Physics 157 Homework 9: due Fri, Nov 29th by 5pm

The focus of this homework set is analyzing damped oscillations. We'd like you to be able to do the following

- Given a graph representing the displacement vs time or the acceleration vs time for damped oscillation, to deduce the time constant for the system or (given the mass) the damping constant b, and to write an equation for the displacement vs time.
- Given a graph representing the displacement vs time or the acceleration vs time for damped oscillation, to determine the energy lost per cycle.
- To determine the damping required to produce critical damping given a graphical or mathematical representation of the oscillations in an undamped or underdamped system.

Your Homework: For the questions below, put your answers into Mastering Physics **and** hand in your written solution to the homework box. Also do the Mastering Physics questions on waves.

New equations specific to damped oscillations:

Displacement vs time for damped oscillation (where amplitude decreases by an equal fraction for equal time intervals):

$$A\, e^{-t/\tau} \cos(\omega t + \phi)$$

Energy is proportional to amplitude squared

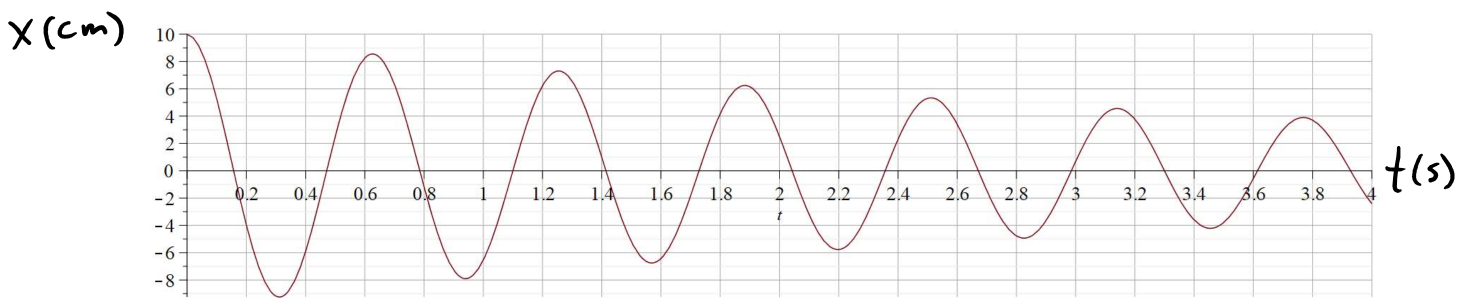
Energy is $\frac{1}{2} k x^2$ at times when x is maximum.

Calculating τ for a system with damping forces $F = -b v$: $\boldsymbol{\tau = 2m/b}$

Frequency for damped oscillation with $F_{NET} = -k x - b v$: $\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4 m^2}}$

Note: When damping is small (i.e. there are many visible oscillations), it's a good approximation just to use $\omega = \sqrt{\frac{k}{m}}$.

Critical damping (no more oscillations) occurs for: $b = 2\sqrt{k\, m}$



Problem 1

The graph shows the displacement vs time for an object of mass 2kg oscillating on a spring.

a) Assuming that we can represent the motion of this object by

$$x(t) = A\,e^{-t/\tau}\cos(\omega t + \phi),$$

what is the time constant τ?

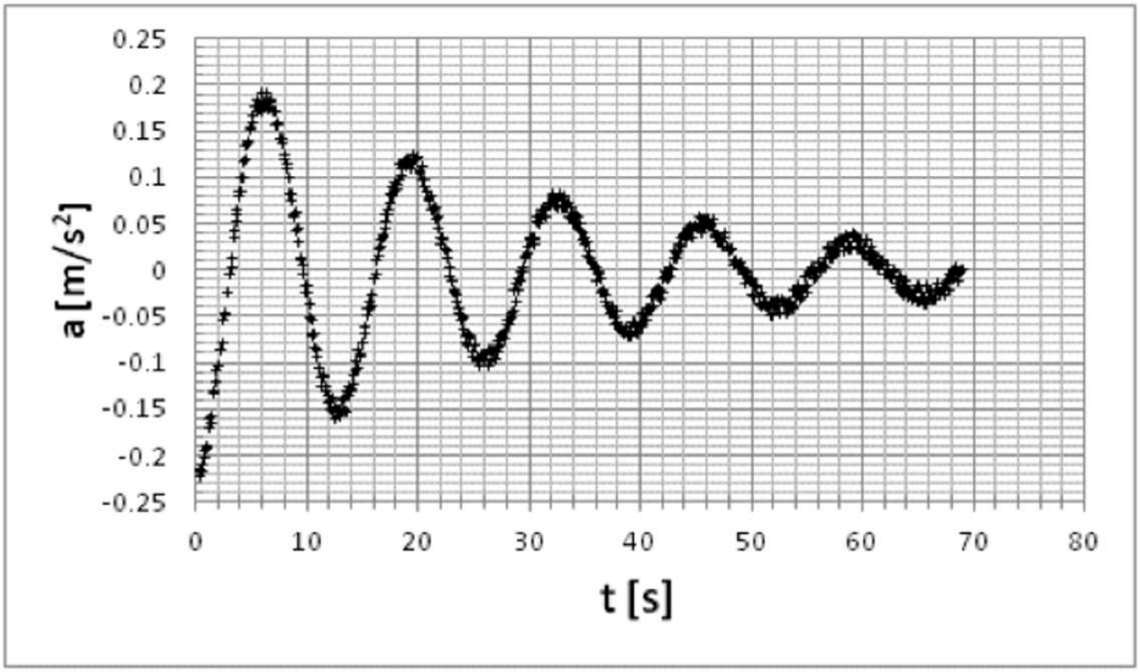
b) What is the frequency ω?

c) What is the initial amplitude A?

d) What is the damping constant b?

e) What is the spring constant k?

f) What is the fraction of energy lost in one complete oscillation?



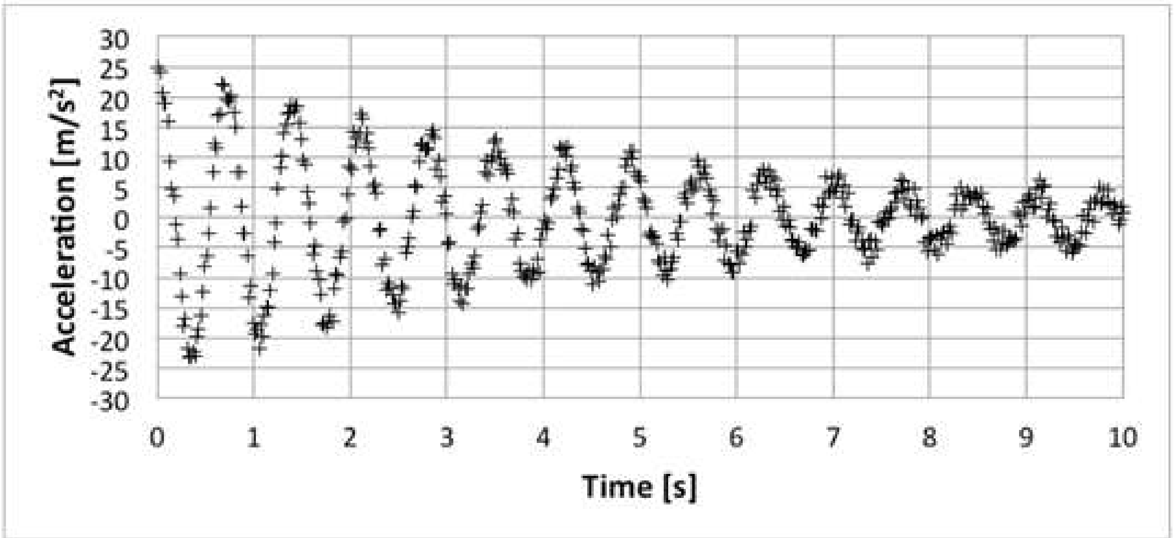
Problem 2

One day, a large drilling platform (M= 9×10^7kg) is floating in the ocean minding its own business when a tsunami passes by. The upward vertical acceleration vs time for the platform after the tsunami passes is measured by an accelerometer and plotted in the graph above.

a) What is the damping constant b for this oscillation?

b) What was the initial vertical displacement of the platform (give a positive answer if it was upwards and a negative answer if it was downwards)?

Problem 3. The engineers working on a car prototype before the shock absorbers were installed had the car pushed down and released and recorded the acceleration as a function of time. The graph of the recording is shown below. The mass of the car was 1500 kg.



a) Assuming that the graph of acceleration as a function of time can be described by

$$a = a_0 \exp(-\frac{t}{\tau}) \cos(\omega t + \phi)$$

find the parameters a_0, τ, ω, and ϕ.

b) What is the spring constant of the springs in this car?

c) What is the damping constant of this movement?

d) How much was the car pushed down?

e) What should be the damping constant of the shock absorbers for this car? (The shock absorbers should achieve critical damping)

Note for part e: "Critical damping" refers to the value b = 2 (k m)$^{1/2}$ above which the displaced system no longer oscillates at all, but simply returns to its equilibrium position via an exponential decay.