Homework 1 Solutions

① A linear relation between groan rate and temperature means that for any change ΔT in temperature, the change in groan rate is

$$\Delta G = m\,\Delta T$$

for some constant m. From the data given in the question, $\Delta G = 35.2 - 30.0 = 5.2$ for $\Delta T = 55°C - 20°C = 35C°$, so we must have

$$m = \frac{\Delta G}{\Delta T} = \frac{5.2}{35C°} = 0.15\,(C°)^{-1}$$

Now comparing $20°C$ to T_{Aq}, we have:

$$G(T_{Aq}) - G(20°C) = m\,(T_{Aq} - 20°C)$$

$$\Rightarrow 1.5 = 0.15\,(C°)^{-1} \cdot (T_{Aq} - 20°C)$$

$$\Rightarrow T_{Aq} - 20°C = 10°C \quad \Rightarrow T_{Aq} = 30°C$$

ALTERNATE SOLUTION:

A linear relationship means that we have:

$$G = mT + b$$

for some constants m and b. Using the data:

$$30 = m\cdot(20°C) + b \qquad ①$$

$$35.2 = m\cdot(55°C) + b \qquad ②$$

Subtracting ② - ① gives: $5.2 = m \cdot 35C°$

$$\Rightarrow m = 0.15\,(C°)^{-1}$$

Using this in ① gives $b = 30 - 20°C\cdot m = 27$.

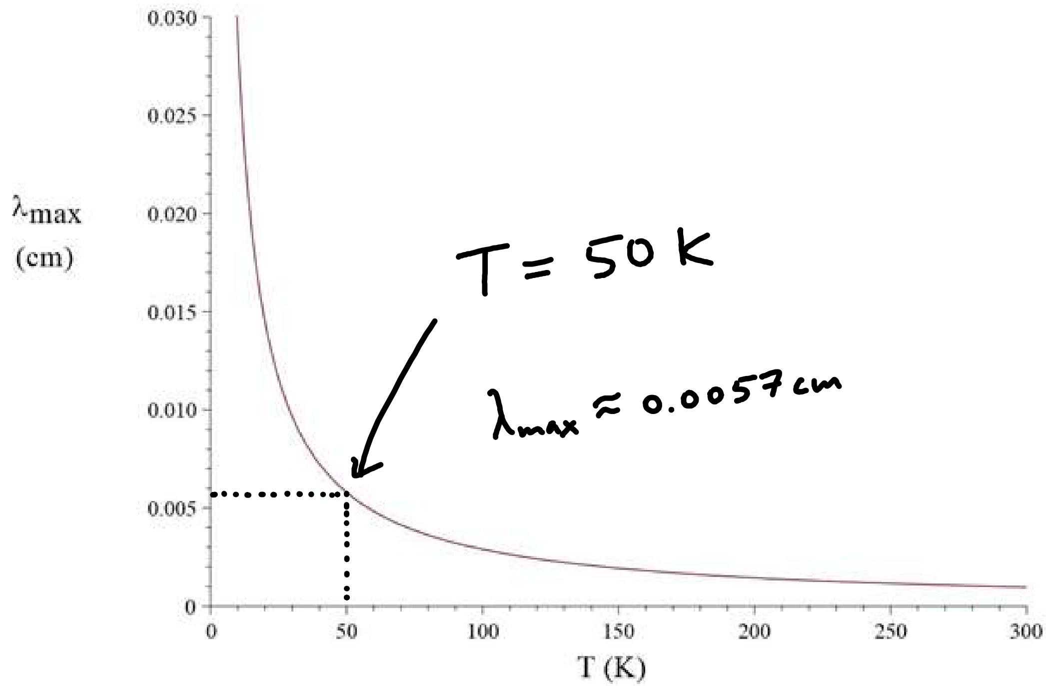
For $G = 31.5$, we then have

$$31.5 = m\cdot(T_{Aq}) + b = (0.15)\cdot T_{Aq} + 27$$

Solving gives $T_{Aq} = 30°C$

(2) We are given that $\lambda_{max} = \frac{C}{T}$ for some constant C.

To find C, we can take a data point from the graph:



We get $C = \lambda_{max} \cdot T \approx 0.0057\,\text{cm} \cdot 50\,\text{K} = 0.28\ \text{cm} \cdot \text{K}$

Then for $\lambda_{max} = 0.107\,\text{cm}$, we get
$$T = \frac{C}{\lambda_{max}} = \frac{0.28\,\text{cm}\cdot\text{K}}{0.107\,\text{cm}} \approx \underline{\underline{2.6\,\text{K}}}$$

This radiation (whose wavelength corresponds to microwaves) is called the cosmic microwave background radiation. It is the radiation left over from very early in the universe when atoms first formed and the universe became transparent (before that, the whole universe was like the interior of the sun).

③ a) We are given that the resistance is

$$R = R_0 (1 + A \cdot T + B \cdot T^2)$$

where T is the temperature in degrees Celcius.

At the ice point of water, $T = 0°C$ and $R = 5.000$ ohms, so $R_0 = 5.000$. Using the other data provided, we get:

$$6.973 = 5.000 \left(1 + A \cdot 100°C + B \cdot (100°C)^2\right) \quad ①$$

$$10.80 = 5.000 \left(1 + A \cdot (327.46°C) + B \cdot (327.46°C)^2\right) \quad ②$$

From ①, we get $B = \dfrac{0.3946}{(100°C)^2} - \dfrac{A}{100°C}$ ③

Plugging this into ② and solving for A, we get:

$$A = 0.004123 \ (C°)^{-1}$$

From ③, $B = -1.776 \times 10^{-6} \ (C°)^{-2}$.

b) If $R = 8.3$ ohms, we have:

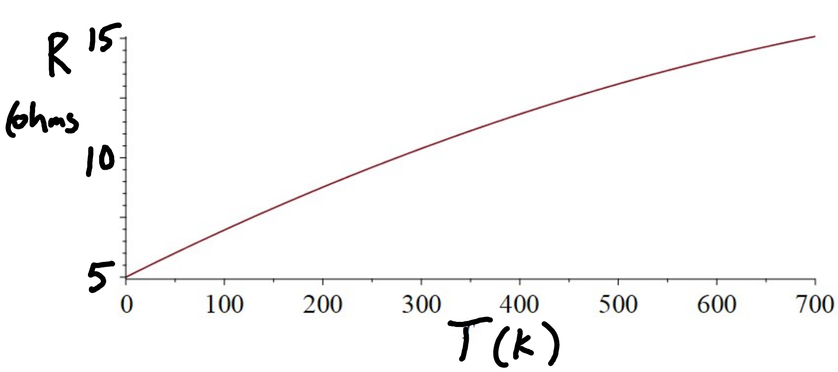
$$8.3 = 5.0 \left(1 + AT + BT^2\right)$$

so: $BT^2 + AT - 0.74 = 0$

$$\Rightarrow T = -\frac{A}{2B} \pm \frac{1}{2B}\sqrt{A^2 + 4 \cdot B \cdot 0.74} = 172.9°C,\ 2149.4°C$$

Our equation is only valid between 0°C and 700°C, so the temperature should be 172.9°C.

c) plot:



(a plot by hand is also fine)