Physics 157 Homework 7: due Wed, Nov 13th by 5pm

In this homework set, you'll begin to practice analyzing systems in simple harmonic motion (SHM). Here are the skills that we'd like you to develop.

- Given a graph of displacement vs time for a simple harmonic oscillator, determine the amplitude, period, angular frequency, and phase for the corresponding function (represented as a cosine).
- Given a graph of displacement, velocity, or acceleration vs time for a simple harmonic oscillator, sketch graphs for the other two quantities. Also, to say where each of these quantities is zero, maximum or minimum and give the maximum and minimum values.
- Given the displacement, velocity, or acceleration represented as a sinusoidal function of time, to calculate the functions corresponding to the other two quantities and sketch graphs of all three quantities.
- Given the graph or functional form of displacement, velocity, or acceleration for a simple harmonic oscillator, to determine the constant k in the force law given the mass of the object, or vice-versa.

Your Homework: Do the mastering physics assignment "Homework 7".

A summary of the formulae you may need:

Restoring force leading to SHM: $F = -kx$

Newton's 2nd Law: $a = F/m$ (this allows us to predict the acceleration based on the net force)

Basic form of simple harmonic motion: $x(t) = A\cos(\omega t + \phi)$ $\omega^2 = k/m$

Here A is the amplitude, ω is the angular frequency, related to the period by $\omega = 2\pi / T$, and ϕ is the phase.

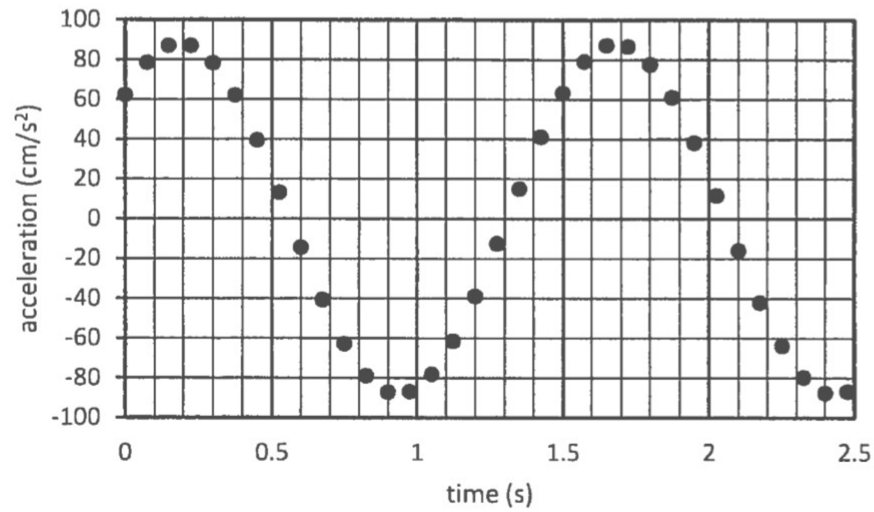
The phase can be determined looking at what fraction of a period the graph is shifted to the left or right by (take the time that the function is maximum and divide by T) , and then multiplying by ±2π, with + for shifts to the left and – for shifts to the right.

Velocity and acceleration: $v(t) = dx/dt$ (slope of displacement graph at time t)

$a(t) = dv/dt$ or $a = -\omega^2 x$

Old midterm problems (see solutions in midterm 2 samples)

Problem 3. A plot of the acceleration as a function of time for a mass on a spring is shown.



a) Writing the position in the form $x(t) = A\ \cos(\omega t + \varphi)$, find the amplitude A, frequency ω, and phase constant φ.

b) On the grid on the next page, plot the velocity as a function of time from $t = 0$ to $t = 2.5$ seconds. (Make sure to label the axes.)

c) If the mass is 100g, what is the spring constant?

3. Two identical springs fixed to a rigid support on one side, with identical 500g masses attached on the other end, sit next to each other as shown in the diagram below. One spring (A) is compressed from its equilibrium position by an amount 2.5cm by pushing on it with a force of 5.0N, the other (B) is stretched from its equilibrium position by the same amount by applying the same force. Both are let go at the same time; consider this t=0. Displacements to the right are positive, and to the left are negative.

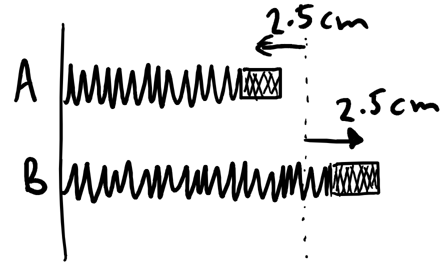
(a) What is the period of oscillation?

(b) What is the maximum acceleration?

(c) Sketch the displacement vs. time for each. Label the graphs with quantitative values on both axes.

(d) Sketch the acceleration vs. time for each. Label the graphs with quantitative values on both axes.

(e) What is the phase difference between the oscillation of B and oscillation of A?



3.

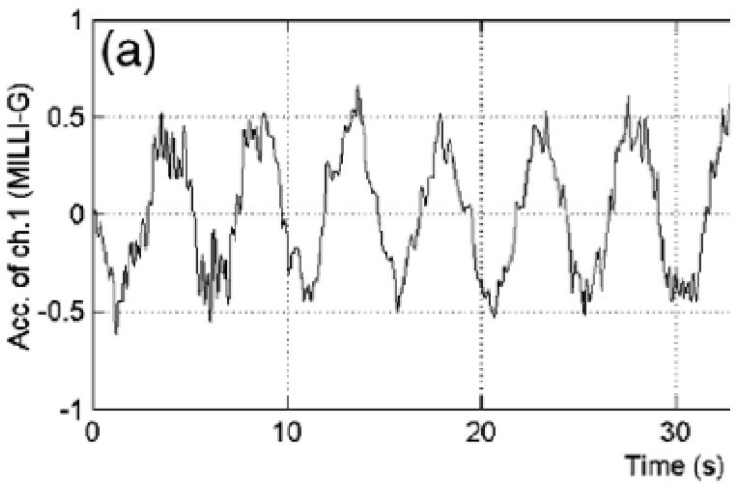


Fig.1. Shows the recording of an accelerometer placed at the height of 298.34 m of the building Di Wang Tower located in Shenzhen, China. This recording of data was taken during a strong wind. Vertical scale is in MILLI-G which is 1/1000 of g (g is the acceleration due to gravity so 1 MILLI-G $= 9.8 \times 10^{-3}\ m/s^2$).

a) If you approximate the displacement of the building as a function of time seconds by $x = A\cos(\omega t + \phi)$, find the values of A, ω, and ϕ.

b) Find the maximum positive velocity of the movement of the building at this height and the time or times it occurred.

c) Find the maximum positive displacement of the movement of the building at this height and the time or times it occurred.

Problem 1. The picture below shows a fragment of the seismogram recorded in Old Harbor (Alaska) during the Earthquake near the Queen Charlotte Islands on October 28, 2012. Using this trace, determine the following quantities for this earthquake:

(a) Period of the oscillation.

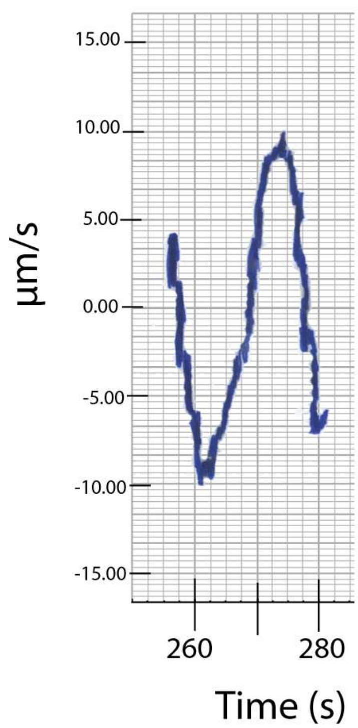
(b) Amplitude of the recorded movement of the seismograph sensor.

(c) Maximum velocity of the movement.

(d) Maximum acceleration of the movement.

(e) At what time was the maximum positive velocity observed?

(f) At what time was the maximum positive acceleration observed?



1. A rescue capsule, like the one used to rescue the trapped miners in Chile's Copiapó mine, is held by a steel cable with a spring constant of 8.0×10^4N/m at its full length, that is at its length when the capsule reaches the trapped miners below ground. When a 70.0 kg miner steps into the capsule to be lifted to safety, it starts oscillating with a period of 0.49s.

a) What is the mass of the rescue capsule?

b) Relative to the position before the miner stepped into the rescue capsule, what is the new equilibrium position around which this oscillation occurs?

c) Write the equation that describes the displacement from the new equilibrium position as a function of time, with t=0 at the moment the miner steps into the capsule. Assume that the miner has stepped into the capsule very gently so that the motion was initiated by the added weight of the miner only.

d) What is the length of the cable if the cross-sectional area is 2.4cm^2 and the Young's modulus of steel is $Y_{steel} = 200\text{GPa}$?

2. A physics professor (weight 60 kg) went bungee jumping with her portable recording accelerometer. After her jump, she plotted some of the recorded values of acceleration as a function of time (see attached figure).

Using this data calculate:

a) The maximum and minimum speed during the up and down oscillations.

b) The amplitude of the oscillation.

c) The elastic constant of the bungee cord.