Physics 157 Homework 2: due Wed, Sept 25th by 5pm

Hand-in boxes are in the Life building outside room 2408. Write your name, student number, and tutorial section on your submission.

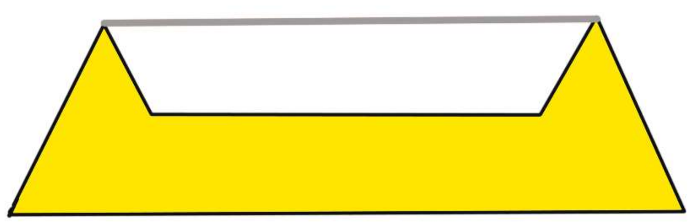
Classes this past week have focused on the thermal expansion of materials and the resulting mechanical forces (stress) that can be caused by this expansion. In this week's homework you will get practice applying the quantitative relationships that govern these effects. Specific skills you will practice include:

- For an object made of some material, to calculate the changes in length or volume that material undergoes in response to changes in temperature and external forces (stress).
- For systems consisting of two different materials, to quantitatively analyze effects resulting from the different expansion rates of different parts.
- For structures involving multiple parts, to calculate the mechanical forces (thermal stress) of various parts of the system on adjacent parts arising from changes in temperature.

For the written questions, explain your work. Don't just write equations and give the answer. For examples, see solutions to last week's homework (though point form is okay).

Question 0: Do the Mastering Physics assignment by the deadline.

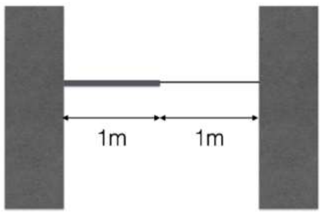
Question 1:



You are the Royal Engineer for the Kingdom of Grrrrrx (pronounced as written). Each year in the kingdom, on the last day of summer, a new Knightship of Grrrrrx is awarded to the winner of the Royal Singing-and-Hopping Race, in which participants (18 years of age and older) must hop and sing through three full laps of the castle perimeter, adhering to the strict regulations of the Royal Singing-and-Hopping Commission.

The race begins when the King of Grrrrrx plucks a single note on the Royal Plucking Instrument, which consists of a single 1mm thick platinum wire stretched between two points on a solid gold frame, as shown in the picture. To achieve the proper note, the wire must be at a tension of 800N. On the morning of the race, you notice the temperature is a chilly 5 degrees Celcius but that the temperature is forecast to rise to 25 degrees Celcius by 2pm when the race starts. It is your duty as Royal Engineer to set the tension of the wire, so that it will play just the right note when plucked at 2pm. To what tension should you set it?

Hint: Assume that the expansion of the gold is unaffected by the presence of the wire. You do not need to know the length of the wire. You should do the Mastering Physics question with the brass rod and wires before this question.



Question 2 (from an old exam):

Two pillars, separated by 2m, have a cord strung between them that consists of two different wires attached to each other, a 5mm diameter nylon wire and a 1mm diameter copper wire. At the initial temperature of 25°C, there is 100N of tension in the wire.

a) If the temperature drops to 5°C, by how much does the joining point of the two wires move?

b) What is the final tension in the wires?

For nylon, we have Y = 3GPa and $\alpha = 5 \times 10^{-5}\ K^{-1}$

For copper, we have Y = 117GPa and $\alpha = 1.6 \times 10^{-5}\ K^{-1}$

Homework tips:

It's usually very helpful to start by drawing before/after pictures and labeling with the various known/unknown quantities. We can introduce variables to gives names to the various known and unknown things. It's often simpler to wait and plug in numbers at the end.

Thermal expansion / thermal stress problems:

For these, it's useful to first consider each part of the system separately. For each part, the change in length is equal to the change in length due to thermal expansion plus the change in length due to the changing forces:

$$\Delta L = \alpha L_0 \Delta T + (\Delta F/A)(L_0/Y)$$

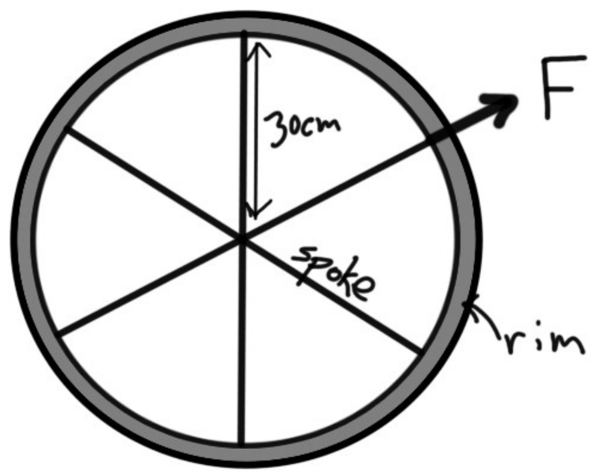
Sometimes, one of these terms will be zero (e.g. if we can ignore external forces on part of the system). This will give you one equation for each part of the system.

Next look at the setup of the problem to understand how the changes in length for various parts are related to each other, and how the various forces are related to each other. For example, changes in length for two parts of the system might be equal to each other, or add to zero. The forces can be related by Newton's third law or by the fact that the net force is zero on static objects. These constraints will give you the remaining equations you need to solve for your unknowns.

Old midterm questions on thermal expansion and thermal stress

(try some if you want extra practice, not to be handed in)

You've just come up with a new design for a lightweight bicycle wheel with an aluminum rim ($\alpha = 13 \times 10^{-6}$) and steel spokes ($\alpha = 7 \times 10^{-6}$, $Y = 20 \times 10^{10}$Pa). At 5 degrees Celcius, there is no tension in spokes, but thermal expansion results in a tensile force as shown. If we want this force to be less than 180N for temperatures up to 35°C, what is the maximum cross sectional area (in mm^2) that the spokes can have?



1. A aluminum rod of density $\rho = 2700\,kg/m^3$ has a length of $3.0\,m$ and cross sectional area $9.0\,cm^2$. It is bolted at both ends between two immobile supports. Initially, there is no tension in the rod because the rod just fits between the supports. Through cooling, the rod loses $3500\,J$ of heat.

a) Find the temperature change from the heat loss.

b) Find the force on the rod that develops from the heat loss.

Useful constants $c_{Al} = 910\,J/(kgK)$, $Y_{Al} = 65 \times 10^9\,Pa$, $\alpha_{Al} = 22 \times 10^{-6} K^{-1}$.

1. A length of nylon fishing line of 2 mm diameter is tied to a length of 0.5 mm diameter stainless steel wire, then stretched between two rigid supports 2 m apart such that the tension in the wires is 20 N and the connection point between the two wires is exactly midway between the posts. The air temperature then goes down by 20°C. Find the distance that the connection moves.

[Youngs modulus for nylon is 3 GPa, its thermal expansion coefficient is 6×10^{-5}/K. Youngs modulus for stainless steel is 180 GPa, its thermal expansion coefficient is 1.7×10^{-5}/K.]

1. The main span of the Lions Gate Bridge has a length of 473 m. On each end there are expansion joints like the one on the photo below. One day, the temperature in Vancouver changed from -4°C to +15°C degrees between 6 AM in the morning and 2 PM in the afternoon.

a) What was the *average* speed of a "tooth" in one of the expansion joints?

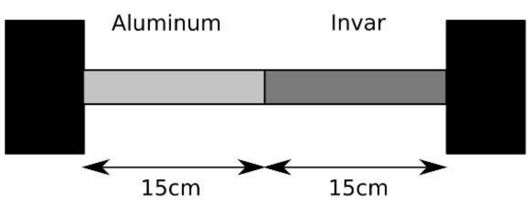
b) At 6 AM in the morning, a piece of tire rubber fell into one of the cracks, filling it completely. The rubber was 10 cm long and had a cross section of 4 cm^2. What was the stress in the rubber at 2 PM?

Clearly state all of the assumptions that you made while solving this problem.

Data: Young modulus of steel 200 GPa
Young modulus of rubber 7 kPa
Linear Expansion coefficient of steel $13 \times 10^{-6} K^{-1}$
Linear Expansion coefficient of rubber $77 \times 10^{-6} K^{-1}$

1. An aluminum rod 15 cm long with a diameter of 1.0 cm is fixed between rigid supports with an Invar rod of the same dimensions such that there is no tension or compression of the rod at 20°C. The rods are attached to each other such that they can push or pull without separating. Invar has a negligibly small thermal expansion coefficient. You place this apparatus in an experiment where heat is added or removed and you measure that the Invar rod is a total of 0.25 mm longer than it was at 20°C.

Data: $\alpha_{Al} = 23 \times 10^{-6} K^{-1}$, $Y_{Al} = 69 GPa$, $\rho_{Al} = 2.7 g/\text{cm}^3$, $c_{Al} = 0.90 J g^{-1} K^{-1}$, $Y_{Invar} = 140 GPa$



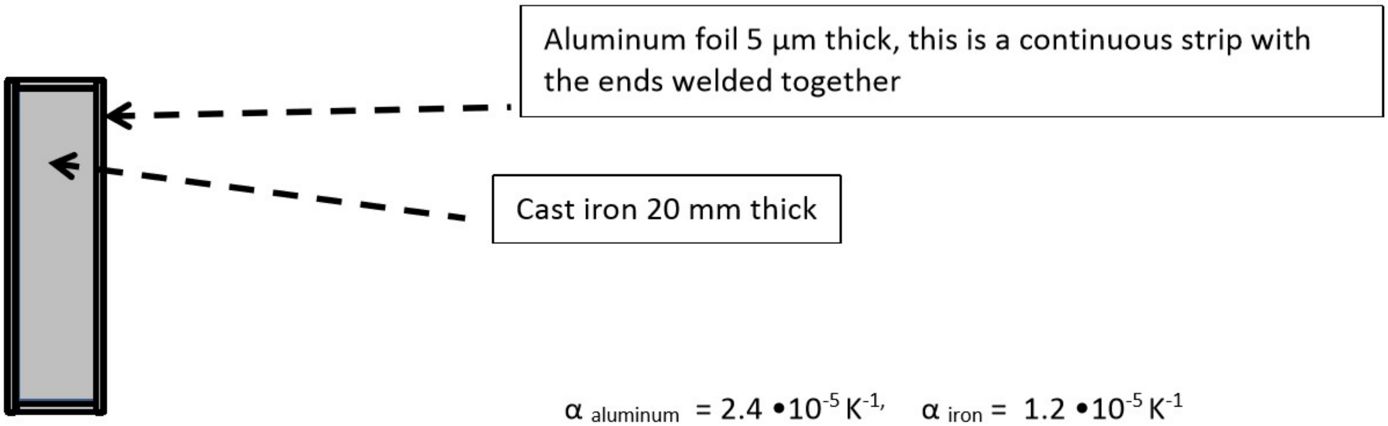
a) Find the final temperature.

b) If the particular Aluminum alloy used has a Yield strength of 200MPa, is the rod likely to be permanently deformed?

c) Was heat added or removed from the Aluminum rod? How much?

Clearly state all of the assumptions that you made while solving this problem.

Problem 2. A block of cast iron was wrapped in the thin aluminum foil as shown on the figure. At room temperature there is no stress on the foil. The whole system is then immersed in liquid nitrogen at -196⁰C. Find the stress on the aluminum foil. Is the foil going to deform permanently? Is it going to break?



$\alpha_{aluminum} = 2.4 \bullet 10^{-5}\, K^{-1},\quad \alpha_{iron} = 1.2 \bullet 10^{-5}\, K^{-1}$

Fig. 2. Stress-strain diagrams for aluminum. Left: Low strain region. Right: Full strain range. Hint: you can get the Young's modulus, proportions limit (elastic limit), and break point from the graphs.