## Physics 157 Tutorial 1 - Young's modulus

Today, we'll think about how the length of an object changes when stretching or compressing forces act on it. This will be important next week when we talk about the forces generated by heating and cooling structures. Work in groups of three or four, but hand in your own worksheet. Ask the TA for help if you are stuck!

You've probably learned about Hooke's Law for a spring. This says that if we exert a pushing or pulling force $F$ on the spring, the change in length $\Delta L$ is related to the force by

$$
\begin{equation*}
\mathrm{F}=\mathrm{k} \Delta \mathrm{~L} \tag{1}
\end{equation*}
$$



Here, F is positive for pulling and negative for pushing. The number $k$ is a constant (the "spring constant") that is larger for stiffer springs.

Actually, almost all materials obey a law like this (as long as the forces aren't too large); they are just usually not as stretchy as a spring, so it's harder to measure $\Delta \mathrm{L}$. For a block of some solid material, like a brick, the constant $k$ in the equation above depends on the material, but also on the dimensions of an object. Let's understand how:


Question 1: First, let's think about how $k$ depends on the cross-sectional area A, shown in the middle figure above.

Suppose we have an object with twice the cross-sectional area as the one in the pictures above, but the same length. If we also apply twice as much compressive force as in the first picture above (shown at the right), we expect that the amount of compression $\Delta L$ will be
a) Twice as much as in the first picture above
b) Half as much as in the first picture above
c) The same as in the first picture above

Why?


Hint: we can't use equation (1) to answer this because the constant $k$ may not be the same for the two cases. Instead, compare the two pictures and think of a direct argument for your answer.

Question 2: For each quantity below, circle one of the options to say how that quantity in the example of question 1 compares to the original case:

| F: | same as before | double | half |
| :--- | :--- | :--- | :--- |
| $\boldsymbol{\Delta L}:$ | same as before | double | half |
| $\mathbf{k}=\mathrm{F} / \boldsymbol{\Delta L}:$ | same as before | double | half |
| A: | same as before | double | half |

Question 3: Based on your answer to question 2, would you say that $k$ is
a) proportional to the area A (i.e. doubling A doubles k )
b) inversely proportional to the area $A$ (i.e. doubling $A$ halves $k$ )
c) independent of the area A (i.e. doubling A doesn't change $k$ )

Check with a TA to make sure you have the right answer here!
Question 4: Now let's understand how $k$ depends on the length/thickness of our object if we keep the cross sectional area fixed. First, imagine a bed made from a thin foam mattress. Now imagine a second bed made from several of these stacked on top of one another. If two people of the same mass lay on each bed, we expect that

a) The second bed will compress significantly less than the first
b) The second bed will compress the same amount as than the first
c) The second bed will compress significantly more than the first

Question 5: The figure below shows an object with twice the length and the same cross-sectional area as the object in the first picture on the previous page. If we apply the same force as in that case, there will be:
a) The same compression as before.
b) Half the compression as before.
c) Twice the compression as before.


Hint: Use your intuition from question 4! Question 5 is like the situation with two mattresses, except things are rotated so the forces are horizontal.

Can you give a convincing argument for your answer?

Question 6: For each quantity below, circle one of the options to say how that quantity in the example of question 5 compares to the original case:

| F: | same as before | double | half |
| :--- | :--- | :--- | :--- |
| $\boldsymbol{\Delta L}:$ | same as before | double | half |
| $\mathbf{k}=\mathrm{F} / \boldsymbol{\mathrm { L } :}$ | same as before | double | half |
| $\mathrm{L}_{0}:$ | same as before | double | half |

Question 7: Based on your answer to question 6, would you say that $k$ is
a) Proportional to the length $L_{0}$ (i.e. doubling $L_{0}$ doubles $k$ )
b) Inversely proportional to the length $L_{0}$ (i.e. doubling $L_{0}$ halves $k$ )
c) Independent of the the length $L_{0}$ (i.e. doubling $L_{0}$ doesn't change $k$ )

Check with a TA to make sure you have the right answer here!

Hopefully, you have convinced yourself that $k$ should be proportional to area (if length is kept fixed) and inversely proportional to length (if area is kept fixed). This allows us to write an equation

$$
\begin{equation*}
\mathrm{k}=\mathrm{Y} \frac{\mathrm{~A}}{\mathrm{~L}_{0}} \tag{2}
\end{equation*}
$$

where $\mathbf{Y}$ is a constant that does not depend on the length or the area. This constant is called the Young's modulus, and is a basic property of the material that our object is made of. It tells us how stiff that material is.

Substituting the expression for $k$ in (2) into equation (1), we get:


The left side is the force per unit area that we apply. This is called the STRESS. The ratio $\Delta \mathrm{L} / \mathrm{L}_{0}$ is fractional change in length of the object. It is called the STRAIN. So equation (3) says that stress is proportional to strain, and the Young's modulus Y is the proportionality constant.


Question 8: Let's put the theory into practice and actually measure the Young's modulus of a material. Since we'll want to be able to measure changes in length that result from modest applied forces, we're going to use a material that's a little squishier than brick: marshmallow.

Pick up from the front of the room: a marshmallow and a cup into which you have measured 100 ml of water. You can use the ruler above if you don't have one.
a) Design an experiment to measure $Y$ : describe
 what you will do and what you will measure:
b) Write your measurement results below. We're just going for an order of magnitude here, so they don't have to be super-accurate. If it's within $25 \%$, that's fine.
c) Calculate Y . In your calculations, use SI units of meters, $\mathrm{kg}, \mathrm{N}$ so that your final result for Y will be in $N / m^{2}$
d) Write down or take a picture of your final result, then hand in your worksheet, along with your cup and marshmallow. It will be marked for participation credit. You can get full credit even if you didn't finish. If you don't have time to finish, record your measurements and finish the calculations later, but please still hand in your worksheet.

You will be asked for your result during next weekend's reading quiz.

