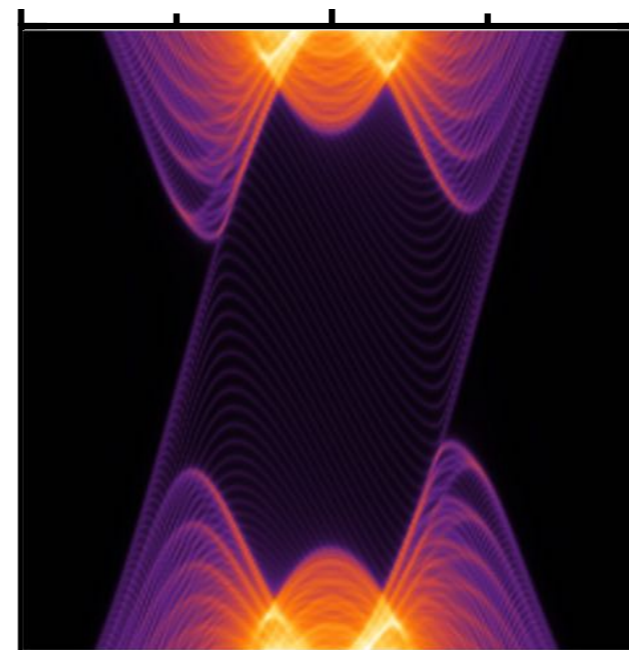
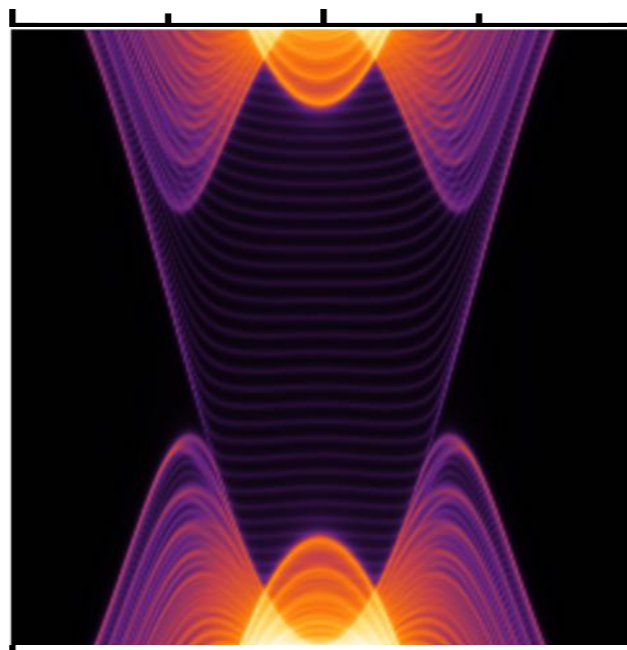


Quantum oscillations without magnetic field

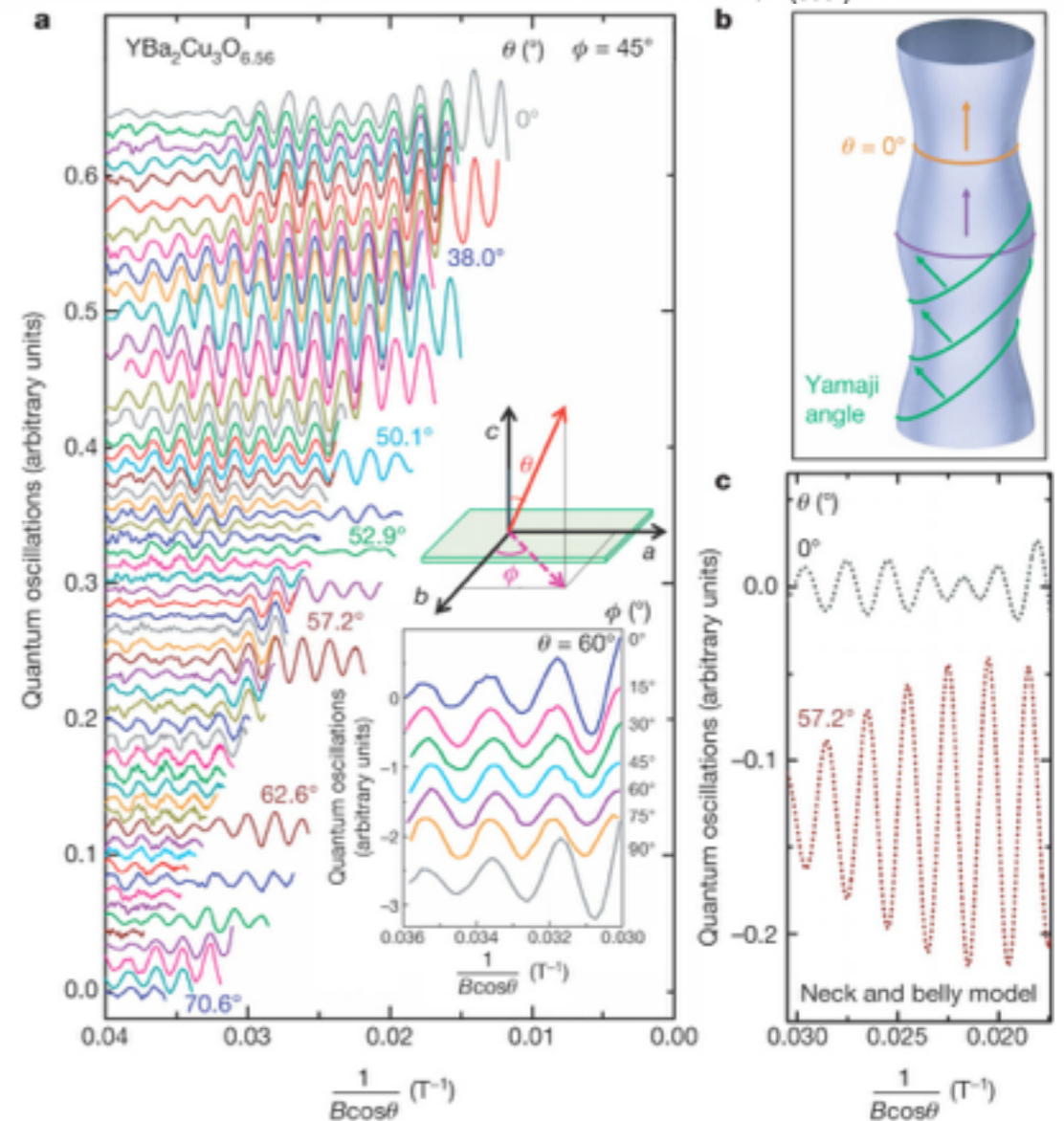
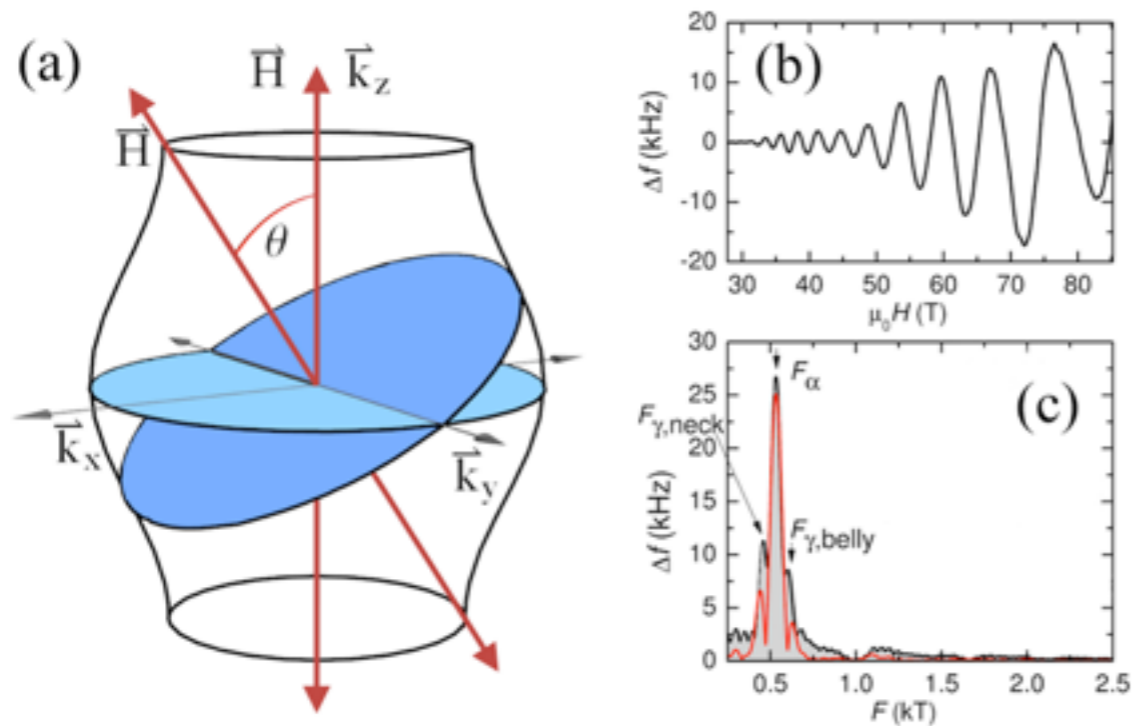
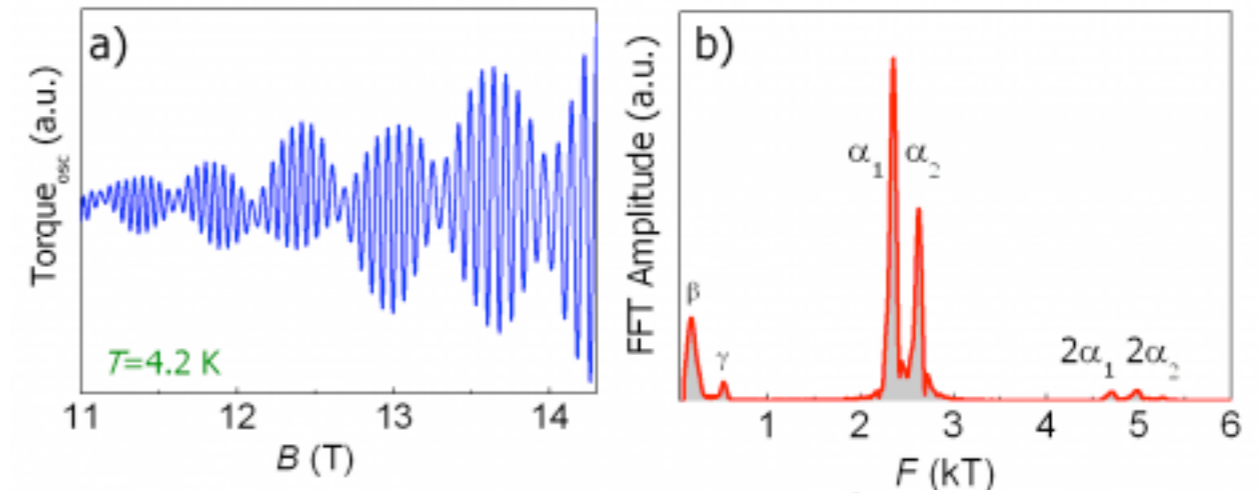
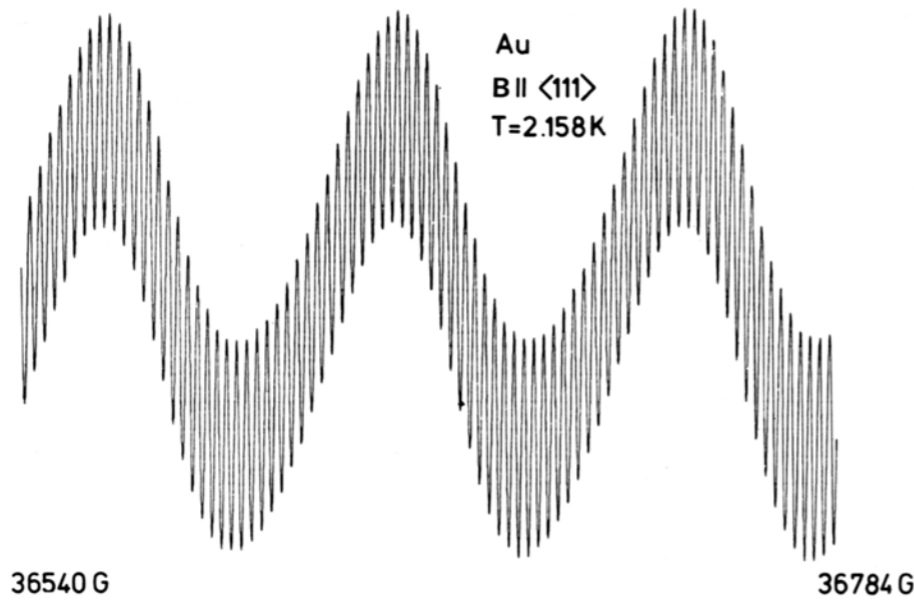
M. Franz, Anffany Chen, Tianyu Liu and D.I. Pikulin

Stewart Blusson Quantum Matter Institute
University of British Columbia

[arXiv:1608.04678](https://arxiv.org/abs/1608.04678), [arXiv:1607.01810](https://arxiv.org/abs/1607.01810)



Quantum oscillations

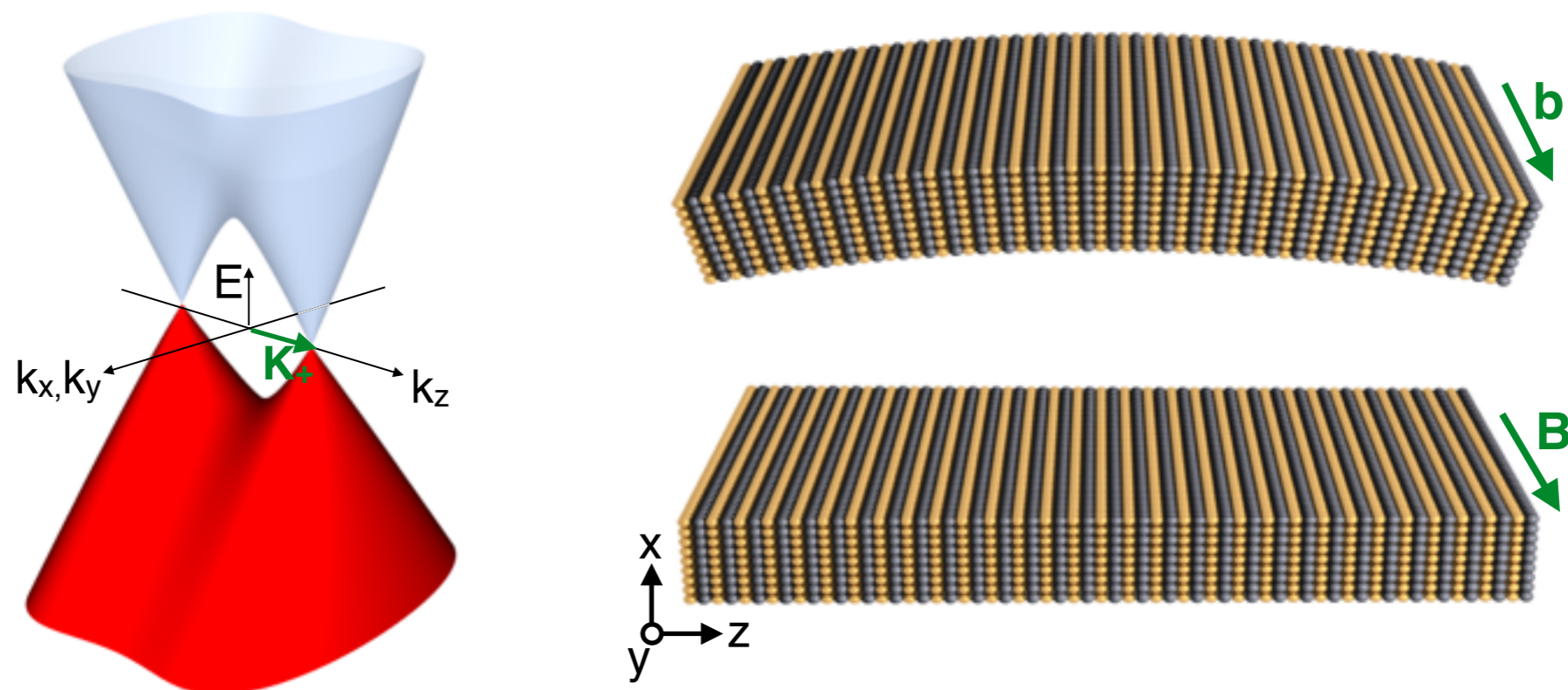


Quantum oscillations normally require magnetic field B

... except when B can be replaced by b .

b = strain-induced pseudomagnetic field

Main result: in Dirac and Weyl semimetals quantum oscillations can be generated by elastic strain in complete absence of magnetic field B



Energy gaps and a zero-field quantum Hall effect in graphene by strain engineering

F. Guinea^{1*}, M. I. Katsnelson² and A. K. Geim^{3*}

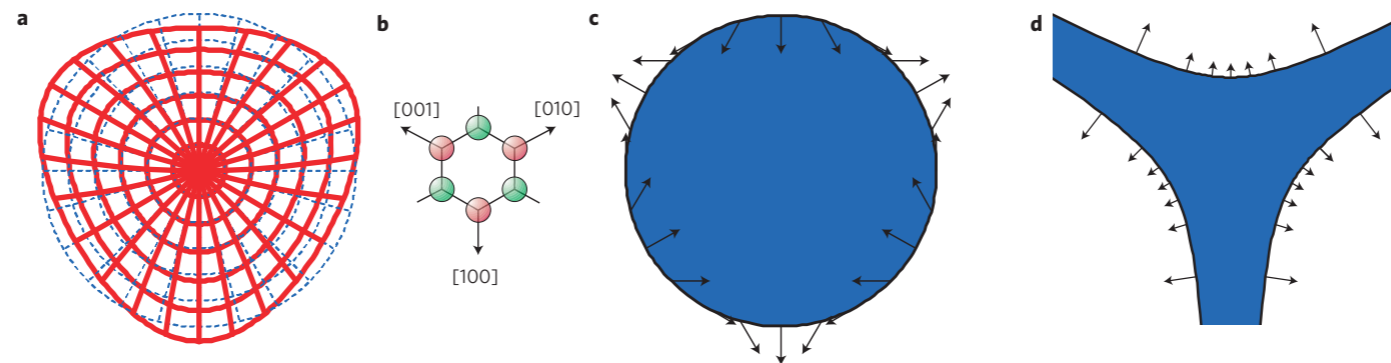


Figure 1 | Designed strain can generate a strictly uniform pseudomagnetic field in graphene. **a**, Distortion of a graphene disc which is required to generate uniform B_S . The original shape is shown in blue. **b**, Orientation of the graphene crystal lattice with respect to the strain. Graphene is stretched or compressed along equivalent crystallographic directions (100). Two graphene sublattices are shown in red and green. **c**, Distribution of the forces applied at the disc's perimeter (arrows) that would create the strain required in **a**. The uniform colour inside the disc indicates strictly uniform pseudomagnetic field. **d**, The shown shape allows uniform B_S to be generated only by normal forces applied at the sample's perimeter. The length of the arrows indicates the required local stress.

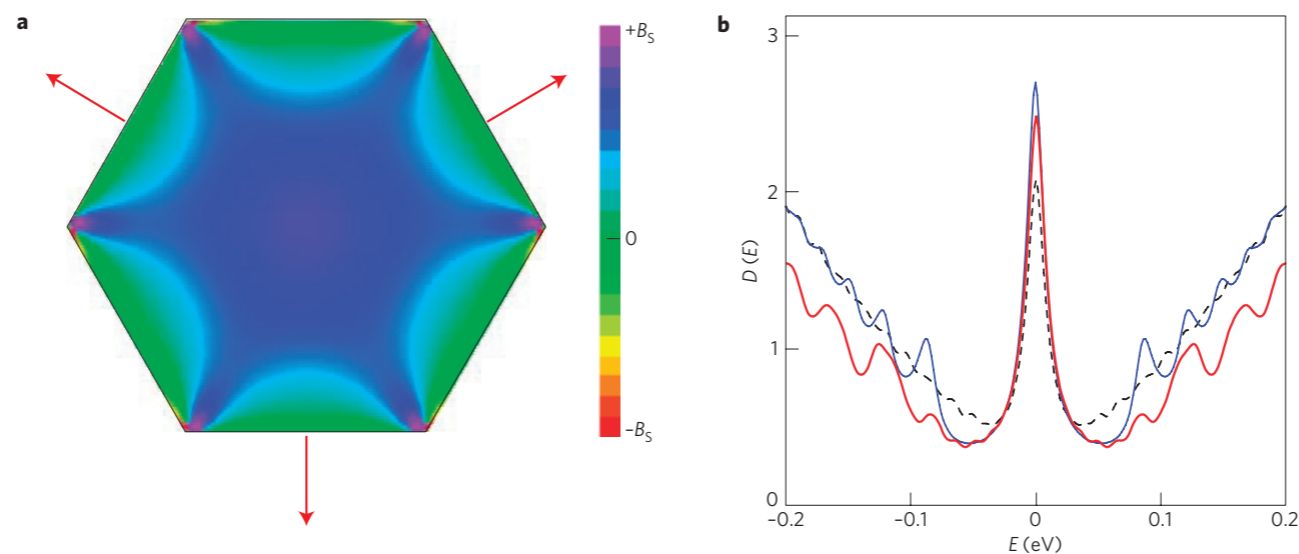
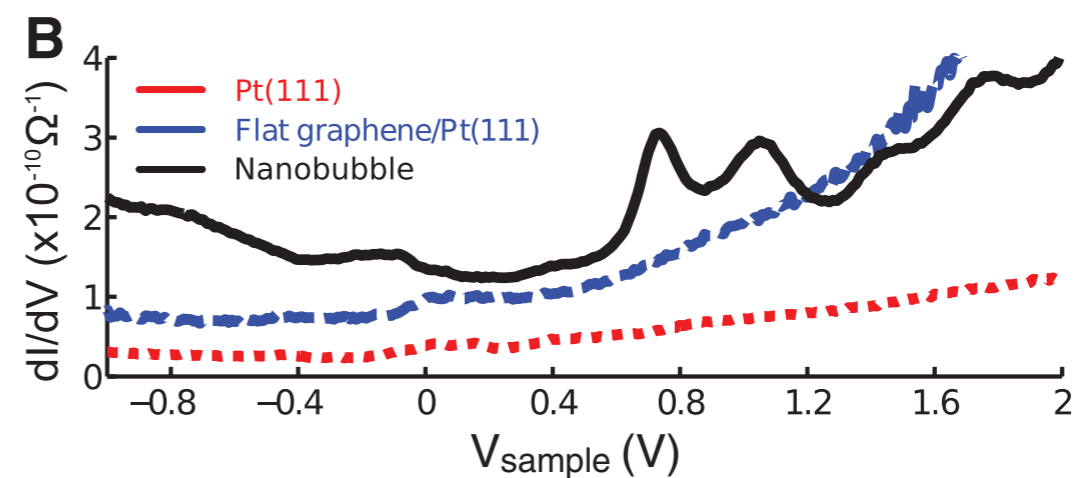
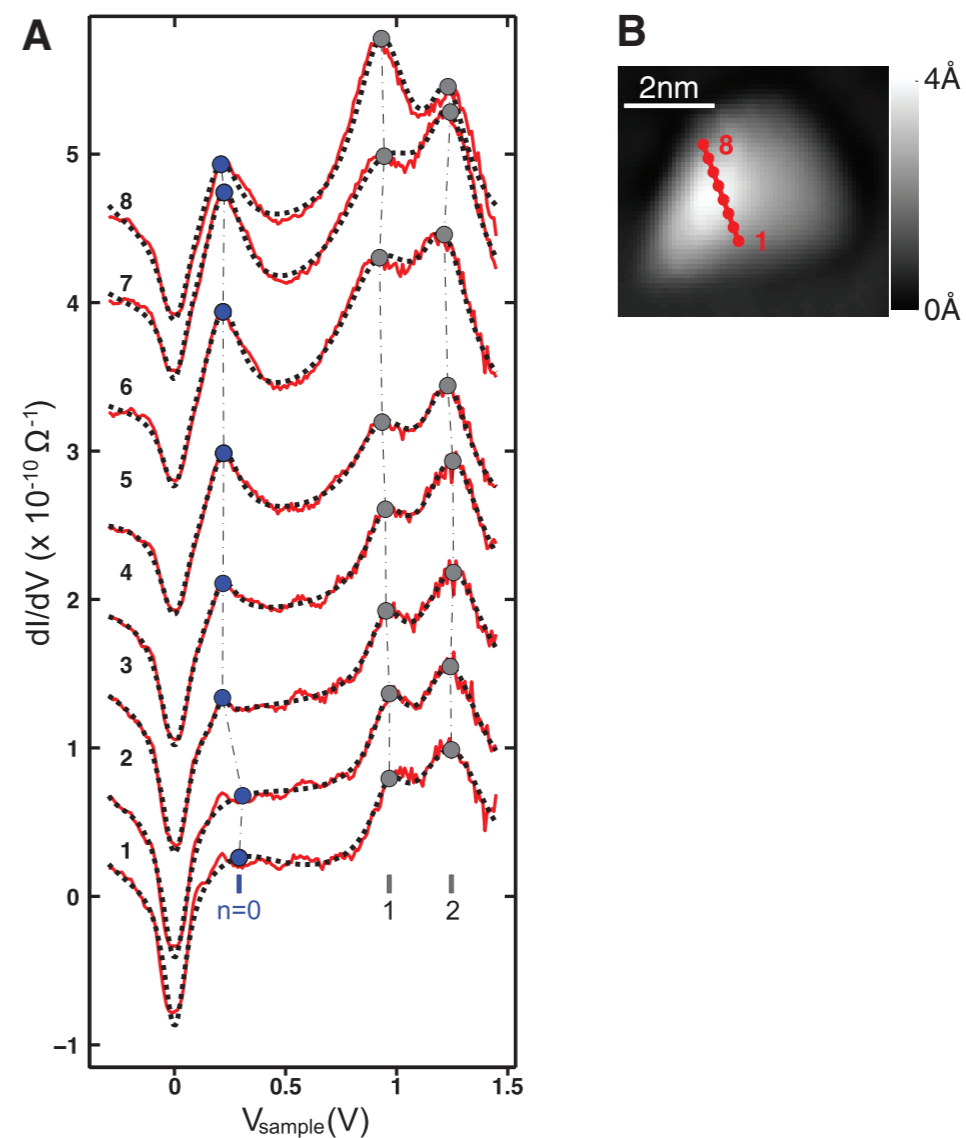
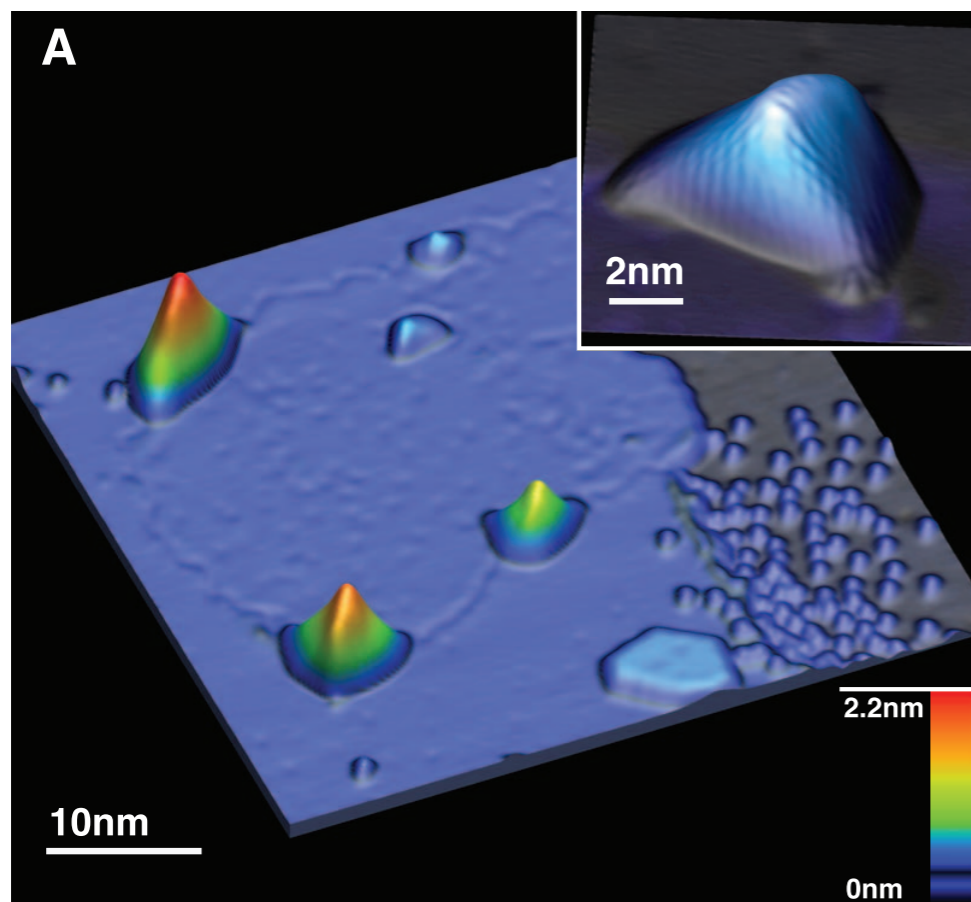


Figure 2 | Stretching graphene samples along (100) axes always generates a pseudomagnetic field that is fairly uniform at the centre. **a**, Distribution of B_S for a regular hexagon stretched by its three sides oriented perpendicular to (100). Other examples are given in the Supplementary Information. **b**, Normalized density of states for the hexagon in **a** with $L = 30$ nm and $\Delta_m = 1\%$. The black curve is for the case of no strain and no magnetic field. The peak at zero E is due to states at zigzag edges. The blue curve shows the Landau quantization induced by magnetic field $B = 10$ T. The pseudomagnetic field with $B_S \approx 7$ T near the hexagon's centre induces the quantization shown by the red curve. Comparison between the curves shows that the smearing of the pseudo-Landau levels is mostly due to the finite broadening $\Gamma = 2$ meV used in the tight-binding calculations (Γ corresponds to submicrometre mean free paths attainable in graphene devices). The inhomogeneous B_S plays little role in the broadening of the first few pseudo-Landau levels (see Supplementary Fig. S4).

Strain-Induced Pseudo-Magnetic Fields Greater Than 300 Tesla in Graphene Nanobubbles

N. Levy,^{1,2*}† S. A. Burke,^{1,*}‡ K. L. Meaker,¹ M. Panlasigui,¹ A. Zettl,^{1,2} F. Guinea,³ A. H. Castro Neto,⁴ M. F. Crommie,^{1,2}§

Recent theoretical proposals suggest that strain can be used to engineer graphene electronic states through the creation of a pseudo-magnetic field. This effect is unique to graphene because of its massless Dirac fermion-like band structure and particular lattice symmetry (C_{3v}). Here, we present experimental spectroscopic measurements by scanning tunneling microscopy of highly strained nanobubbles that form when graphene is grown on a platinum (111) surface. The nanobubbles exhibit Landau levels that form in the presence of strain-induced pseudo-magnetic fields greater than 300 tesla. This demonstration of enormous pseudo-magnetic fields opens the door to both the study of charge carriers in previously inaccessible high magnetic field regimes and deliberate mechanical control over electronic structure in graphene or so-called "strain engineering."



Back to 3D: Model for Cd_3As_2 and Na_3Bi

In the basis of spin-orbit coupled states

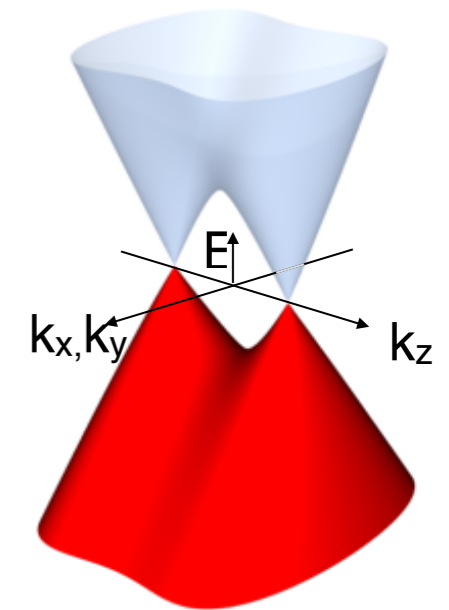
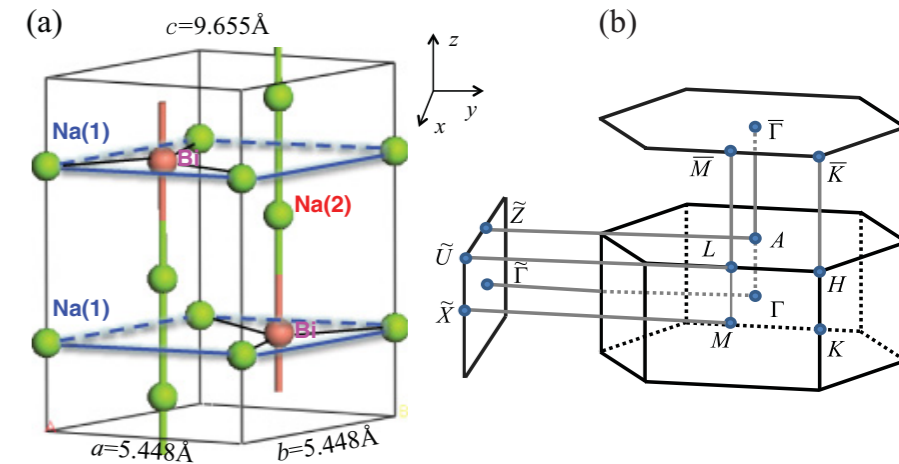
$$|P_{\frac{3}{2}}, \frac{3}{2}\rangle, |S_{\frac{1}{2}}, \frac{1}{2}\rangle, |S_{\frac{1}{2}}, -\frac{1}{2}\rangle, |P_{\frac{3}{2}}, -\frac{3}{2}\rangle$$

the tight-binding model reads:

$$H^{\text{latt}} = \epsilon_{\mathbf{k}} + \begin{pmatrix} h^{\text{latt}} & 0 \\ 0 & -h^{\text{latt}} \end{pmatrix}$$

$$h^{\text{latt}}(\mathbf{k}) = m_{\mathbf{k}}\tau^z + \Lambda(\tau^x \sin a_x k_x + \tau^y \sin a_y k_y),$$

$$m_{\mathbf{k}} = t_0 + t_1 \cos ak_z + t_2(\cos ak_x + \cos ak_y)$$

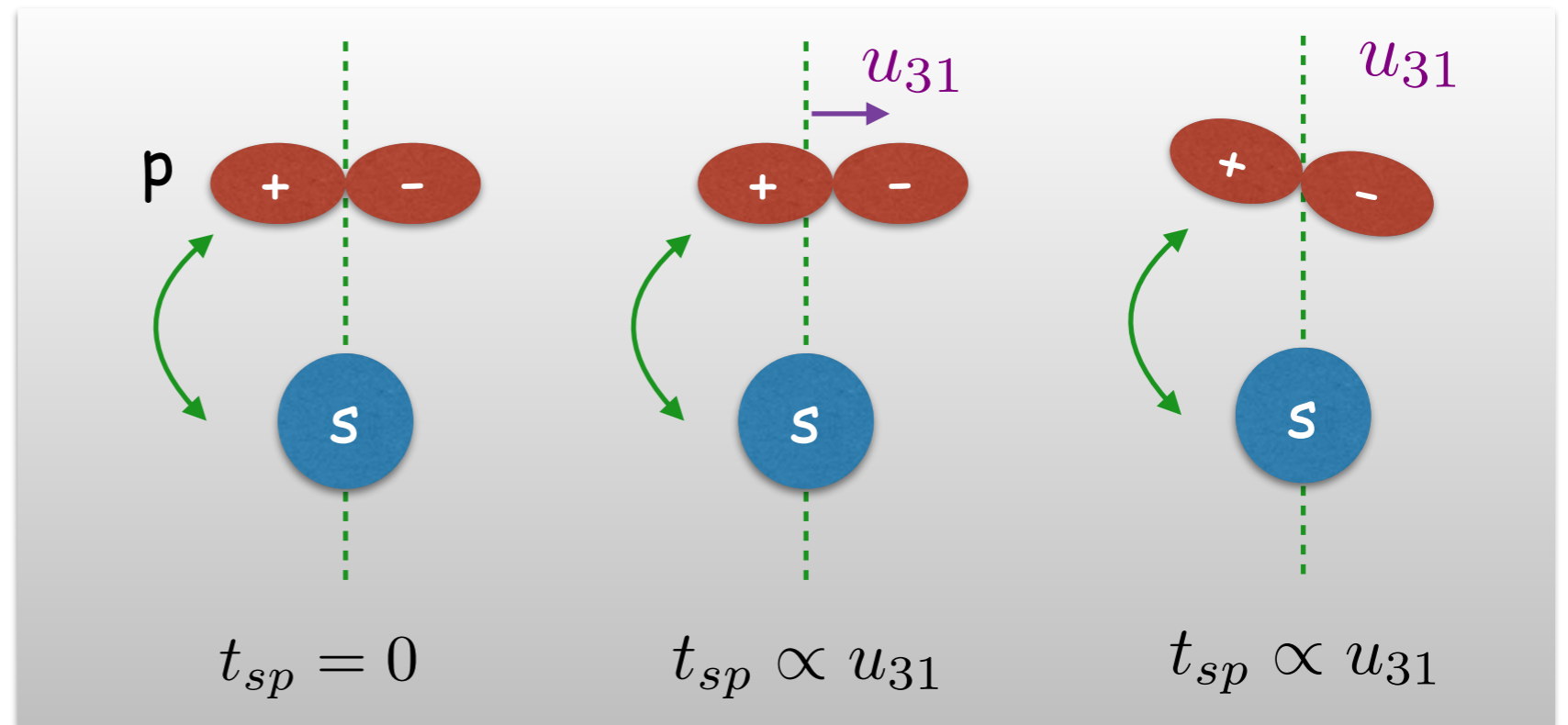
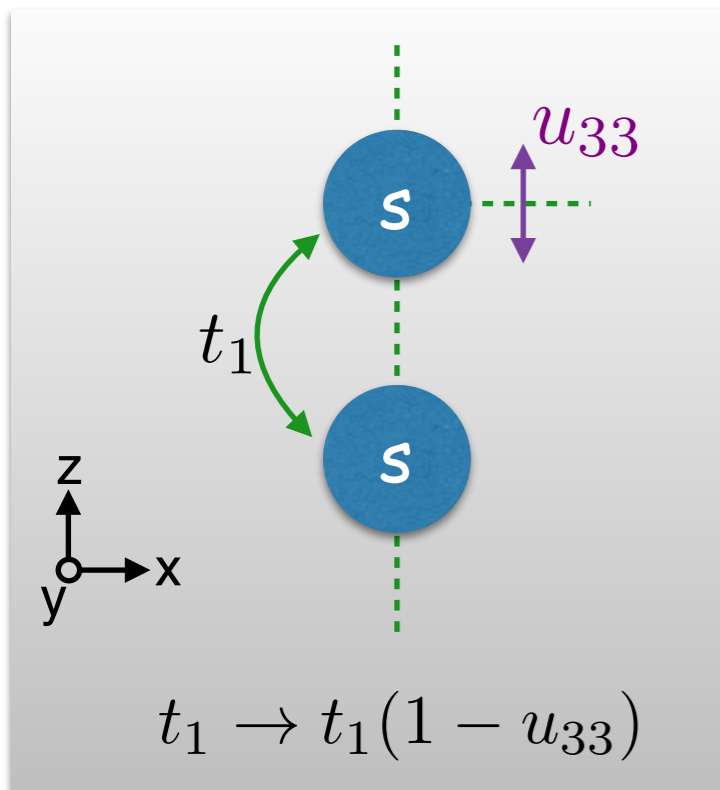


Effect of elastic strain on tunnelling amplitudes

The most important effect of elastic strain is incorporated by

$$t_1 \tau^z \rightarrow t_1 (1 - u_{33}) \tau^z + i\Lambda \sum_{j \neq 3} u_{3j} \tau^j,$$

where $u_{ij} = \frac{1}{2}(\partial_i u_j + \partial_j u_i)$ is the strain tensor and \mathbf{u} is the displacement field.



Strain = chiral gauge potential at low energies

Derive low-energy theory by expanding $h^{\text{latt}}(\mathbf{k})$ near Dirac points $\mathbf{k} = \mathbf{K}_{\pm} + \mathbf{q}$

$$h_{\eta}(\mathbf{q}) = v_{\eta}^j \tau^j \left(\hbar q_j - \eta \frac{e}{c} \mathcal{A}_j \right), \quad \eta = \pm$$

Here $v_{\eta} = \hbar^{-1} a(\Lambda, \Lambda, -\eta t_1 \sin aQ)$ and

$$\vec{\mathcal{A}} = -\frac{\hbar c}{ea} (u_{13} \sin aQ, u_{23} \sin aQ, u_{33} \cot aQ).$$

$$\mathbf{K}_{\pm} = (0, 0, \pm Q)$$

Components u_{3j} of the strain tensor act on Dirac fermions as chiral gauge field.

Strain-induced field strength estimate

Displacement field

$$\mathbf{u} = (0, 0, 2\alpha xz/d)$$

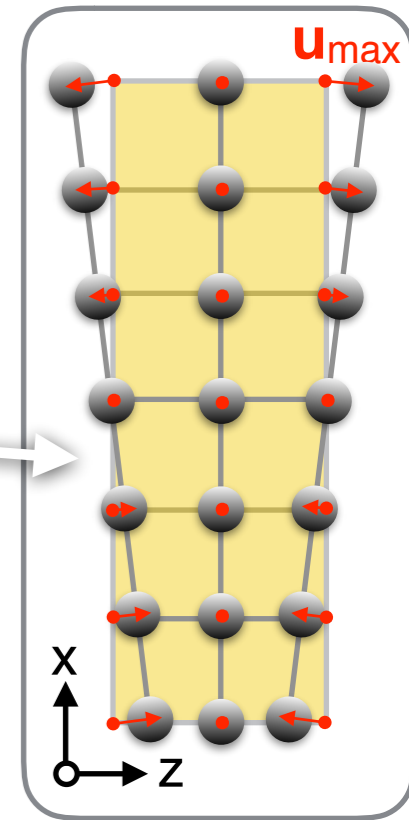
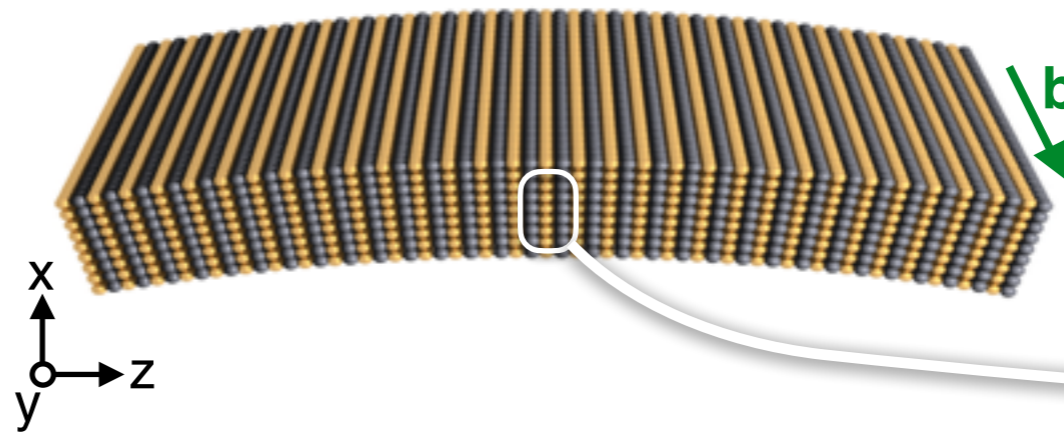
gives strain tensor

$$u_{33} = 2\alpha x/d$$

and pseudomagnetic field

$$\mathbf{b} = \nabla \times \vec{\mathcal{A}} = \hat{y} \left(\frac{2\alpha}{d} \right) \frac{\hbar c}{ea} \cot aQ \simeq \hat{y} \alpha \times 246 \text{T}$$

for Cd_3As_2 parameters with $aQ \simeq 0.132$.



$$\alpha = u_{\text{max}}/a$$

What is the maximum strain Cd_3As_2 film can sustain?

ARTICLE

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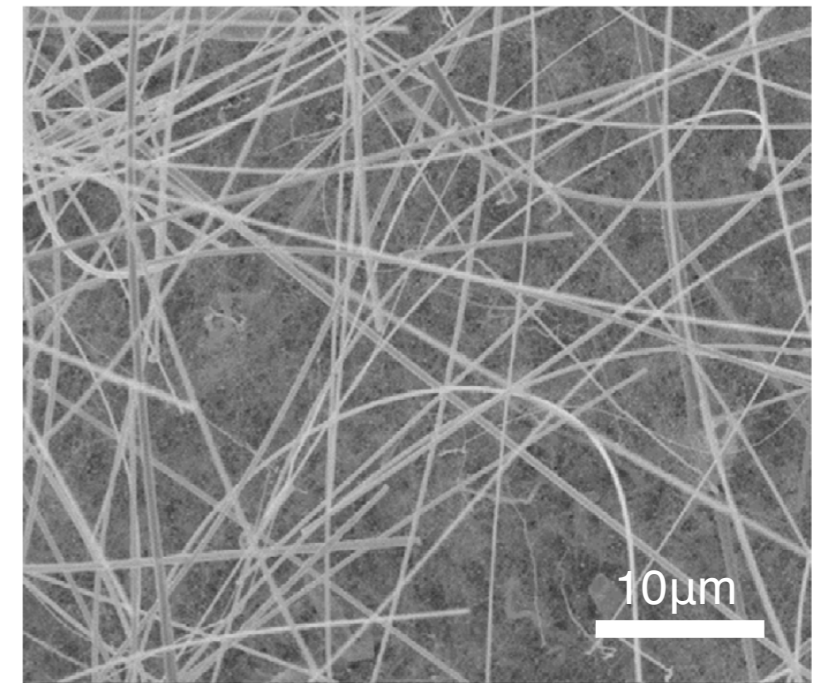
DOI: 10.1038/ncomms10137

OPEN

Giant negative magnetoresistance induced by the chiral anomaly in individual Cd_3As_2 nanowires

Cai-Zhen Li^{1,*}, Li-Xian Wang^{1,*}, Haiwen Liu², Jian Wang^{2,3}, Zhi-Min Liao^{1,3} & Da-Peng Yu^{1,3}

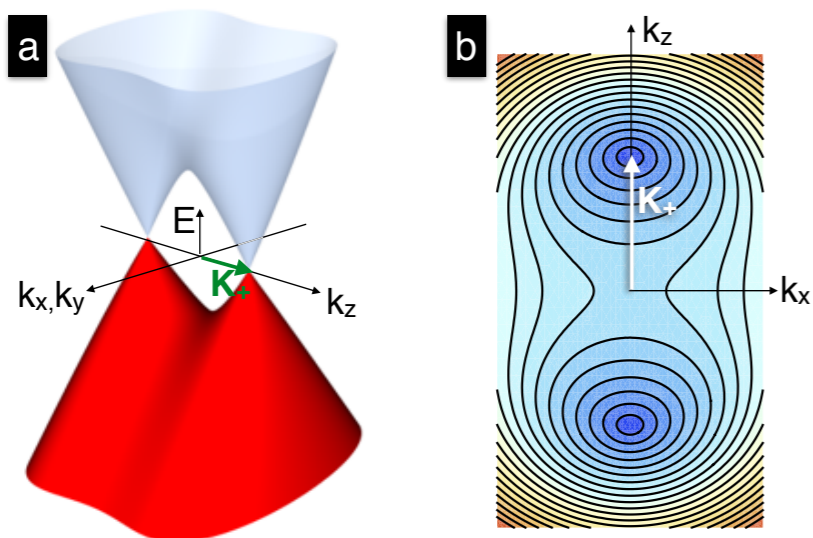
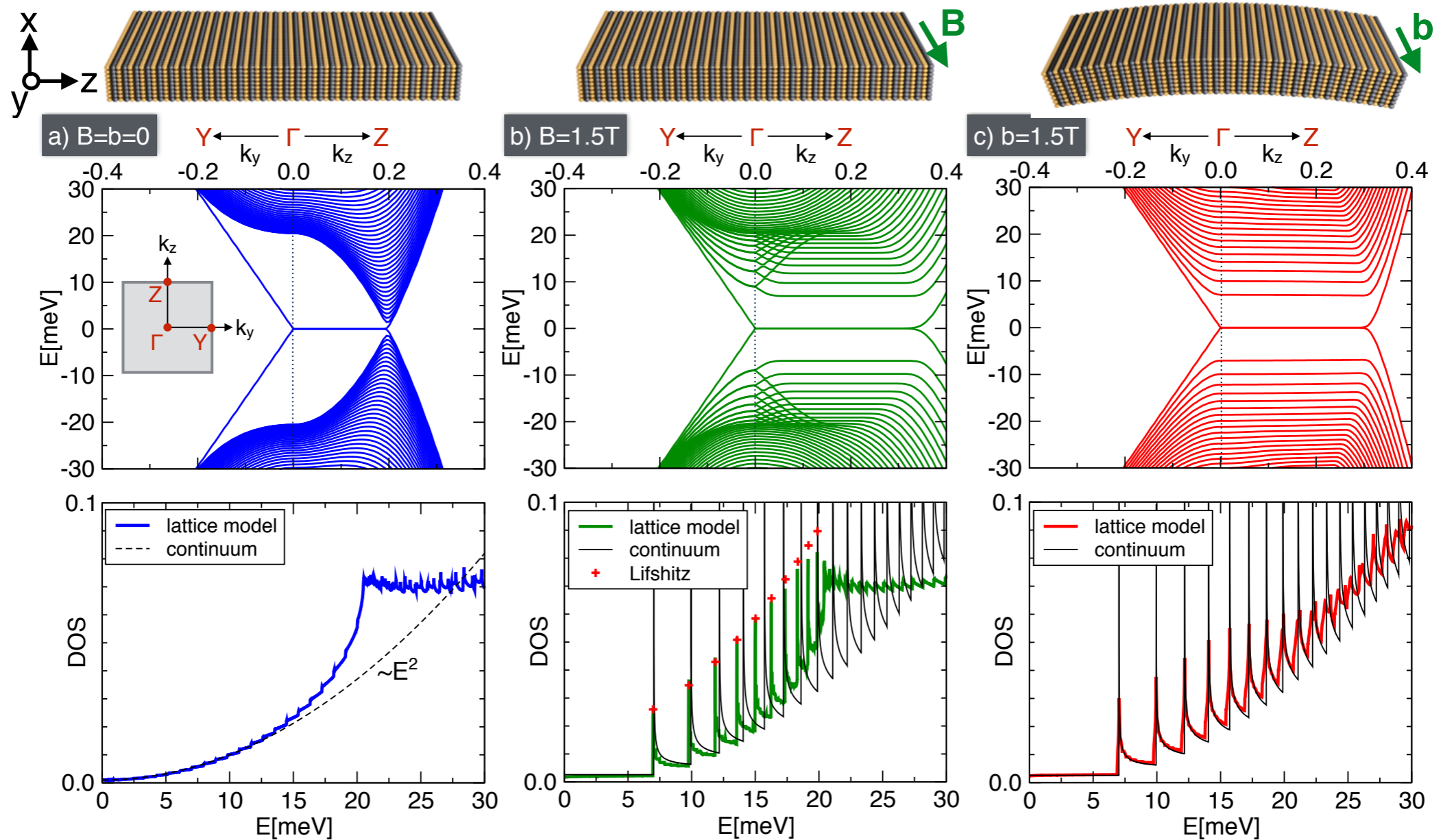
a



From the radius R of the bend and diameter d of nanowires we estimate that distortion α of several percent can be achieved. This gives maximum estimated pseudomagnetic field of $10\text{-}15\text{T}$ in Cd_3As_2 .

This should be sufficient to observe strain-induced quantum oscillations.

Numerical results for Cd_3As_2 lattice model



Dirac Landau levels:

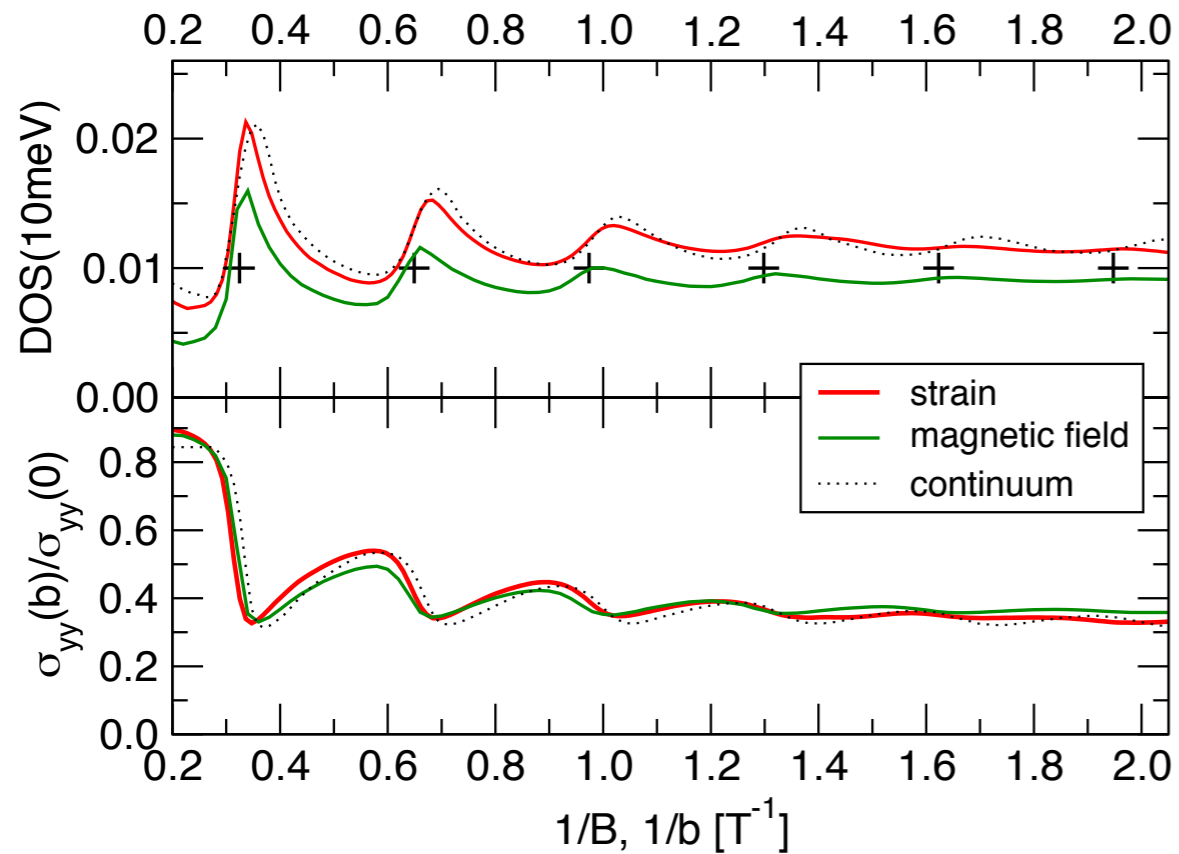
$$E_n(k_y) = \pm \hbar \sqrt{v_y^2 k_y^2 + 2n v_x v_z \frac{e|B|}{\hbar c}}, \quad n = 1, 2, \dots,$$

Lifshitz-Onsager quantization:

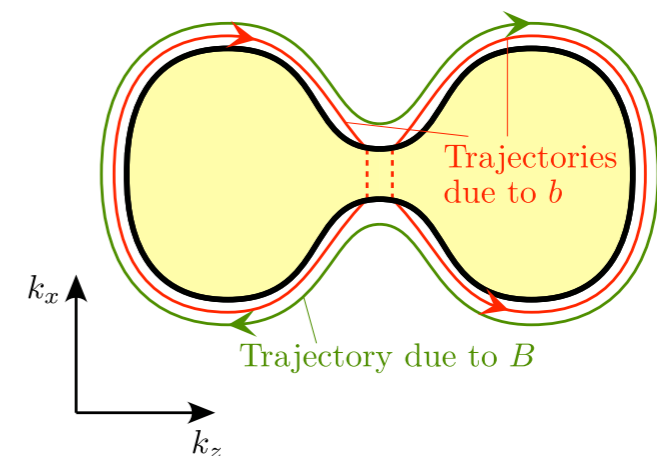
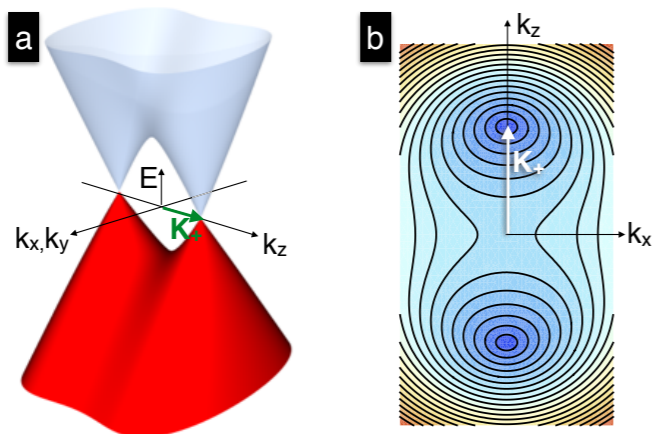
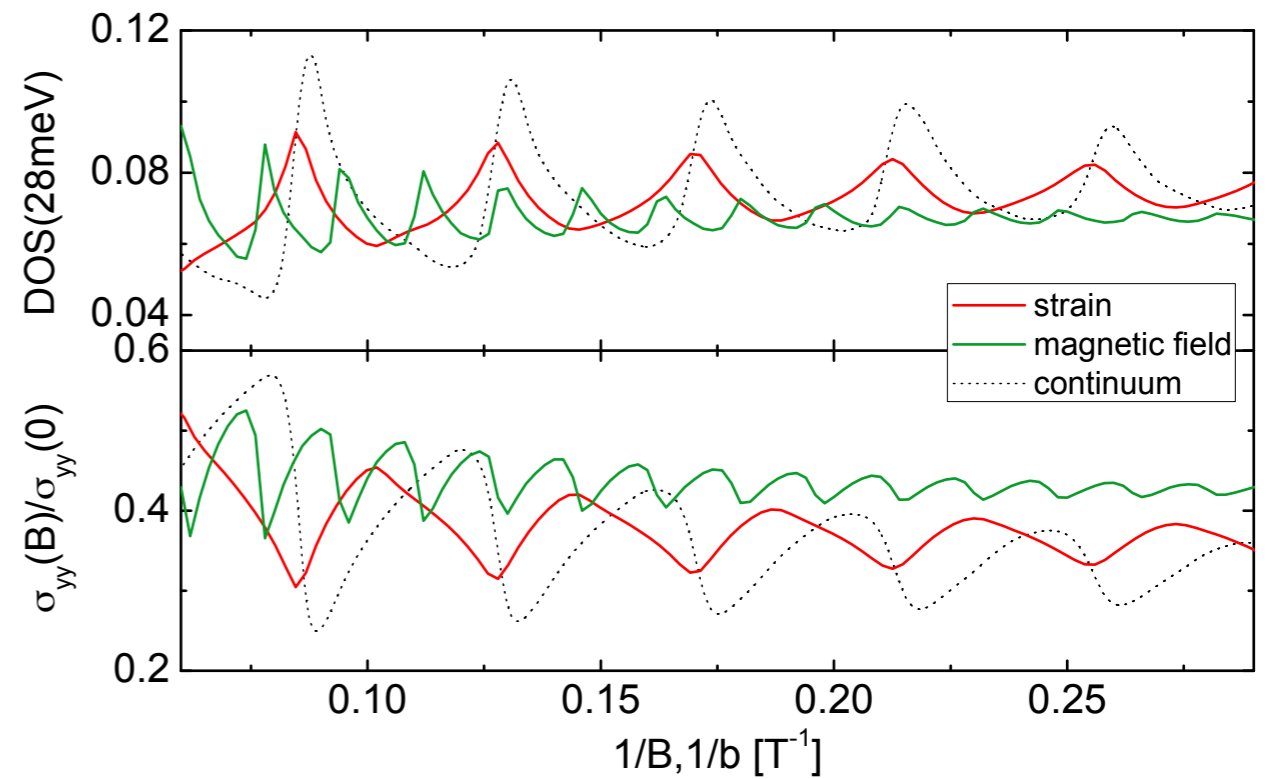
$$S(E_n) = 2\pi n (eB/\hbar c)$$

de Haas-van Alphen and Shubnikov-de Haas oscillations

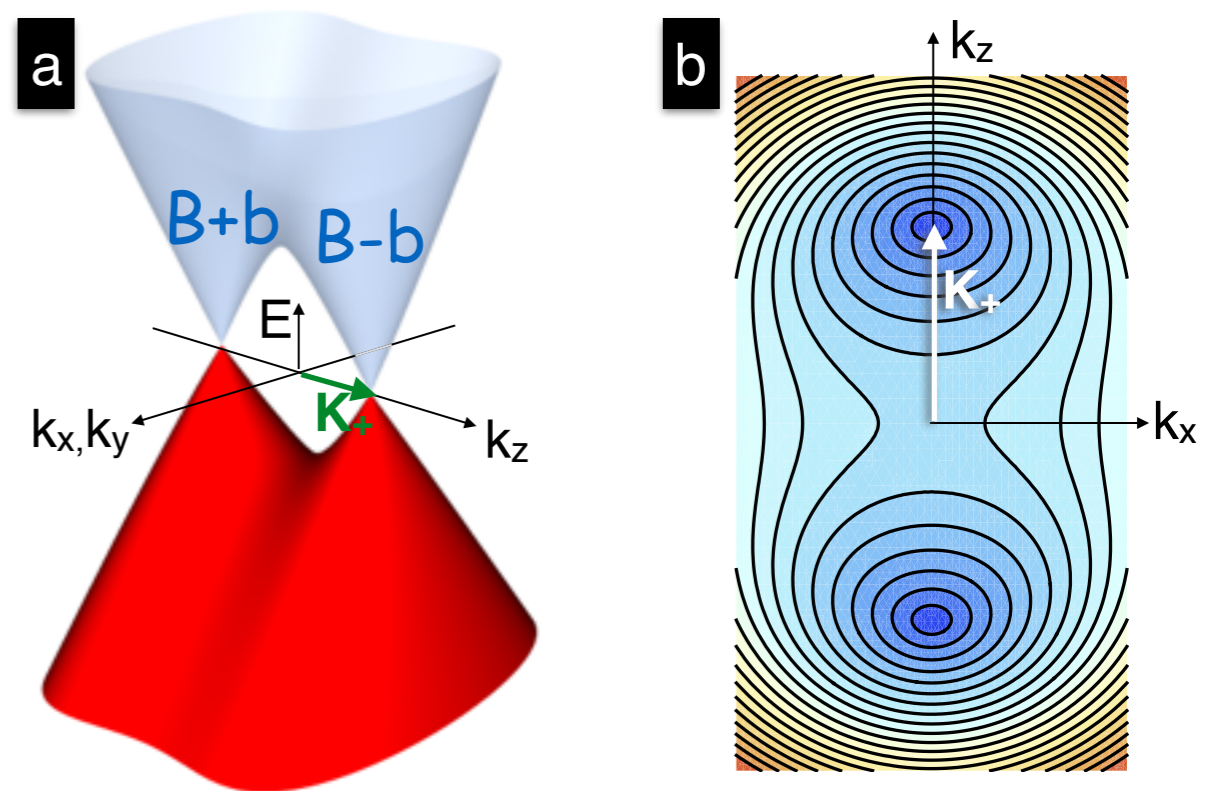
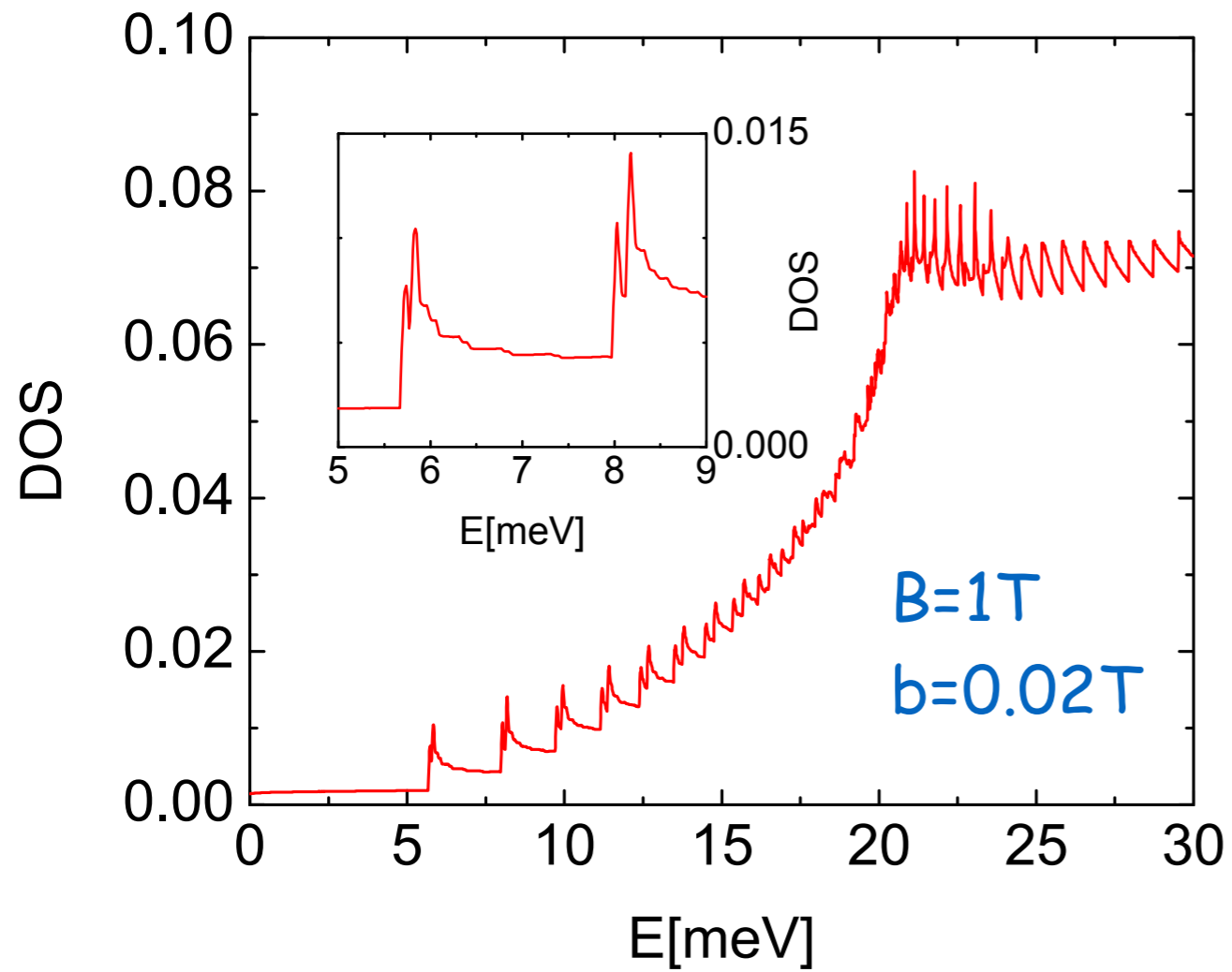
Below E_{Lif}



Above E_{Lif}

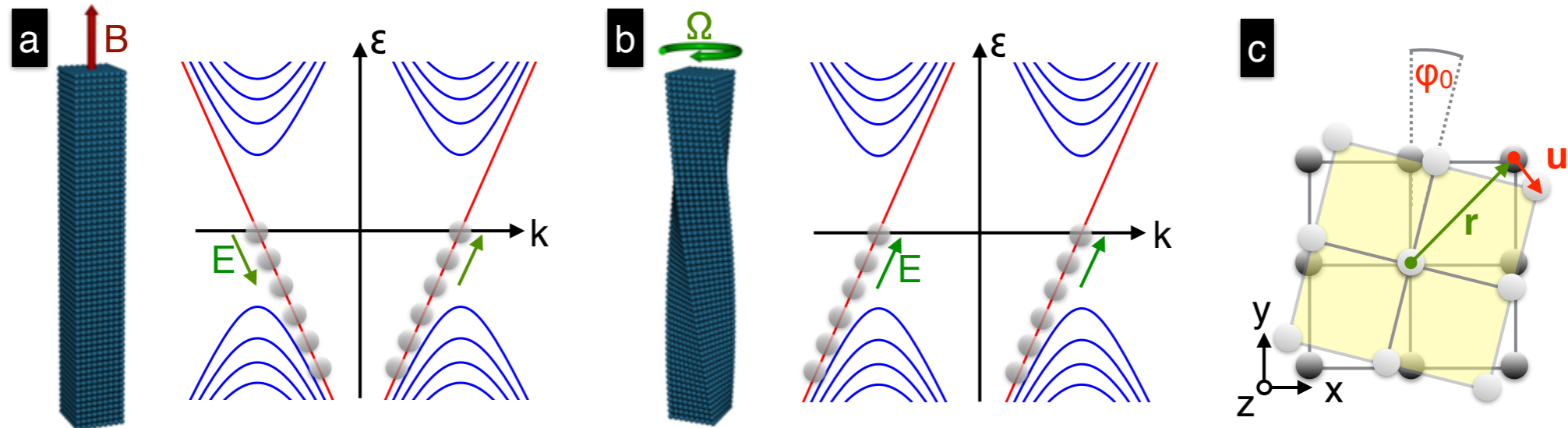


Equivalence of B and b for low-energy electrons



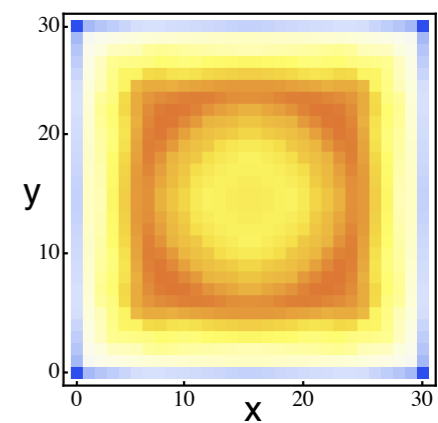
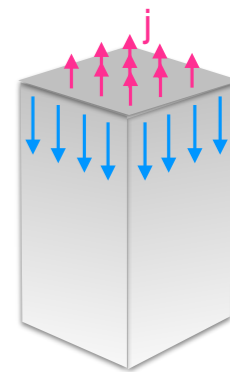
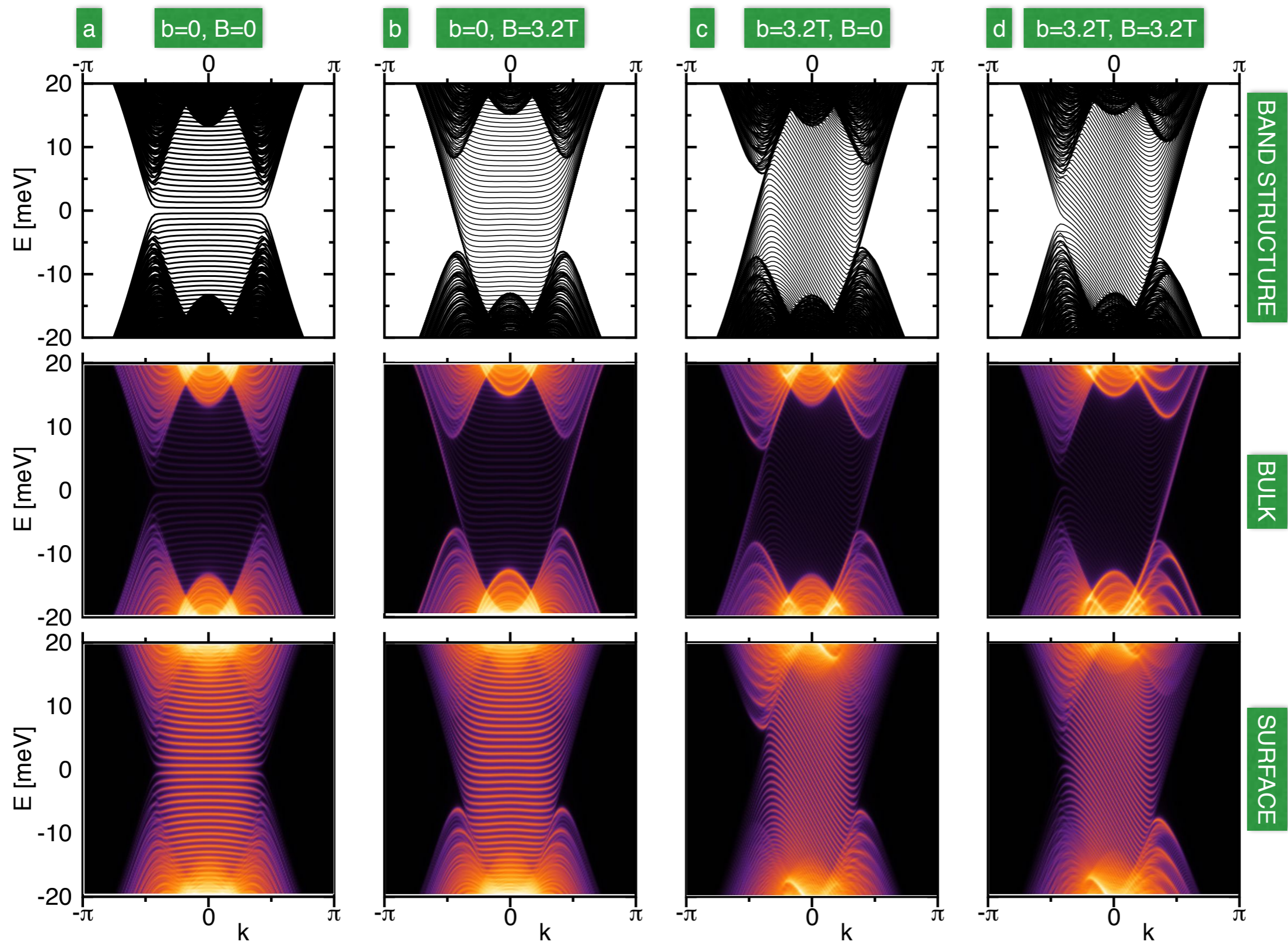
Chiral anomaly from strain-induced EM fields

arXiv:1607.01810



$$\partial_t \rho_5 + \nabla \cdot \mathbf{j}_5 = \frac{e^2}{2\pi^2 \hbar^2 c} (\mathbf{E} \cdot \mathbf{B} + \mathbf{e} \cdot \mathbf{b}),$$

$$\partial_t \rho + \nabla \cdot \mathbf{j} = \frac{e^2}{2\pi^2 \hbar^2 c} (\mathbf{E} \cdot \mathbf{b} + \mathbf{e} \cdot \mathbf{B}).$$



Conclusions

- Similar to graphene elastic strain in Dirac and Weyl semimetals acts as chiral gauge potential
- This gives rise to **quantum oscillations** in complete absence of magnetic field
- Also generates an interesting novel manifestation of the chiral anomaly

