature.


your hand
0°C
100°C
CHILL-O-RAMA
copper
styrofoam


your hand
small thermal gradient
big thermal gradient
0°C
H_1
H_2
100°C
CHILL-O-RAMA
Copper
Styrofoam
T < 1°C

$$\boxed{H = kA\frac{T_H - T_C}{L}}$$



$k_1 > k_2$

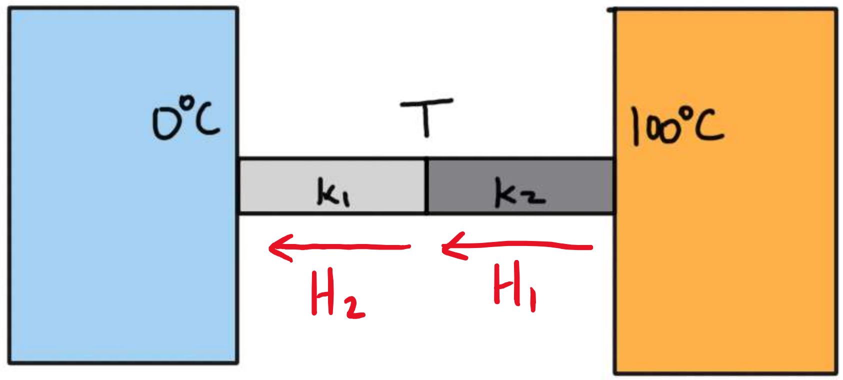
Calculate T in terms of k_1 and k_2

Hint: what are H_1 and H_2 and how are they related to each other?

Click A if you have an answer

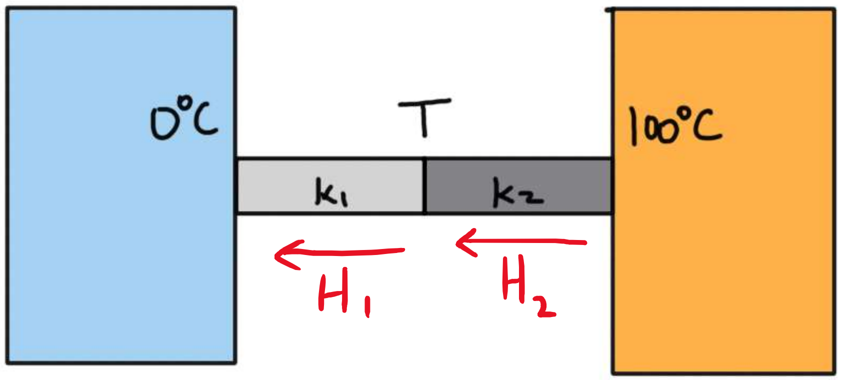
Click B if you are stuck

$$\boxed{H = kA\frac{T_H - T_C}{L}}$$



$k_1 > k_2$

Calculate T in terms of k_1 and k_2



$k_1 > k_2$

+ steady flow

- Energy conservation $\Rightarrow H_1 = H_2$

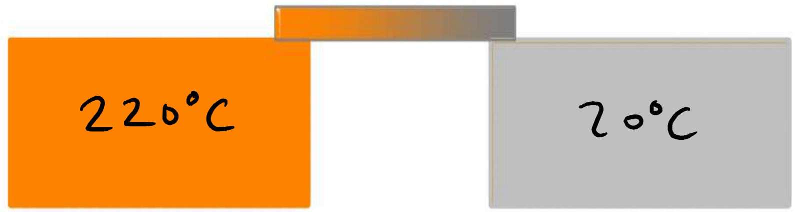
- $$k_1 \cdot A \cdot \frac{T - 0°C}{L} = k_2 \cdot A \cdot \frac{100° - T}{L}$$

$$k_1 (T - 0°C) = k_2 (100° - T)$$

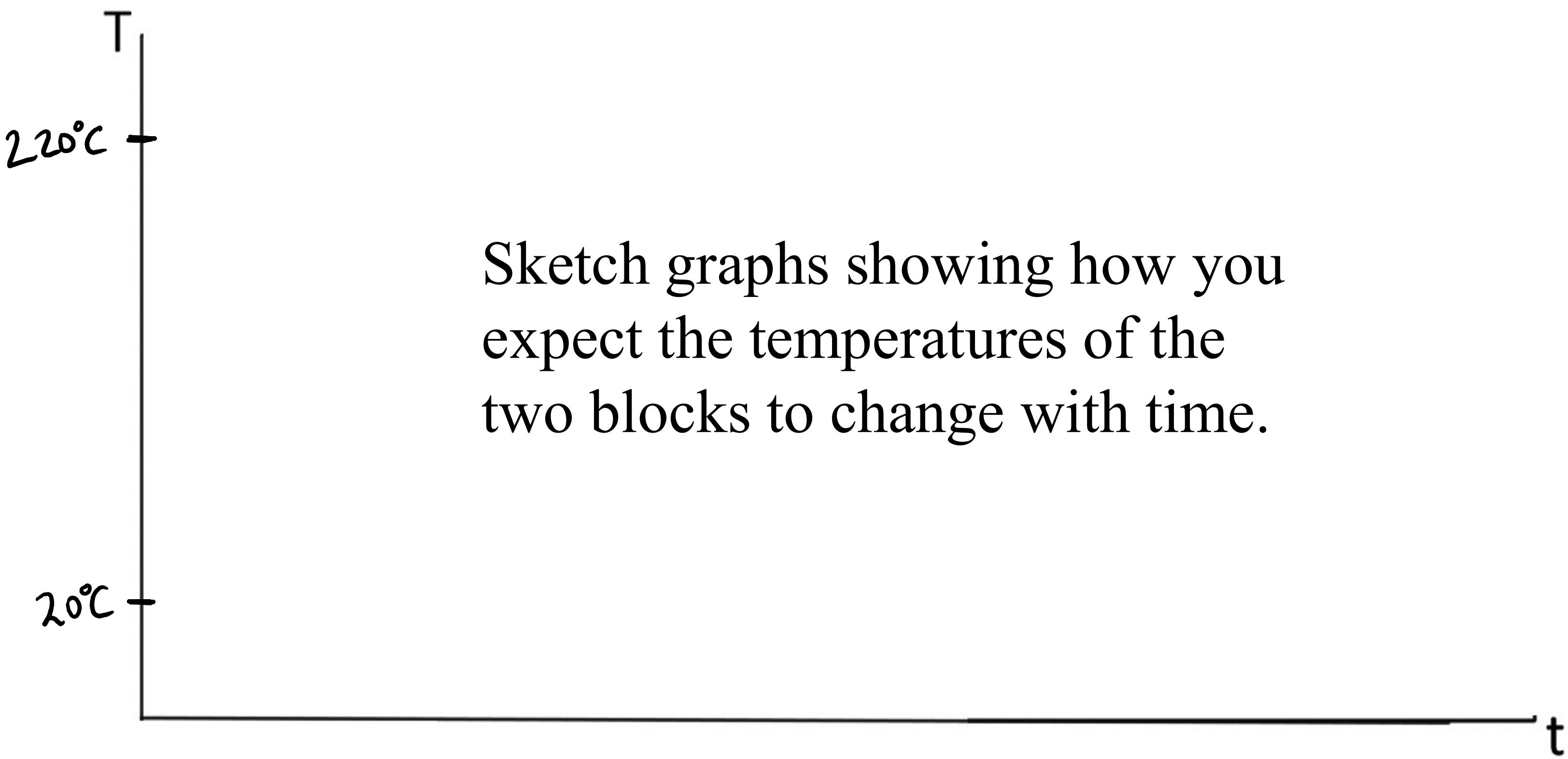
bigger

smaller

$$T = \frac{k_2}{k_1 + k_2} \times 100°C$$

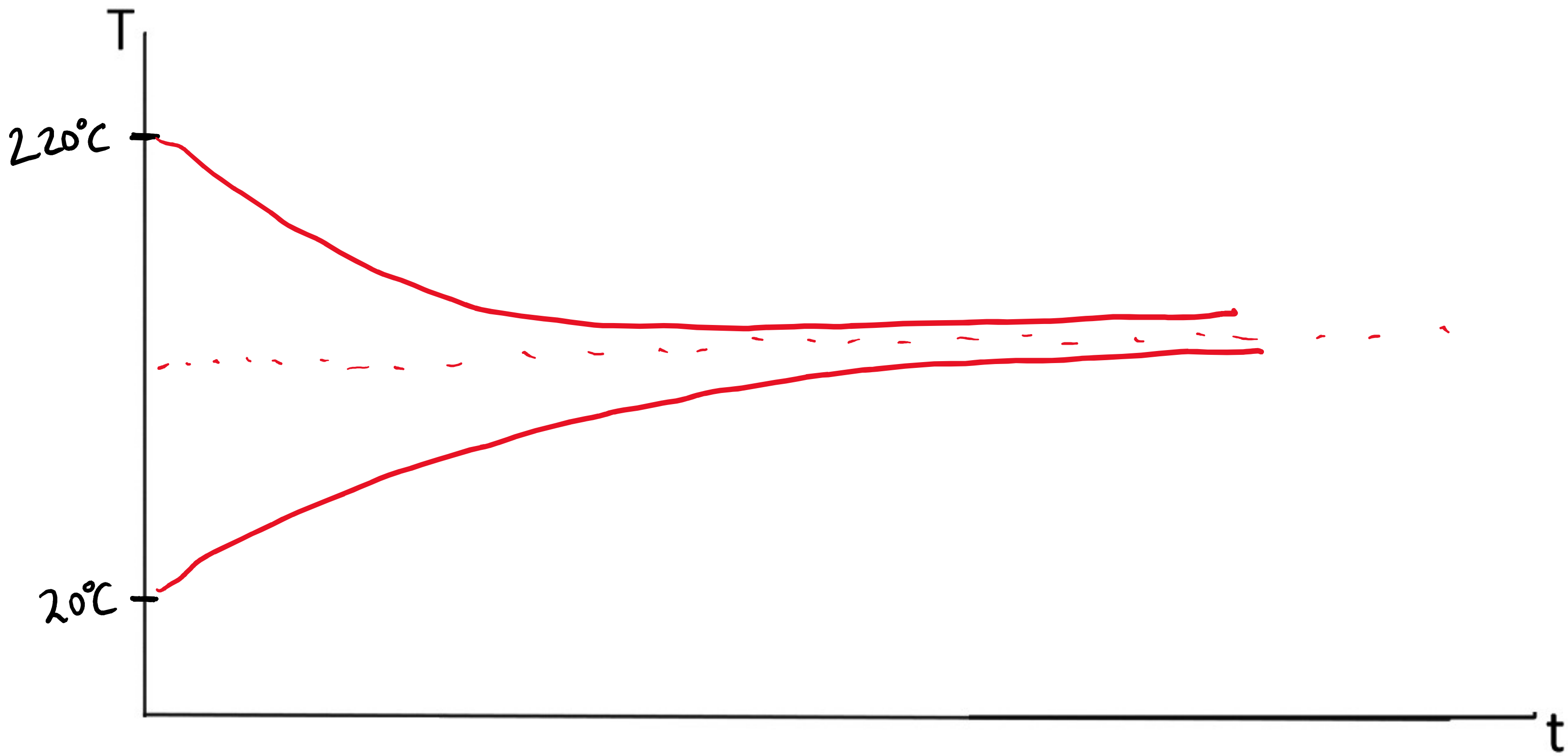


A block of aluminum at room temperature (T_1 = 20℃) is connected to another equivalent block of aluminum at (T_2 = 220℃) by another strip of aluminum (that has been in place for a while).

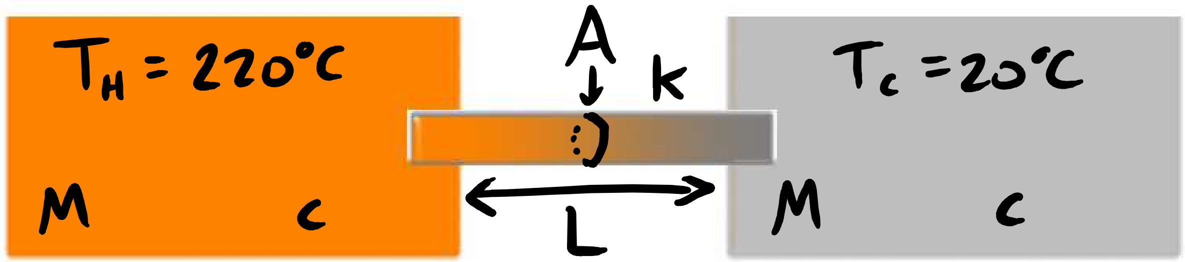


Sketch graphs showing how you expect the temperatures of the two blocks to change with time.

Sketch graphs (one for each block) showing how you expect the temperatures of the two blocks to behave as a function of time.



Let's understand this quantitatively



$M = 0.1\,kg$

$c = 900\,J/kg$

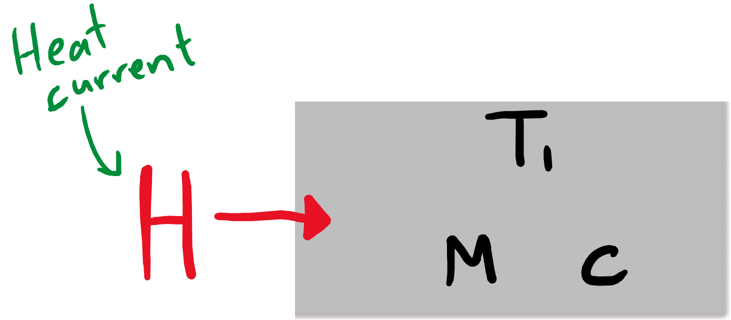
$k = 200\,\frac{W}{m}\cdot K$

$A = 0.1\,cm^2$

$L = 1\,cm$

What is the change in temperature dT of the cooler block that occurs in a small time dt = 1 second?

Strategy: first look at parts separately

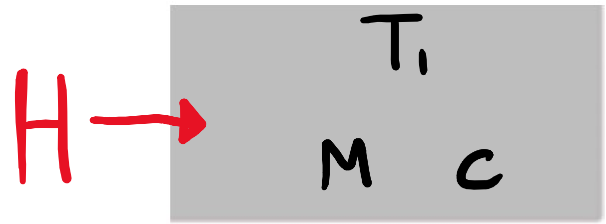


$$\boxed{Q = Mc\Delta T}$$

A heat current **H** flows into the cooler block. In a time **dt**, what is the change **dT** in the temperature of this block (in terms of dt and the quantities shown)?

A) $\dfrac{H}{Mc}$ B) $\dfrac{H\,dt}{Mc}$ C) $\dfrac{H}{Mc\,dt}$ D) H dt E) H / dt

Hint: how much heat enters the block during this time?



$$\boxed{Q = Mc\,\Delta T}$$

A heat current **H** flows into the cooler block. In a time **dt**, what is the change **dT** in the temperature of this block (in terms of dt and the quantities shown)?

Hint: how much heat enters the block during this time?

In time dt, heat added is $Q = H\,dt$.

Heat per time

time

$$\boxed{Q = Mc\Delta T}$$

A heat current **H** flows into the cooler block. In a time **dt**, what is the change **dT** in the temperature of this block (in terms of dt and the quantities shown)?

Hint: how much heat enters the block during this time?

In time dt, heat added is $Q = H\,dt$.

We have $dT = \dfrac{Q}{Mc}$.

So: $dT = \dfrac{H}{Mc}\,dt$

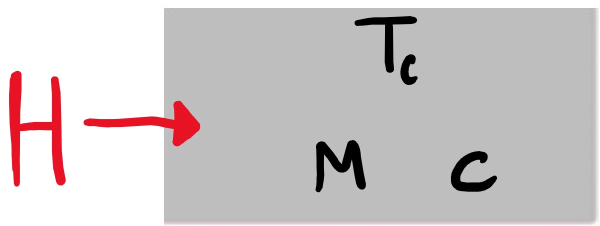
(answer B)

What is the change in temperature dT of the cooler block that occurs in a small time dt?

$$\boxed{Q = Mc\,\Delta T}$$

$$\boxed{H = kA\frac{T_H - T_c}{L}}$$

Cool block:



$$dT = \frac{H}{Mc} \cdot dt$$

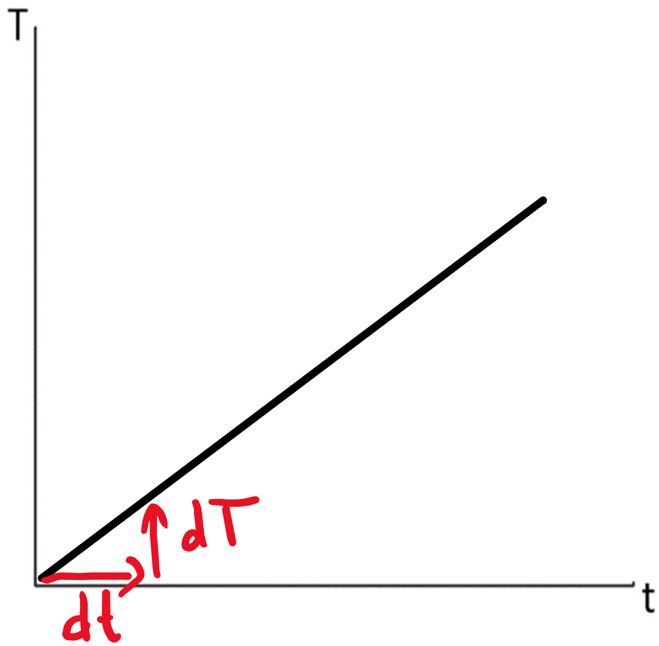
Strip:

$$H = k \cdot A \cdot \frac{T_H - T_c}{L}$$

(all Hs same by energy conservation)

Combine:

$$dT = \frac{kA}{McL} \cdot (T_H - T_c) \cdot dt$$

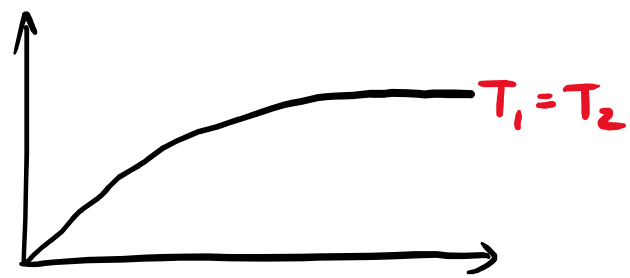


Slope is $\frac{dT}{dt}$

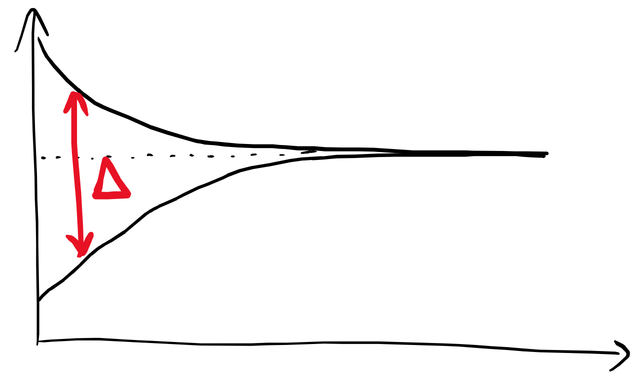
From previous slide:

$$\frac{dT}{dt} = \frac{kA}{McL} \cdot (T_2 - T_1)$$

decreases as T_2 gets closer to T_1:



$\Delta = T_2 - T_1$ decreases twice as fast as T_1 increases:

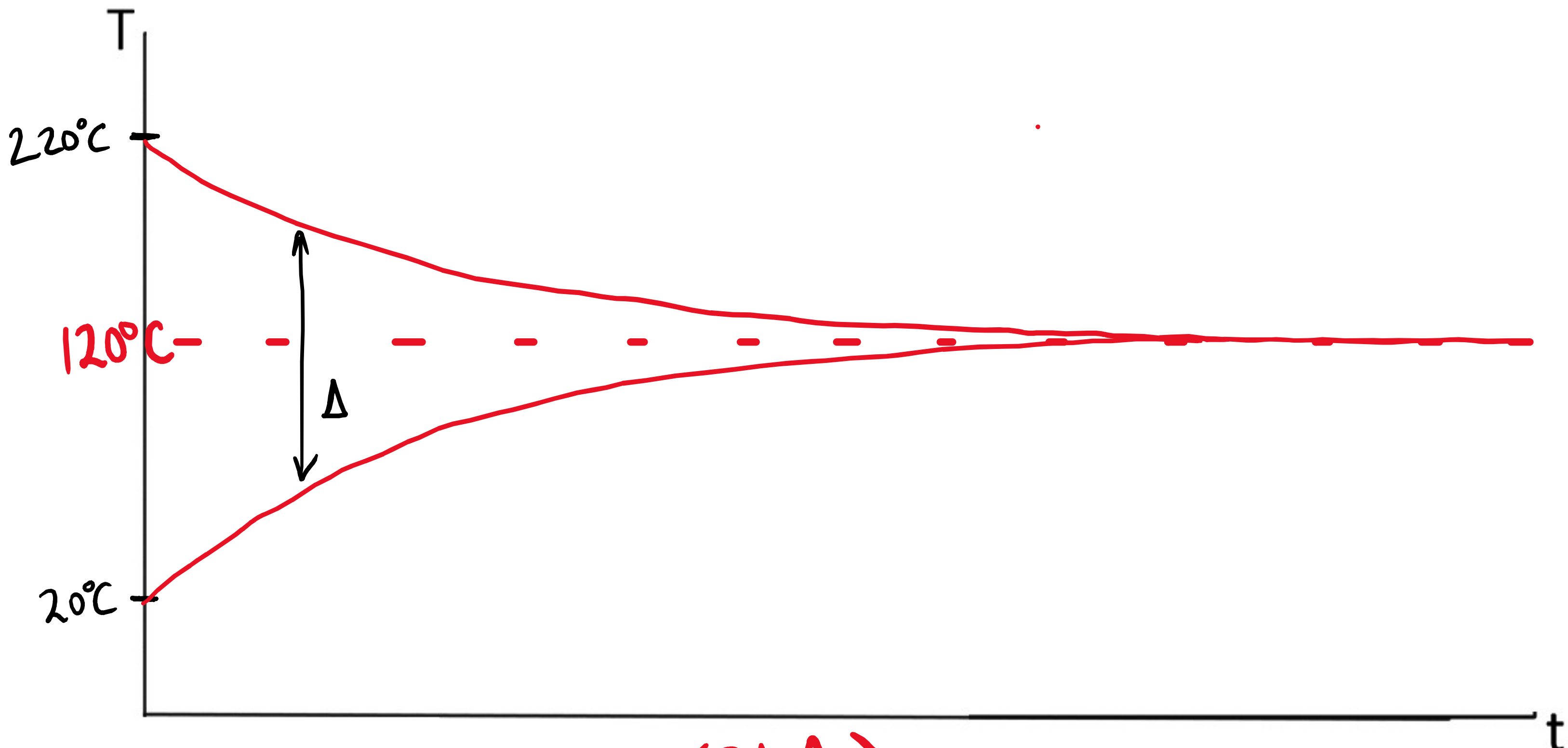


$$\frac{d\Delta}{dt} = -\frac{2kA}{McL} \cdot \Delta$$

Rate of decrease of Δ is proportional to Δ.

Math: this means $\Delta(t)$ is an EXPONENTIAL

$$\Delta(t) = \Delta_{t=0} \cdot e^{-\frac{2kA}{McL} \cdot t}$$



$$\Delta(t) = 200^\circ \times e^{-\left(\frac{2kA}{McL}\right)t}$$