

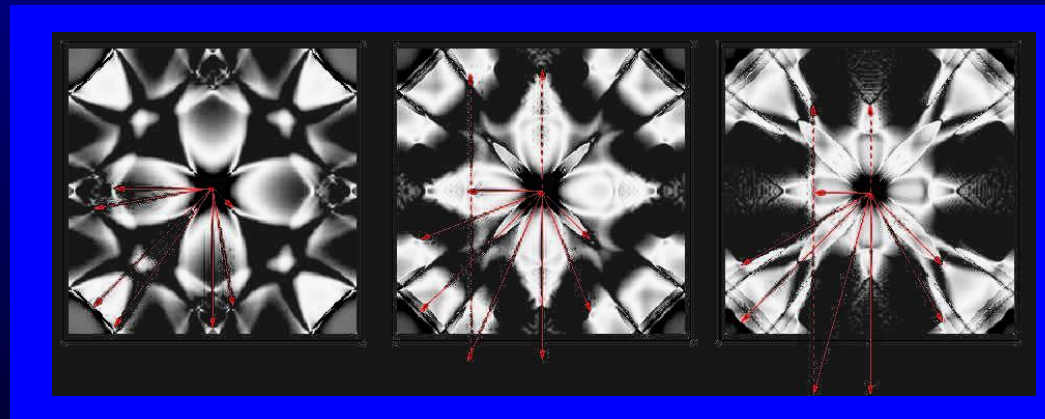
Quasiparticle interference in the pseudogap phase of cuprate superconductors

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March 25, 2004



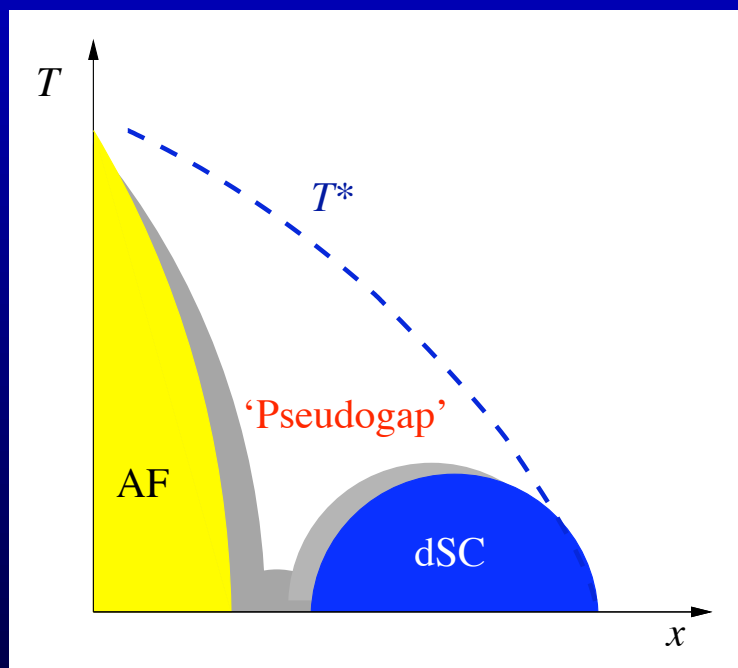
In collaboration with: T. Pereg-Barnea (KITP & UBC)

Pseudogap: the key mystery

Pseudogap is a nonsuperconducting phase intermediate between the **AF insulator** and *d-wave* superconductor.

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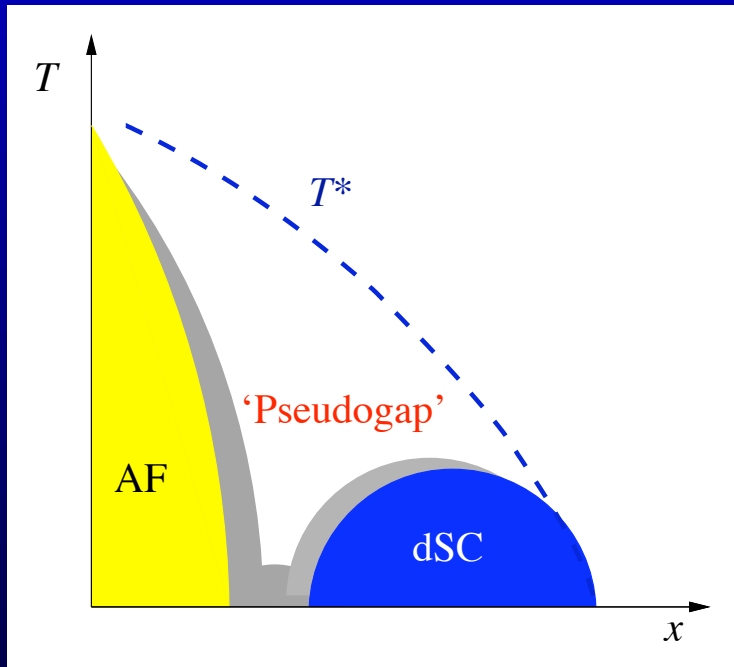
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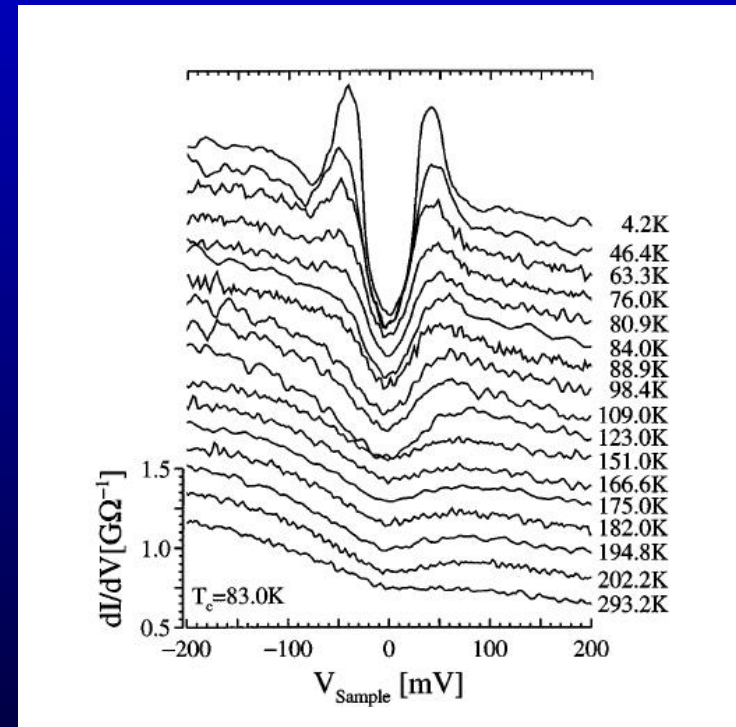
Phase diagram of cuprates. [For exp. review see [Timusk and Statt, Rep. Prog. Phys. 62, 61 \(1999\).](#)]

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Gap in the single-particle DOS above T_c [tunneling data from [Renner *et al.*, PRL 80, 149 \(1998\)](#)]

Two schools of thought on the origin of pseudogap

Ascribe the pseudogap phenomenon to:

- Remnants of superconducting order
 - ★ Emery and Kivelson, Nature **374**, 434 (1995).
 - ★ Franz and Millis, PRB **58**, 14572 (1998)
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 - ★ Laughlin, cond-mat/0209269
- Static or fluctuating competing order in p-h channel (SDW, CDW, DDW, ...)
 - ★ Zhang, Science **275**, 1089 (1997)
 - ★ Varma, PRL **83**, 3538 (1999)
 - ★ Vojta, Zhang, and Sachdev, PRB **62**, 6721 (2000)
 - ★ Chakravarty, Laughlin, Morr, and Nayak, PRB **63**, 094503 (2001)

Who is right?

Experimental determination of the origin of the pseudogap phase has proven elusive. At present believable experiments can be found to support **either scenario**.

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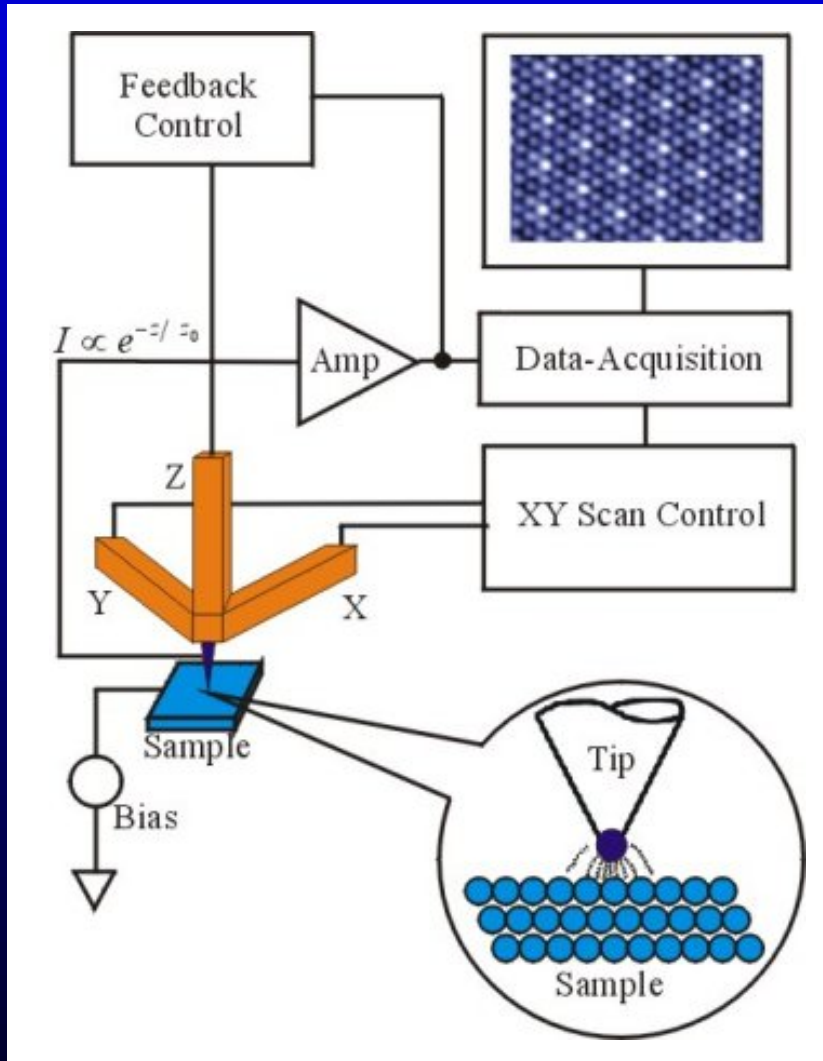
Need a decisive “smoking gun” experiment

Our proposal: use the recently developed technique of Fourier Transform scanning tunneling spectroscopy (FT-STs).

- Pereg-Barnea and Franz, PRB **68**, 180506(R) (2003)
- Pereg-Barnea and Franz, cond-mat/0401594

STM Basics

[<http://people.ccmr.cornell.edu/~jcdavis/stm>]



STM measures differential conductance

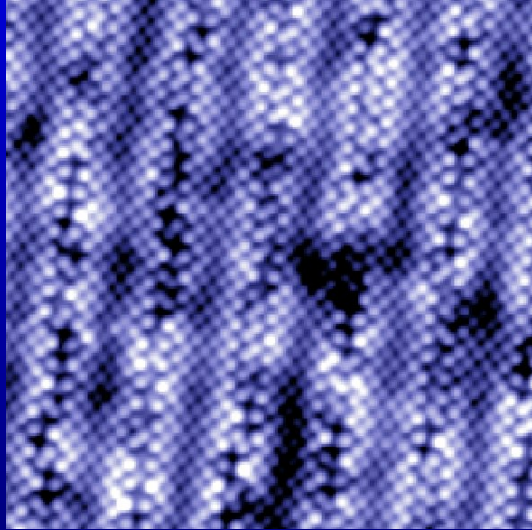
$$n(\mathbf{r}, \omega) \simeq \left(\frac{dI(\mathbf{r}, eV)}{dV} \right)_{eV=\omega},$$

with potentially atomic resolution.

To reasonable approximation $n(\mathbf{r}, \omega)$ is proportional to the **Local Density of States** (LDOS) of the sample at point \mathbf{r} directly under the STM tip.

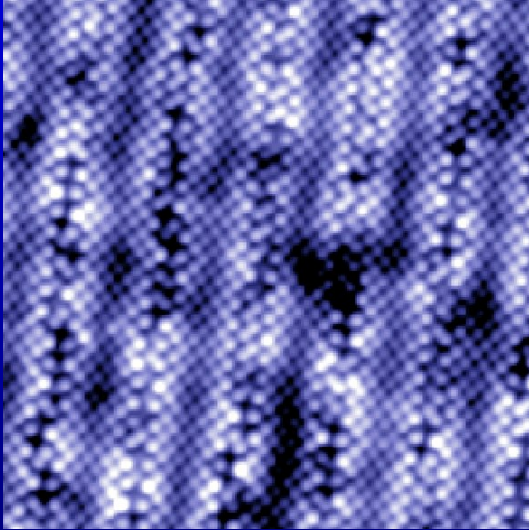
Tunneling spectroscopy in cuprates

Topography of BiSCCO:

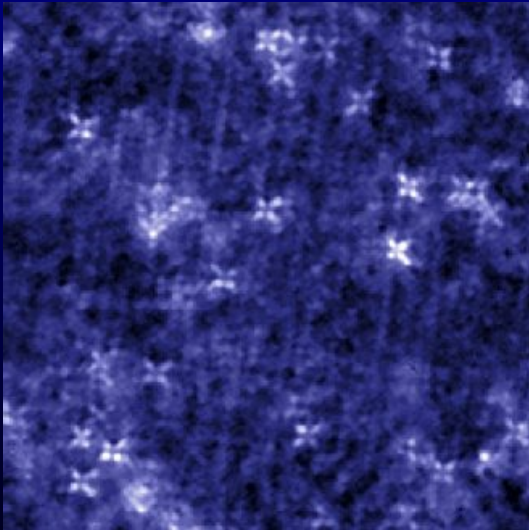


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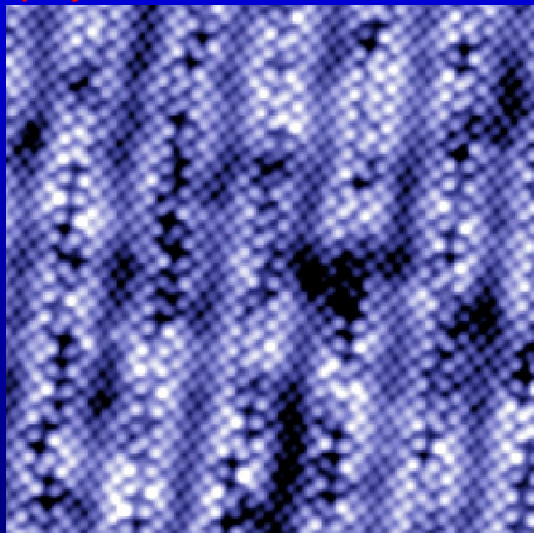


Spectroscopy of Ni impurities:

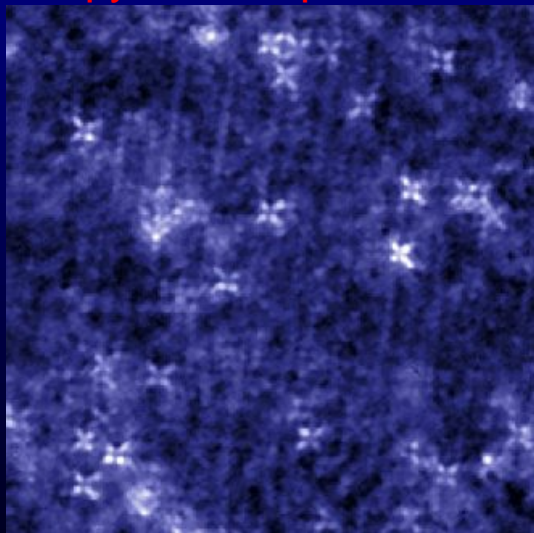


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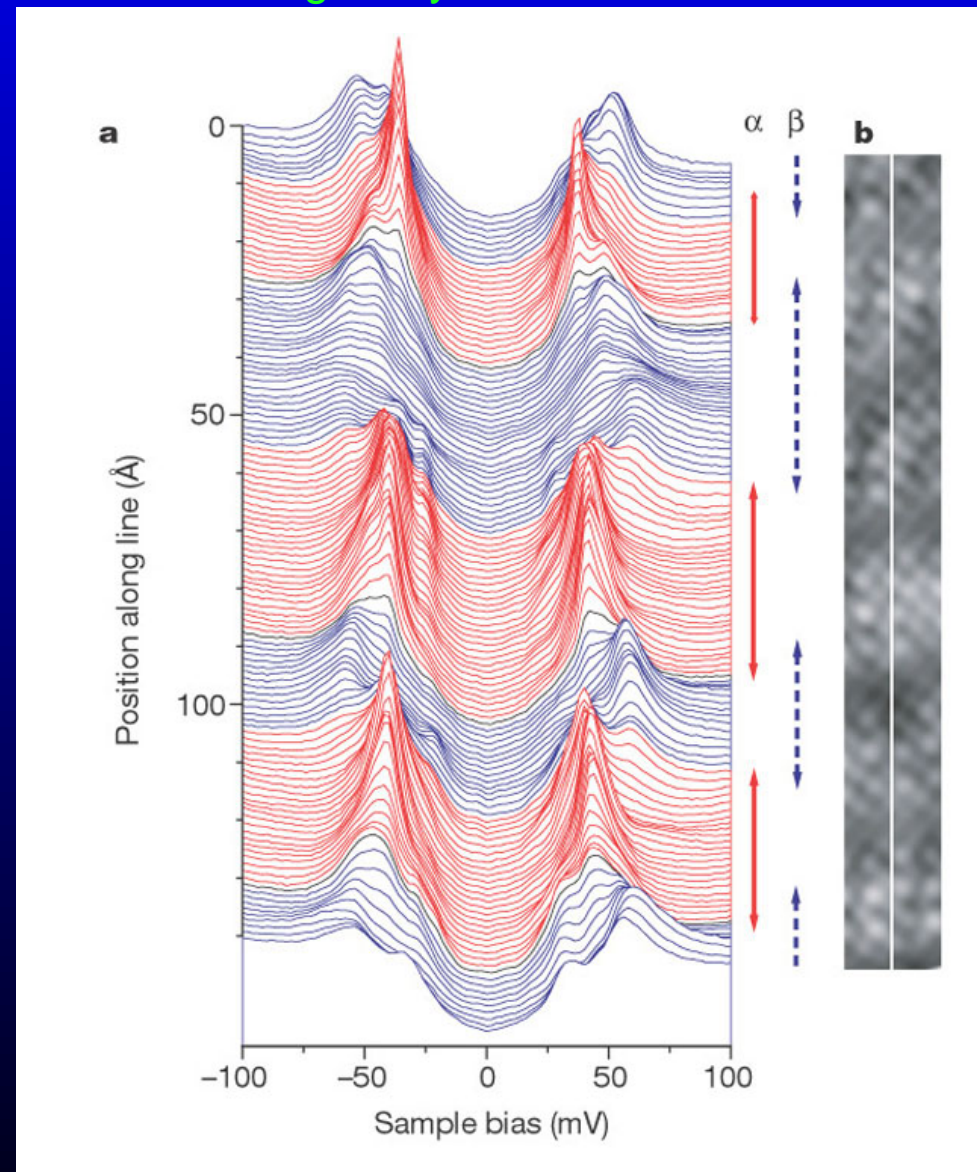
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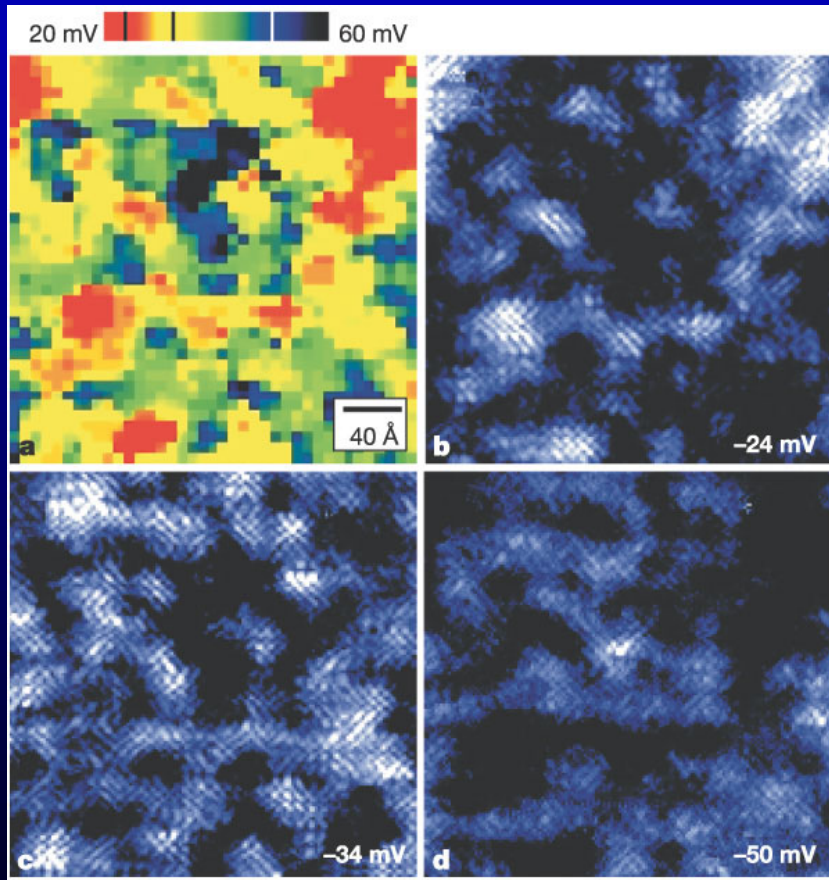


LDOS inhomogeneity:



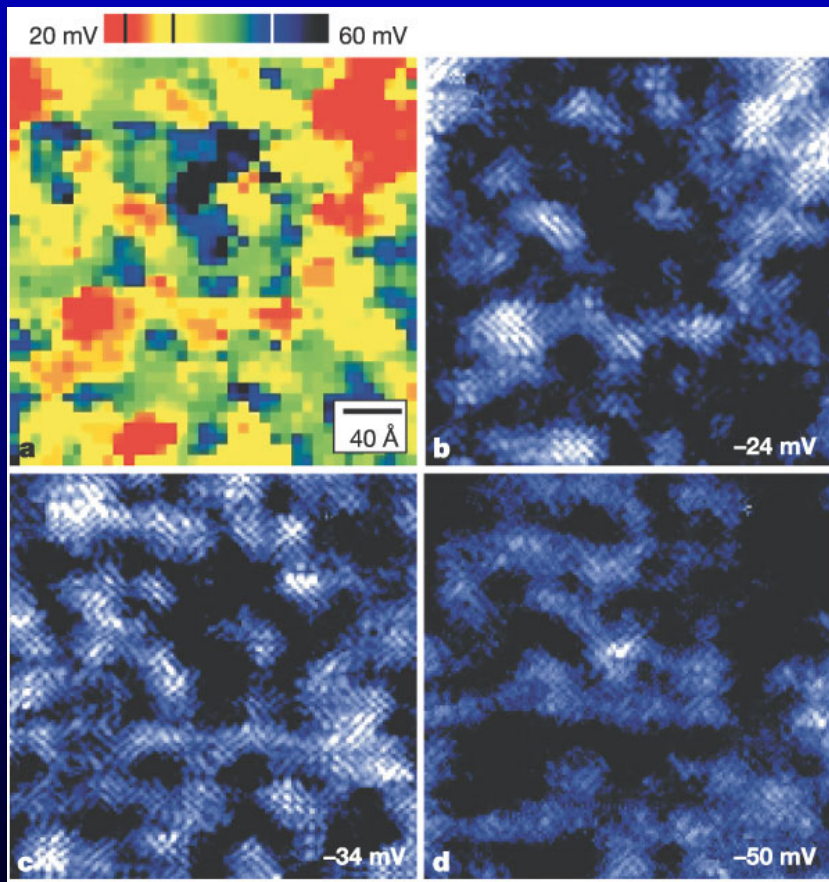
FT-STs: “Fourier Transform Scanning Tunneling Spectroscopy”

Periodic patterns in LDOS at fixed energy are sometimes observed:

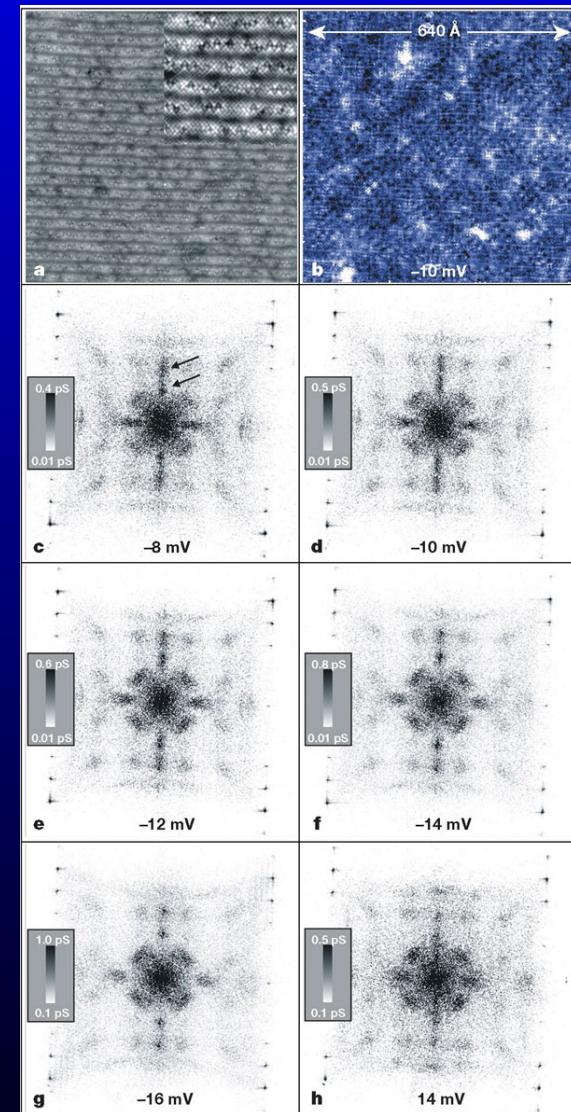


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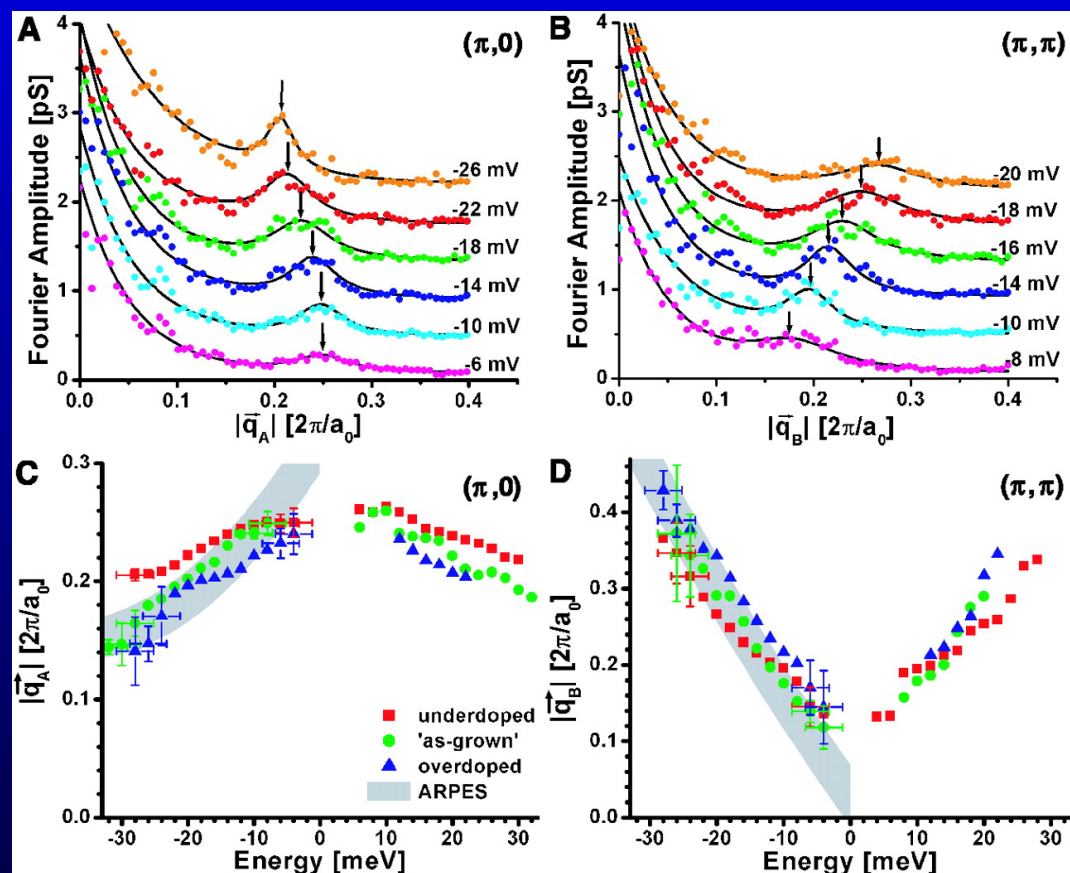
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→
FT



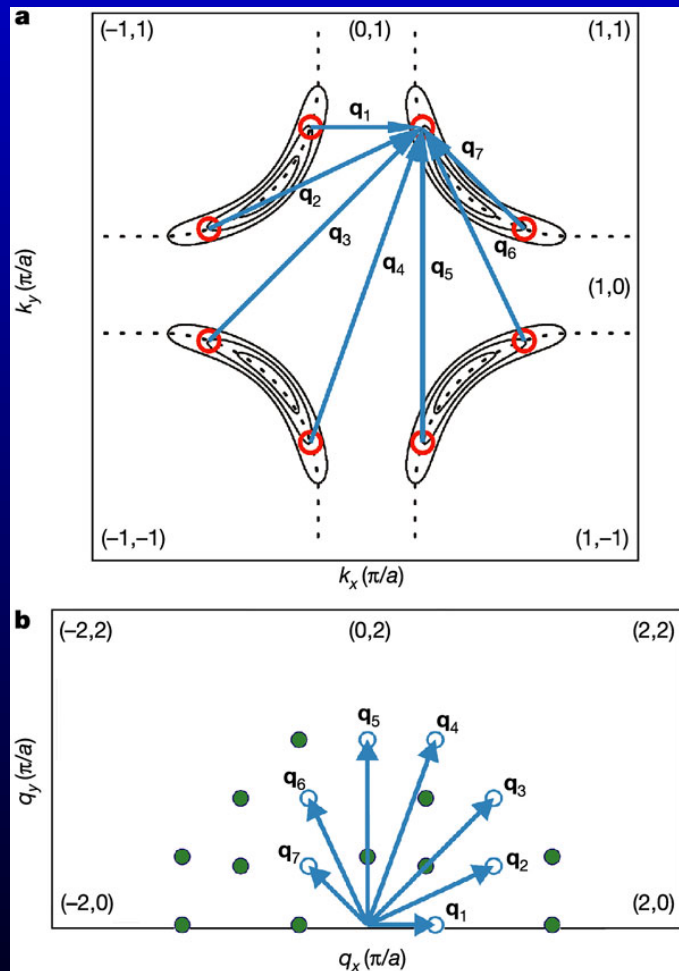
FT-STS peaks disperse as a function of applied bias



- K. McElroy *et al.*, Nature **422**, 592 (2003).
- J.E. Hoffman *et al.*, Science **297**, 1148 (2002).

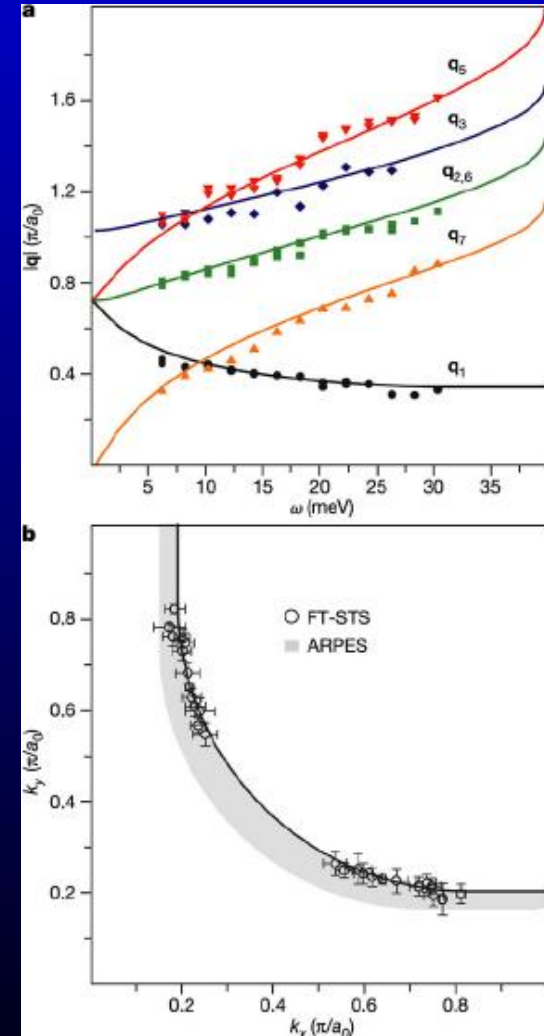
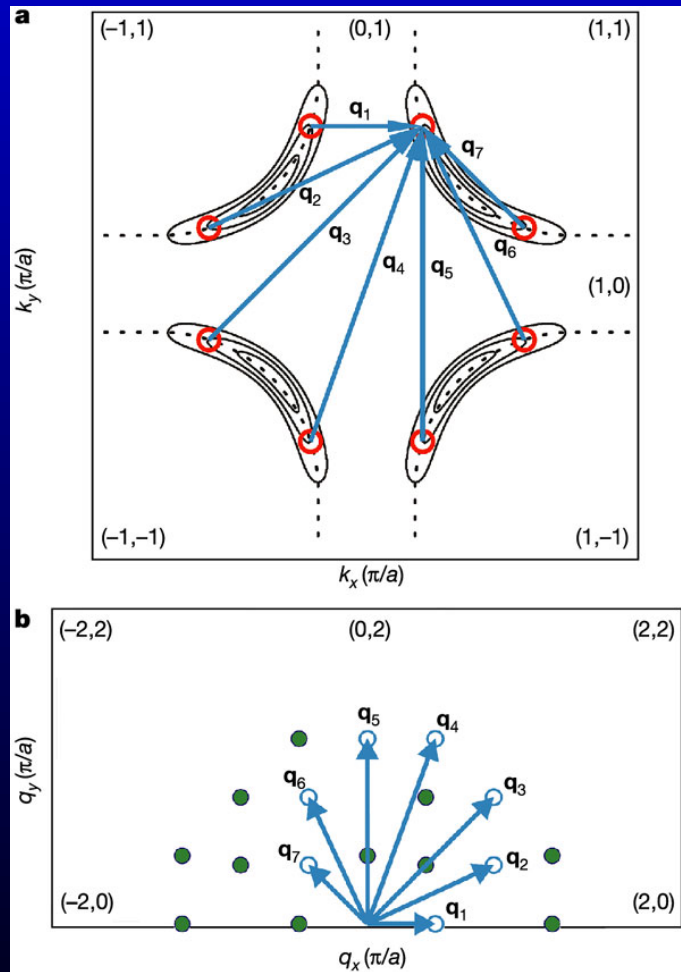
The “Octet Model”

The octet model asserts that the peaks in FT-STs are due to quasiparticle scattering between the regions of **high DOS**.



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→ IDENTIFICATION OF PSEUDOGAP ORDER

Theory of FT-STS

STM measures the quantity [Wang and Lee, PRB 67, 020511(2003)]

$$n(\mathbf{r}, \omega) = -\frac{1}{\pi} \text{Im}[G_{11}(\mathbf{r}, \mathbf{r}, \omega) + G_{22}(\mathbf{r}, \mathbf{r}, -\omega)],$$

where $G(\mathbf{r}, \mathbf{r}', \omega)$ is a full electron propagator. In the presence of **disorder potential V** we can write

$$G(\mathbf{k}, \mathbf{k}', \omega) = G^0(\mathbf{k}, \omega) \delta_{\mathbf{k}, \mathbf{k}'} + G^0(\mathbf{k}, \omega) \hat{T}_{\mathbf{k}\mathbf{k}'}(\omega) G^0(\mathbf{k}', \omega),$$

with $G^0(\mathbf{k}, \omega) = [\omega - \sigma_3 \epsilon_{\mathbf{k}} - \sigma_1 \Delta_{\mathbf{k}}]^{-1}$ the bare Green's function and $\hat{T}_{\mathbf{k}\mathbf{k}'}(\omega)$ the T-matrix that satisfies the Lippman-Schwinger equation

$$\hat{T}_{\mathbf{k}\mathbf{k}'}(\omega) = \hat{V}_{\mathbf{k}\mathbf{k}'} + \sum_{\mathbf{q}} \hat{V}_{\mathbf{k}\mathbf{q}} G^0(\mathbf{q}, \omega) \hat{T}_{\mathbf{q}\mathbf{k}'}(\omega).$$

FT-STs measures $n(\mathbf{q}, \omega)$, a spatial Fourier transform of $n(\mathbf{r}, \omega)$.

It is useful to consider a limit of **weak disorder** (i.e. Born limit) in which one can express the non-uniform part $\delta n(\mathbf{q}, \omega)$ [Capriotti *et al.* PRB **68**, 014508 (2003)]

$$\delta n(\mathbf{q}, \omega) = -\frac{1}{\pi} |V_{\mathbf{q}}| \text{Im} [\Lambda_{11}(\mathbf{q}, \omega) + \Lambda_{22}(\mathbf{q}, -\omega)],$$

where, for scattering in the charge channel,

$$\Lambda(\mathbf{q}, \omega) = \sum_{\mathbf{k}} G^0(\mathbf{k}, \omega) \sigma_3 G^0(\mathbf{k} - \mathbf{q}, \omega).$$

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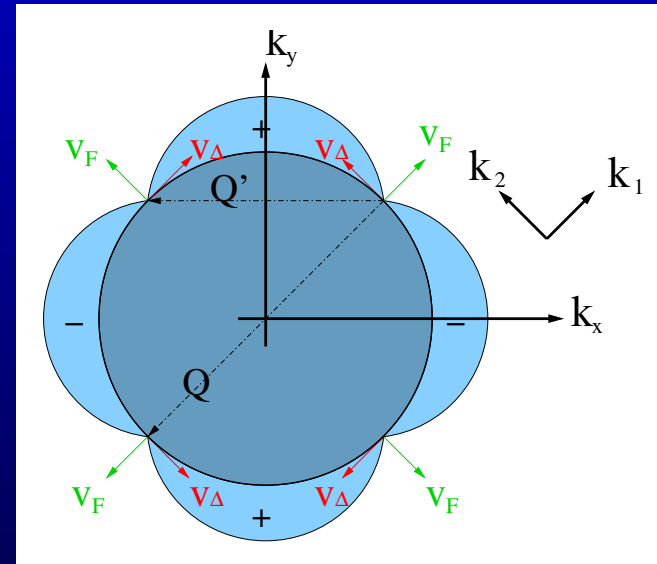
For weak disorder FT-STS provides information about the underlying electron order

Nodal approximation: importance of coherence factors

One finds

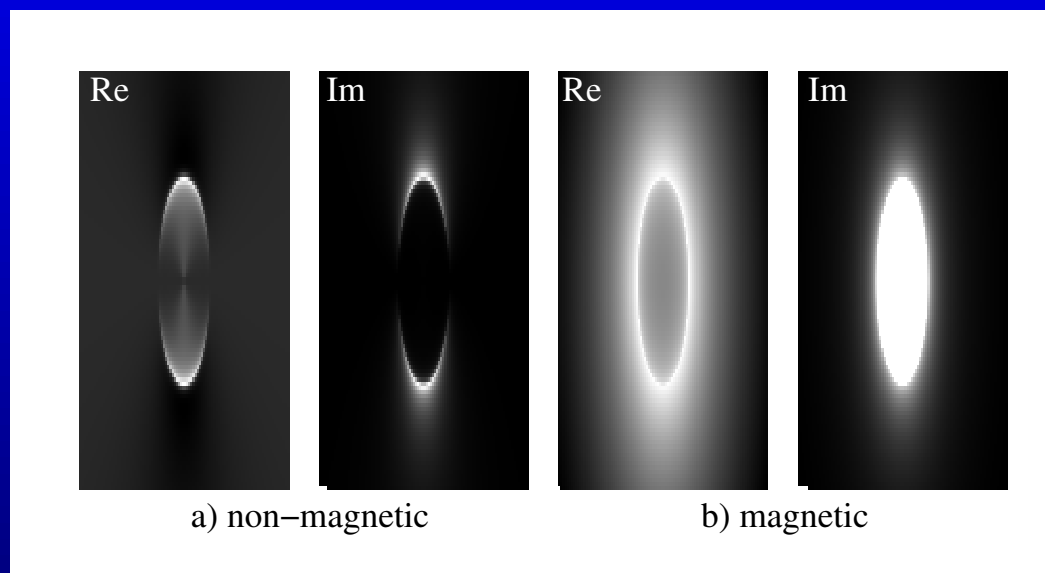
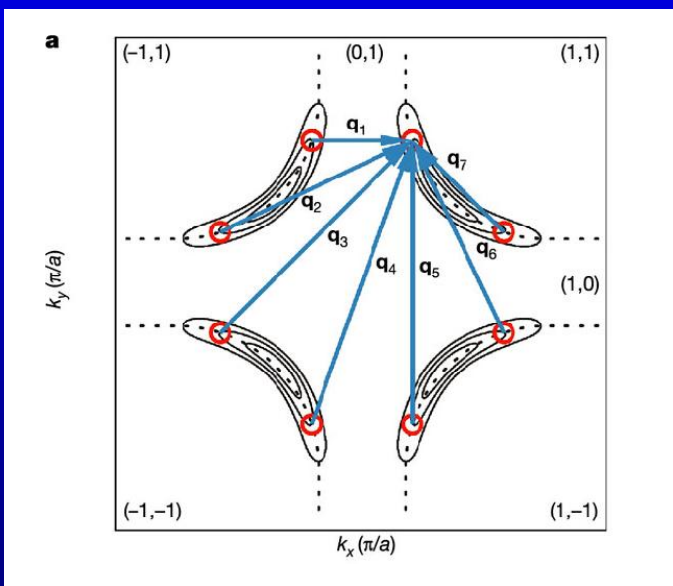
$$\Lambda(\mathbf{q}, i\omega) = \frac{1}{L^2} \sum_{\mathbf{k}} \frac{(i\omega + \epsilon_+)(i\omega + \epsilon_-) - \Delta_+ \Delta_-}{(\omega^2 + E_+^2)(\omega^2 + E_-^2)},$$

with $\epsilon_{\pm} = \epsilon_{\mathbf{k} \pm \mathbf{q}/2}$, $\Delta_{\pm} = \Delta_{\mathbf{k} \pm \mathbf{q}/2}$
and $E_{\pm} = \sqrt{\epsilon_{\pm}^2 + \Delta_{\pm}^2}$. Linearize
near the nodes to obtain



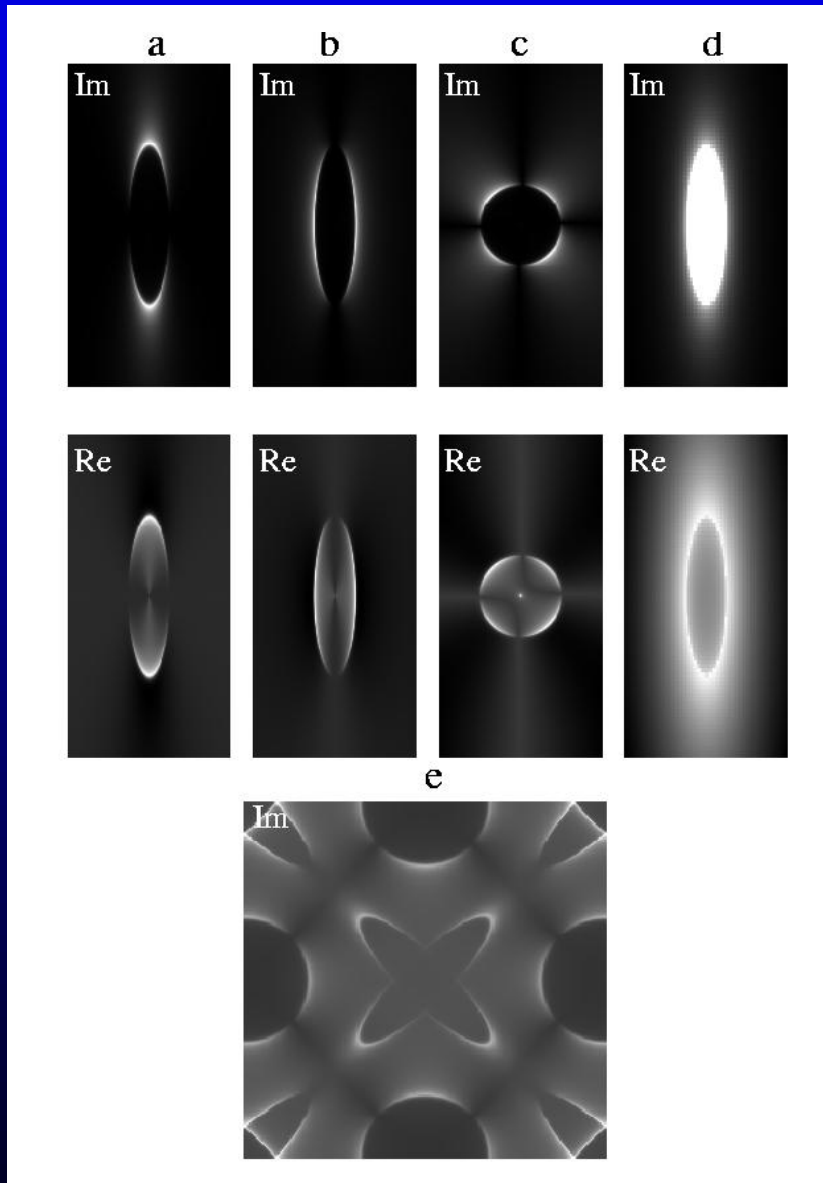
$$\Lambda_{\text{lin}} = \frac{1}{v_F v_{\Delta}} \int \frac{d^2 k}{(2\pi)^2} \frac{-\omega^2 + (k_1^2 - k_2^2) - (\tilde{q}_1^2 - \tilde{q}_2^2)}{[\omega^2 + (\mathbf{k} + \tilde{\mathbf{q}})^2][\omega^2 + (\mathbf{k} - \tilde{\mathbf{q}})^2]}.$$

For intranodal scattering we thus get



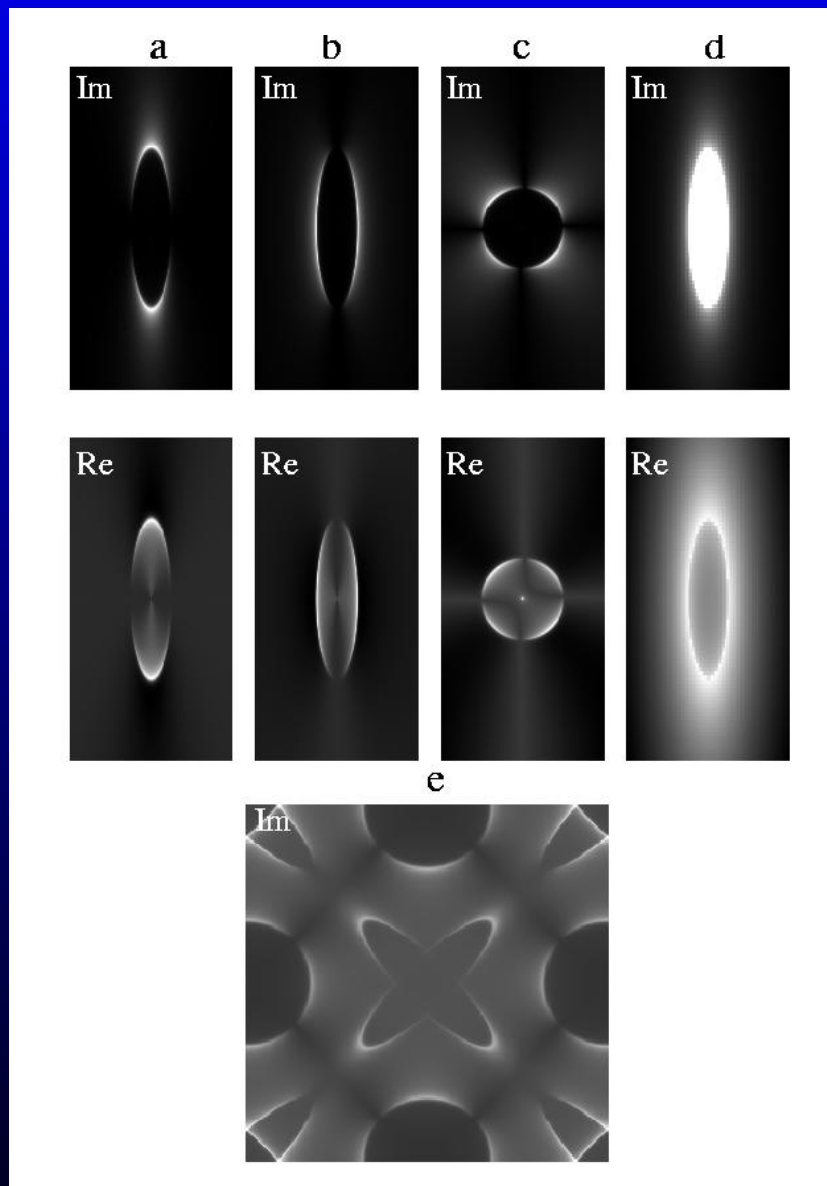
Magnetic and non-magnetic scattering differ only in the **coherence factors**, DOS is exactly the same. Yet, the FT-STS patterns are *qualitatively different!*

The full picture



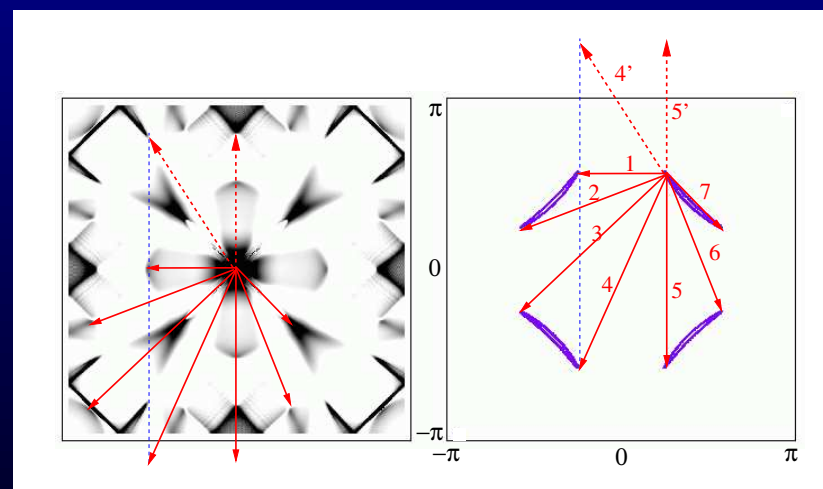
One can analyze various intra-node processes similarly in the linearized approximation to obtain the full picture.

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Alternately, one can evaluate $\Lambda(\mathbf{q}, \omega)$ **exactly** using numerical techniques:



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- If the pseudogap is primarily due to some p-h order then we expect a **fundamentally different** patterns above T_c .
- In the following we illustrate this general thesis on the comparison between QED₃ theory of phase disordered d SC and d -density wave (DDW) scenario for pseudogap.

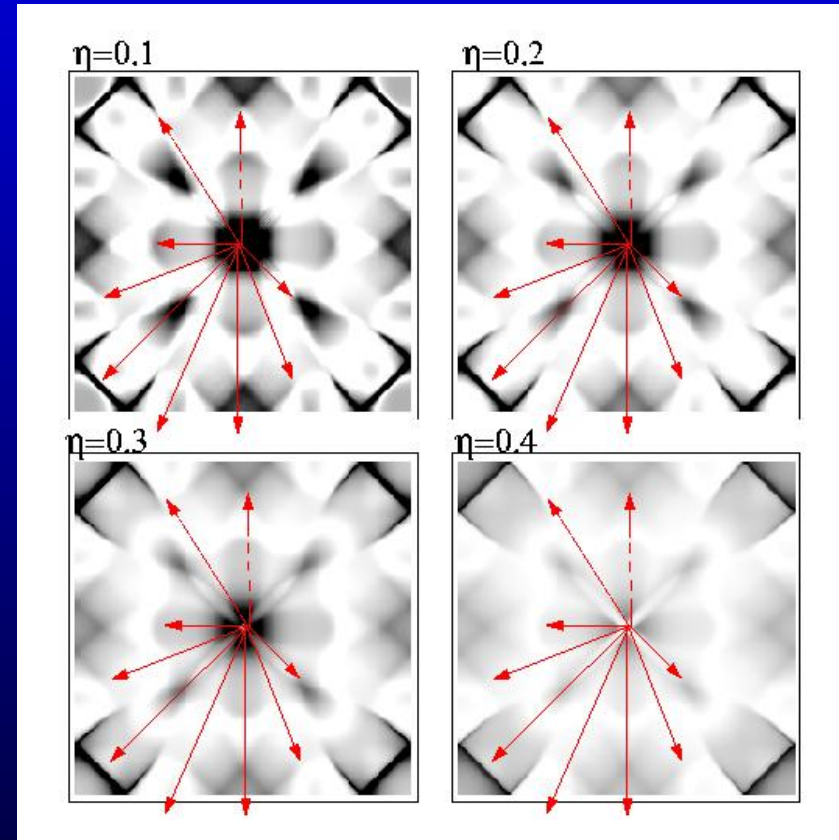
QED₃

[Franz and Tešanović, PRL **87**, 257003 (2001)]

This theory describes fermionic excitations in a **phase-disordered *d*-wave superconductor**. The electron propagator reads

$$G^0(\mathbf{k}, i\omega) = \lambda^{-\eta} \frac{i\omega + \epsilon_{\mathbf{k}}\sigma_3}{[\omega^2 + \epsilon_{\mathbf{k}}^2 + \Delta_{\mathbf{k}}^2]^{1-\eta/2}},$$

where λ is a high energy cutoff and η is the anomalous dimension exponent which encodes the physics of phase fluctuations. η is a small positive number, whose precise value is still under debate.



DDW

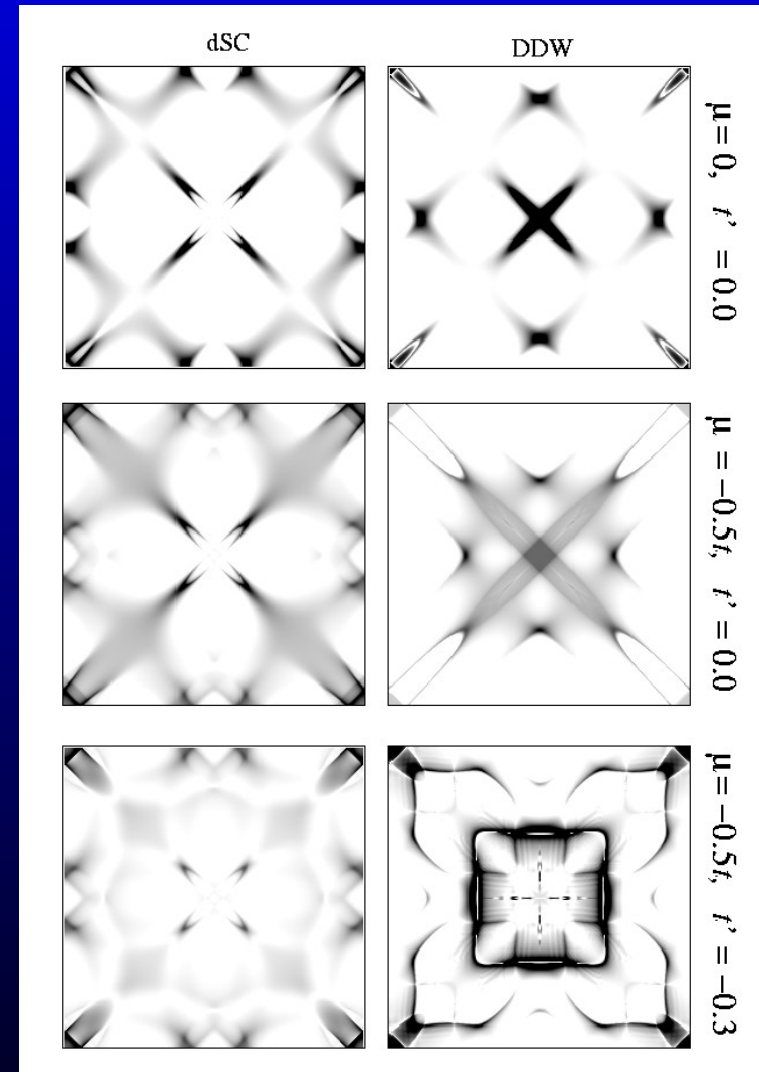
[Chakravarty, Laughlin, Morr, and Nayak PRB **63**, 094503 (2001)]

Also known as the “flux phase”, this theory describes the pseudogap as a mean-field state with staggered pattern of currents, breaking the translational symmetry of the square lattice. We have

$$G^0(\mathbf{k}, i\omega) = [(i\omega - \epsilon'_\mathbf{k}) - \epsilon''_\mathbf{k}\sigma_3 - D_\mathbf{k}\sigma_2]^{-1},$$

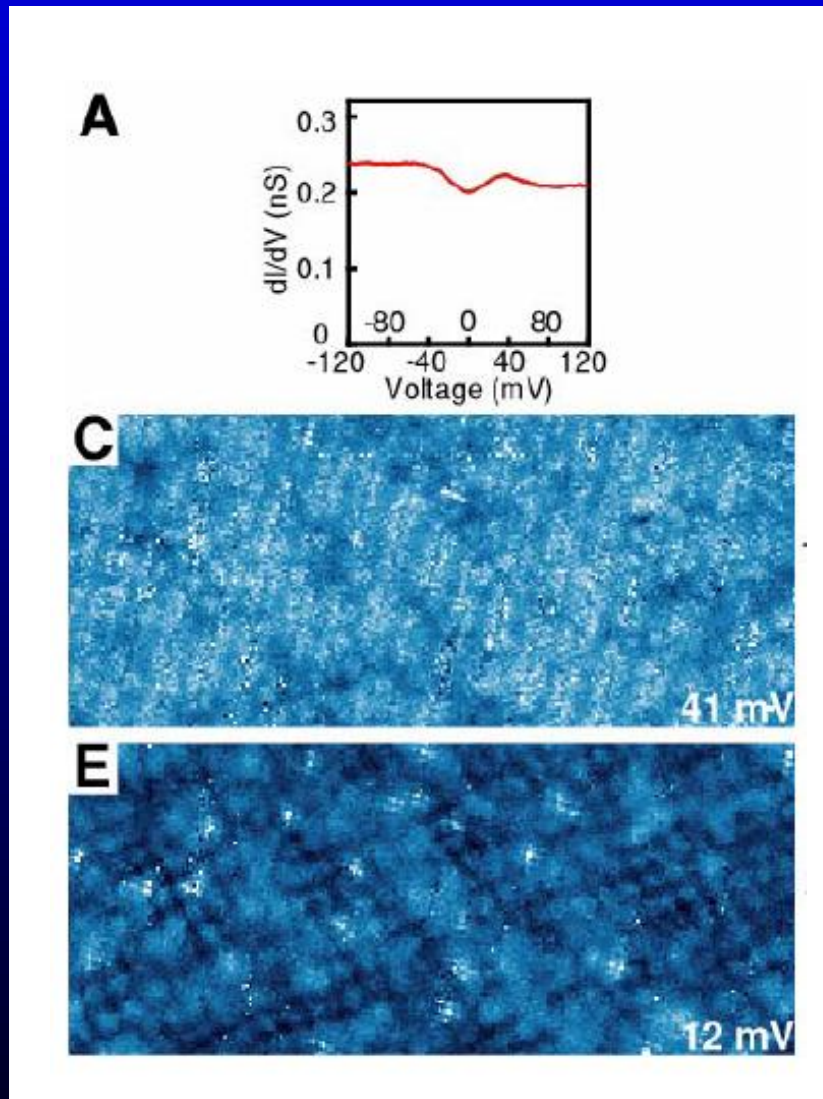
with $\epsilon'_\mathbf{k} = \frac{1}{2}(\epsilon_\mathbf{k} + \epsilon_{\mathbf{k}+\mathbf{Q}})$, $\epsilon''_\mathbf{k} = \frac{1}{2}(\epsilon_\mathbf{k} - \epsilon_{\mathbf{k}+\mathbf{Q}})$, and the DDW gap $D_\mathbf{k} = \frac{1}{2}D_0(\cos k_x - \cos k_y)$.

At half filling ($\mu = 0$) and with nn dispersion ($t' = 0$) DDW has **the same DOS** as the *d*SC.



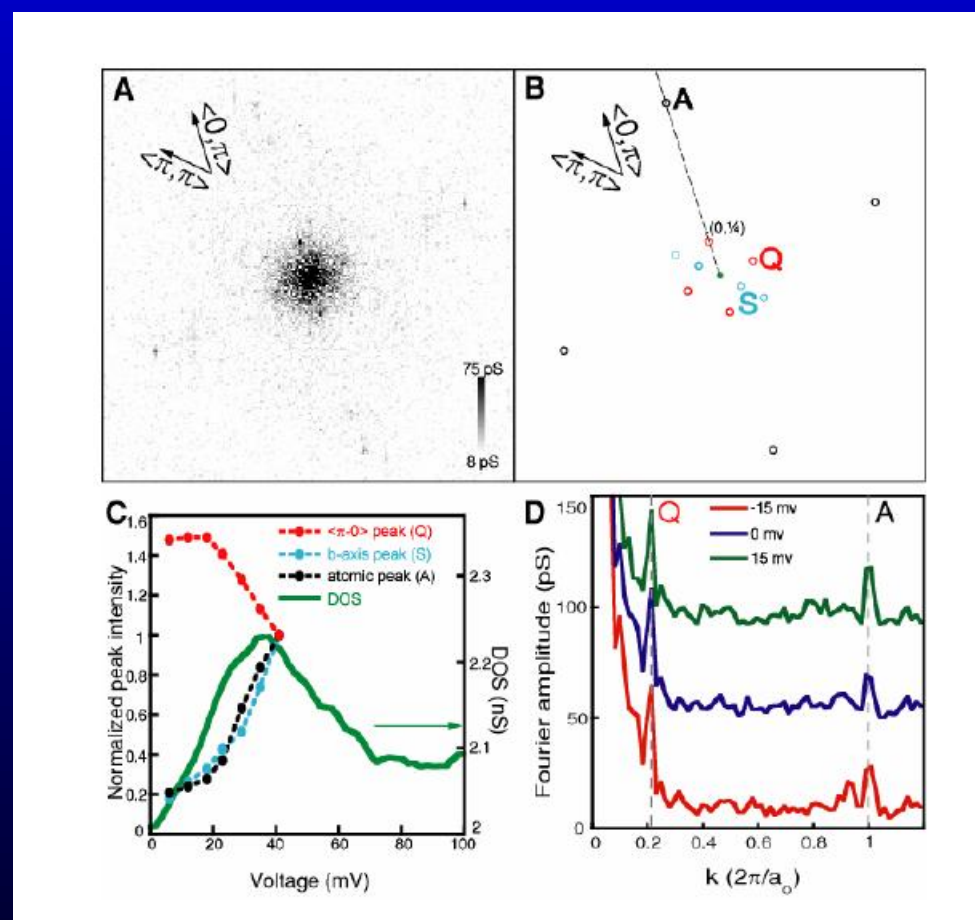
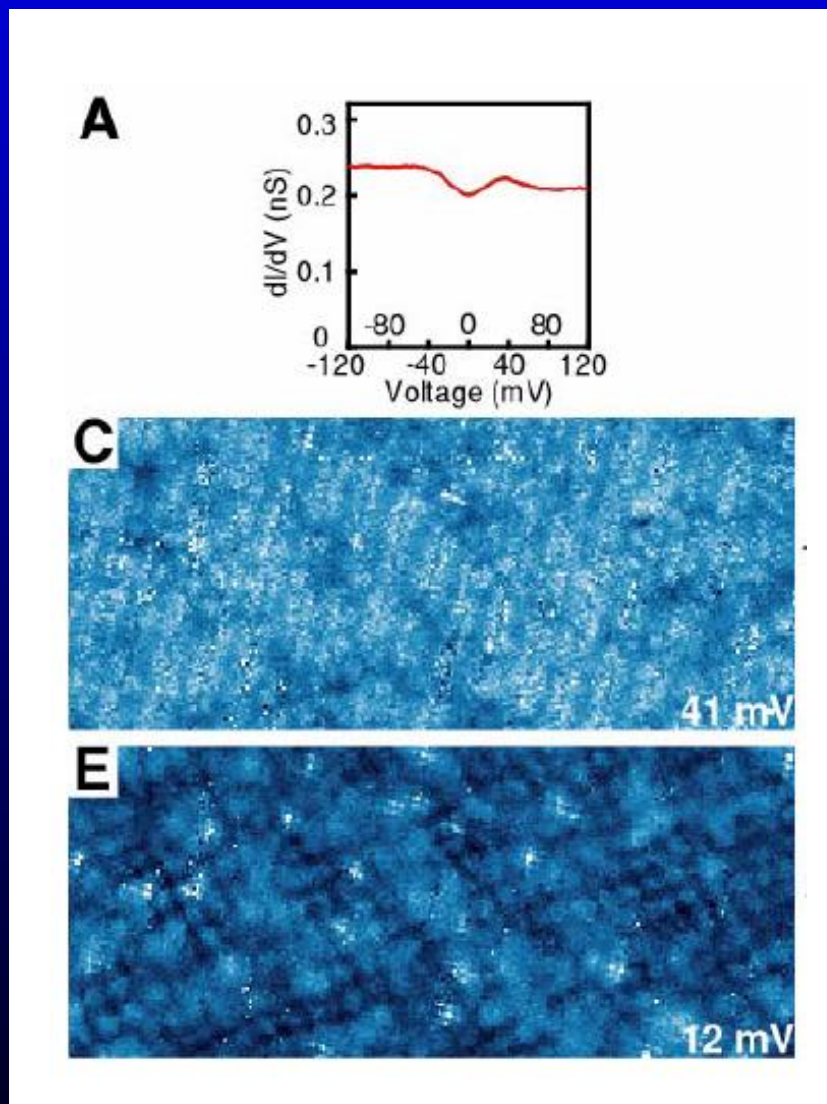
Reality check: FT-STs at 100K in BiSCCO

Vershinin *et al.*, Science Express, 12 Feb. 2004



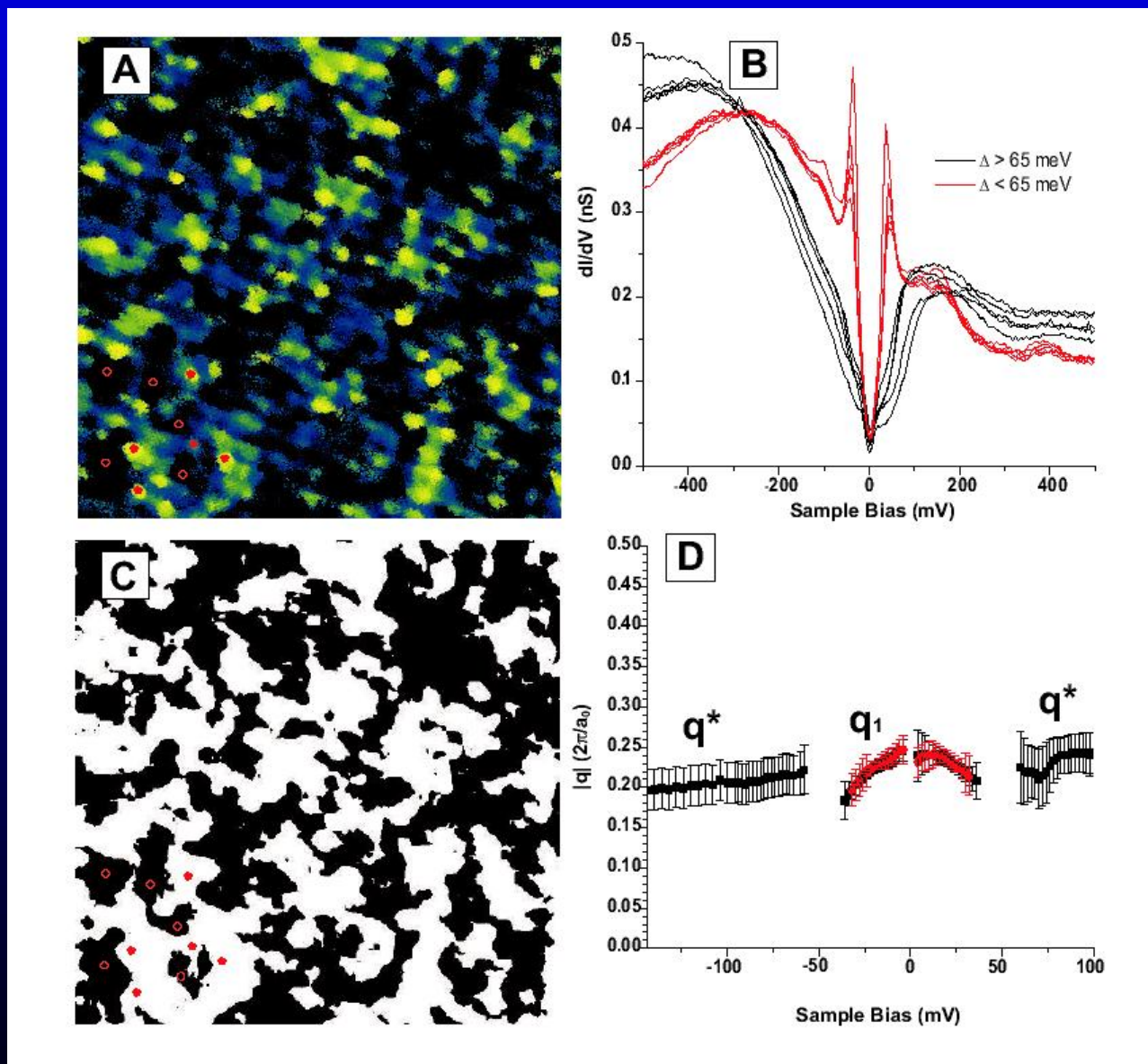
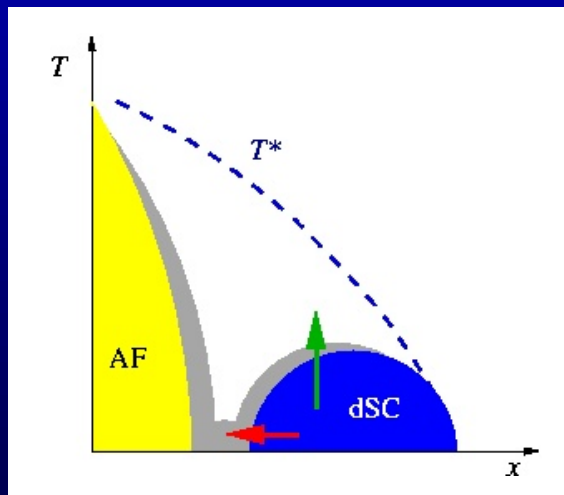
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FT-STs in strongly underdoped BiSCCO at 100mK

McElroy *et al.*, unpublished



Origin of the static pattern

Several theoretical proposals:

- **Density wave of Cooper pairs** [H.-D. Chen, O. Vafek, A. Yazdani, S.-C. Zhang, cond-mat/0402323]

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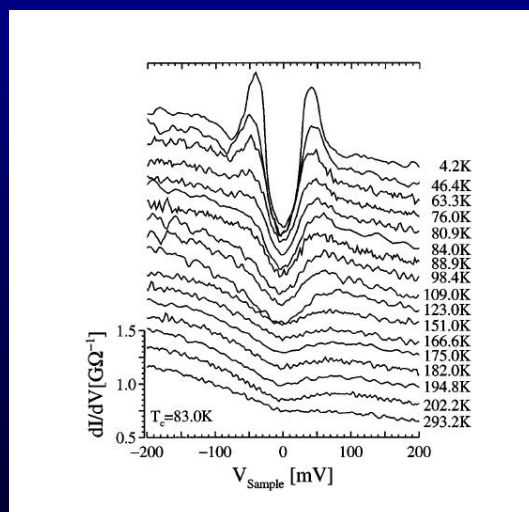
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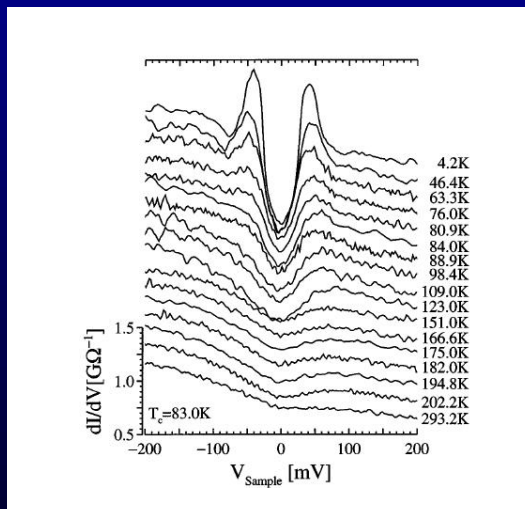


Renner (1998)

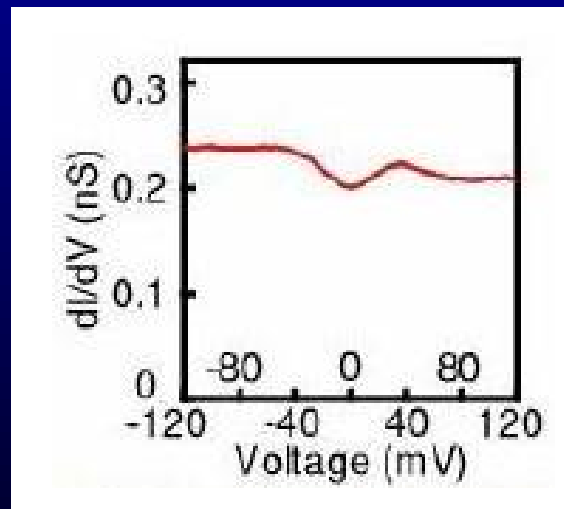
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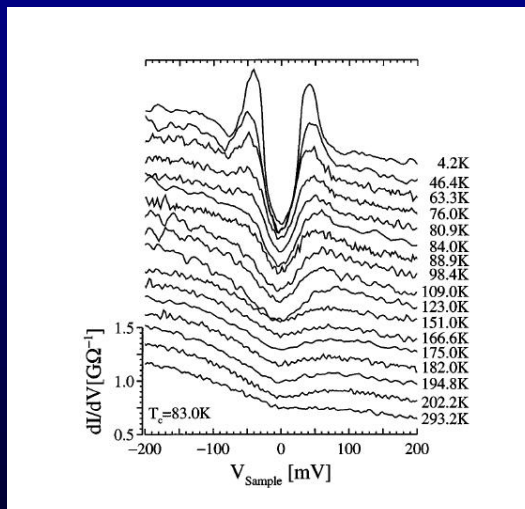


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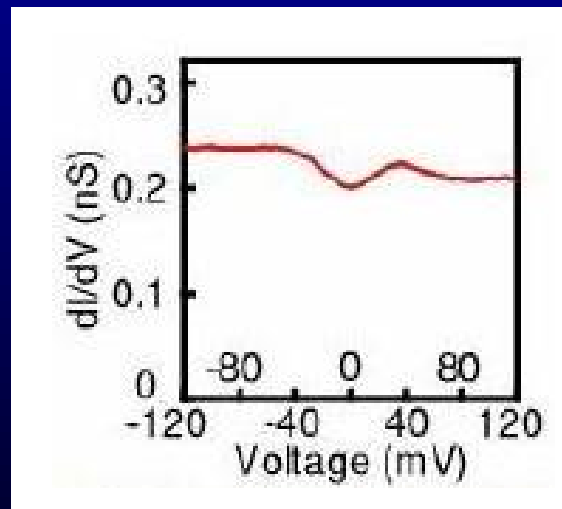
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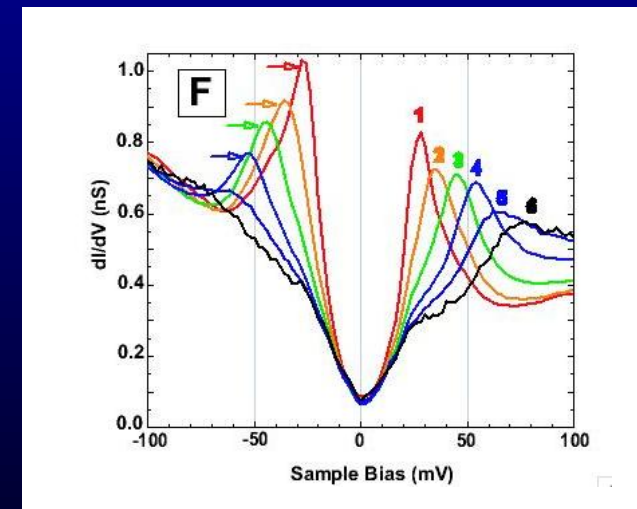
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Renner (1998)



Vershinin (2004)



McElroy (2004)

Conclusions

- By analyzing the quasiparticle interference patterns in the nodal approximation we gained some crucial insights into FT-STs in the superconducting state.
- FT-STs is sensitive to **both** the quasiparticle DOS and the coherence factors.
- This sensitivity can be used to determine the nature of the condensate responsible for the pseudogap phenomenon in the cuprates.
- Several experimental groups are now actively pursuing related projects.