## Physics 157 Homework 5: due Wed, Oct $23^{\text {th }}$ (online by 8:59pm)

In this homework set, you'll get some practice with problems involving processes involving gases, heat, work, internal energy, and the First Law of Thermodynamics. We hope this will help you develop the following specific skills:

- To use the ideal gas law to predict how certain thermodynamics variables (pressure, volume, temperature) will change given the changes in the other variables.
- To calculate work for various processes based on forces and displacement or pressure and changes in volume. To calculate work in a process represented as a path on a PV-diagram.
- To calculate the change in internal energy of a gas given changes in temperature.
- To calculate heat added to a gas by making use of the First Law of Thermodynamics (via a calculation of work and internal energy).
- To calculate changes in $\mathrm{P}, \mathrm{V}, \mathrm{T}, \mathrm{U}, \mathrm{W}$, and Q for processes involving constant pressure, constant volume, constant temperature or zero heat exchange (i.e. adiabatic processes).


## Your homework: do Mastering Physics Assignment 5 (no written part)

Everything you need is summarized below. Some of these things may be unfamiliar at first; if you get stuck, read through these tips again and see what is relevant.

## Important formulae and tips:

The ideal gas law relates the pressure of a gas to its volume, temperature, and number of moles $\mathbf{n}$ via:

$$
\mathbf{P V}=n \mathbf{R T}
$$

When we have some process that changes some of these quantities but keeps the number of moles fixed, it is often useful to relate the initial quantities to the final quantities by

$$
P_{1} V_{1} / T_{1}=P_{2} V_{2} / T_{2}
$$

This simplifies even more when pressure, volume, or temperature are fixed during the process. For example, with constant temperature, we have $\mathbf{P}_{\mathbf{1}} \mathbf{V}_{\mathbf{1}}=\mathbf{P}_{\mathbf{2}} \mathbf{V}_{\mathbf{2}}$.

During any process, we define $\mathbf{W}$ to be the work done by the gas. For a constant pressure process, this is equal to

$$
\mathbf{W}=\mathbf{P} \Delta \mathbf{V}
$$

so the work is positive when the gas expands, and negative when the gas is compressed. When the pressure is changing, we need to calculate the work by breaking the process up
into small parts with approximately constant pressure, and using that the work for each infinitesimal part is

$$
\mathrm{dW}=\mathrm{PdV}
$$

The total work can then be written as an integral between initial and final volumes:

$$
\mathrm{W}=\int_{V_{i}}^{V_{f}} P(V) d V
$$

where $\mathrm{P}(\mathrm{V})$ is the function that gives the pressure P at volume V . A simple way to understand this is that it $\mathbf{W}$ is the area under the graph of $\mathbf{P}$ vs $\mathbf{V}$, taken between the initial and final points with a $+/-$ sign if the volume increases/decreases.

To calculate the change in internal energy of a gas, we use that:

$$
\Delta U=n C_{v} \Delta T
$$

Here, $\mathbf{C}_{v}$ is the molar specific heat (equal to $3 / 2 R$ for a monatomic ideal gas). Finally, to calculate the heat added during a process, we usually make use of the First Law of Thermodynamics

$$
Q=\Delta U+W
$$

and calculate $\mathbf{Q}$ by first calculating $\mathbf{\Delta U}$ and $\mathbf{W}$.
Using these techniques, we can derive the following useful results for special processes:

## Constant volume:

$\mathrm{T} / \mathrm{P}$ is constant, $\mathrm{W}=0$

## Constant pressure:

$T / V$ is constant, $W=P \Delta V, Q=n C_{p} \Delta T$ where $C_{p}=C_{V}+R$

## Constant temperature:

PV is constant, $\Delta \mathrm{U}=0, \mathrm{~W}=\mathrm{nRT} \ln \left(\mathrm{V}_{\mathrm{f}} / \mathrm{V}_{\mathrm{i}}\right)$

## Adiabatic:

$Q=0, P V^{v}=$ constant where $\gamma=C_{p} / C_{V}, W=-\Delta U$

## Old midterm problems:

3. A bicycle tire can (but rarely!) become over-pressurized and blow out when the cyclist applies the brakes to reduce her speed when travelling down steep hills. A particular road bike tire is rated to 120 psi ( 924 kPa absolute pressure) and is initially filled to this pressure. The pressure specification has a safety margin of $25 \%$ before failure occurs. What amount of heat transferred into the air in the tire would cause failure? The volume of the tire is $1 \mathrm{~L}\left(0.001 \mathrm{~m}^{3}\right)$. You can assume the volume change is negligible and treat the air as a diatomic ideal gas $\left(c_{v}=\frac{5}{2} R\right)$.
4. A cylinder filled with Neon gas has a base area of $0.300 \mathrm{~m}^{2}$ with a 50.0 kg piston that can move freely but prevents the gas from escaping. Above the piston, the cylinder is open to the atmosphere. The cylinder is being heated, causing the piston to rise at a constant speed. An uncalibrated pressure gauge reads a constant value for the pressure of the Neon gas throughout the process. At $27.0^{\circ} \mathrm{C}$, the piston is at a height of 20.0 cm . After 10 minutes, the piston has moved by 10.0 cm .

Note: $\mathrm{C}_{V}=12.5 \mathrm{Jmol}^{-1} \mathrm{~K}^{-1}$ for Neon and atmospheric pressure is $1.01 \times 10^{5} \mathrm{~Pa}$.
a) Sketch pressure vs. volume for the Neon gas during this process and indicate the direction by an arrow.
b) Calculate the pressure of the Neon gas.
c) How much work was done by the Neon gas in 10 minutes?
d) How much heat was added to the Neon gas in 10 minutes?

3. A diatomic ideal gas follows the cycle shown below consists of two constant pressure (isobaric) processes and two constant volume (isochoric) processes. $P_{A}=3 \mathrm{~atm}, P_{C}=$ $1 \mathrm{~atm}, V_{D}=1 \mathrm{~L}, V_{B}=4 \mathrm{~L}, T_{1}=200 \mathrm{~K}$.
a. What is the temperature at point 3 ?
b. If 3200 J of heat are added to the system to go from $1 \rightarrow 2$, what is the change in internal energy in going from $1 \rightarrow 2$ ?
c. What is the total work done in going around the cycle, in the direction of the arrows?
d. How much heat is exchanged in the cycle (going around in the direction of the arrows)?
e. What is the total work done in going around the cycle, in the direction opposite to the arrows?
f. Is the work done on, or by the gas for each of these directions (as in c. and e.)?


Problem 3. In order to design a better car engine, an engineer studies the heat and cooling of an ideal gas in different processes shown in the plot.
a. Find the work for each of the processes:
i. Process $A$ is the path 1 to 2 then 2 to 3 .
ii. Process $B$ is the path 1 to 3 .
iii. Process $C$ is the path 1 to 4 then 4 to 3 .
b. Rank the processes $A, B$, and $C$ from lowest to highest magnitude of heat transfer $|\mathrm{Q}|$.

2. A pulmonologist, a physician who is specialist in human lung issues, can use a machine called a Spirometry to measure pV curves for humans. When breathing deeply average human lungs produce the pV curves which can be approximated by the one shown in the Figure 1. The Spirometry measures the gauge pressure not the absolute pressure.
a) How much work is done on the air by the average human over one cycle.
b) One proposed design of an iphone charger is to use human breathing to charge it during sleep and other activities (Figure 2). An iphone battery has 18kJ of energy. Humans take on average 10 complete breaths per minute. Calculate how many hours does an average person need to sleep in order to charge an iphone using this charger.
c) Will this be practical way to charge an iphone for everyday use?

Note: $1 \mathrm{~mm} \mathrm{Hg}=0.13332 \mathrm{kPa}$


Figure 1


Figure 2

