REVIEW # 2 ANSWERS

3.6 $f_\pm(\varphi) = A\, e^{\pm 2\pi i n}$, $n = 0, 1, 2, \ldots$ Doubly degenerate except for $n=0$.

3.22 $\langle\alpha| = -i\langle 1| - 2\langle 2| + i\langle 3|$, $\langle\beta| = -i\langle 1| + 2\langle 3|$

$\langle\alpha|\beta\rangle = 1+2i$, $\langle\beta|\alpha\rangle = 1-2i$

$$A = \begin{pmatrix} 1 & 0 & 2i \\ 2i & 0 & -4 \\ -1 & 0 & -2i \end{pmatrix} \quad \text{Not Hermitian.}$$

3.37 Eigenvectors $|s_1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $|s_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $|s_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

a) $|S(t)\rangle = e^{-ict/\hbar}|s_1\rangle$

b) $|S(t)\rangle = \frac{1}{\sqrt{2}}\left[e^{-i(a+b)t/\hbar}|s_2\rangle + e^{-i(a-b)t/\hbar}|s_3\rangle\right] = e^{-iat/\hbar}\begin{pmatrix} \cos bt/\hbar \\ 0 \\ -i\sin bt/\hbar \end{pmatrix}$

3.38 a) H: $E_1 = \hbar\omega$, $E_2 = E_3 = 2\hbar\omega$; $|h_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $|h_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $|h_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

A: $a_1 = 2\lambda$, $a_2 = \lambda$, $a_3 = -\lambda$; $|a_1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $|a_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $|a_3\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

B: $b_1 = 2\mu$, $b_2 = \mu$, $b_3 = -\mu$; $|b_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $|b_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $b_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

b) $\langle H\rangle = \hbar\omega\left(|c_1|^2 + 2|c_2|^2 + 2|c_3|^2\right)$

$\langle A\rangle = \lambda\left(c_1^* c_2 + c_2^* c_1 + 2|c_3|^2\right)$ $\langle B\rangle = \mu\left(2|c_1|^2 + c_2^* c_3 + c_3^* c_2\right)$

c) $|S(t)\rangle = e^{-2i\omega t}\begin{pmatrix} c_1 e^{i\omega t} \\ c_2 \\ c_3 \end{pmatrix}$

H: $P_1 = |c_1|^2$, $P_2 = P_3 = |c_2|^2 + |c_3|^2$

A: $P_1 = |c_3|^2$, $P_2 = \frac{1}{2}\left(|c_1|^2 + |c_2|^2 + c_1^* c_2\, e^{-i\omega t} + c_2^* c_1\, e^{i\omega t}\right)$, $P_3 = 1 - P_1 - P_2$

B: $P_1 = |c_1|^2$, $P_2 = \frac{1}{2}\left(|c_2|^2 + |c_3|^2 + c_2^* c_3 + c_2 c_3^*\right)$, $P_3 = 1 - P_1 - P_2$

4.14 $r = a$

4.15 a) $\Psi(\vec{r}, t) = -\frac{i}{\sqrt{2\pi a}\, 4a^2}\, r e^{-r/2a} \sin\theta \sin\varphi\, e^{-iE_2 t/\hbar}$

b) $\langle V \rangle = -\frac{\hbar^2}{4ma^2} = \frac{1}{2} E_1 = -6.8\,eV$

4.16 $E_n(Z) = Z^2 E_n, \quad a(Z) = a/Z, \quad R(Z) = Z^2 R$

For $Z = 2$: $\lambda_1 = 2.28 \times 10^{-8}\,m$, $\lambda_2 = 3.04 \times 10^{-8}\,m$ (ultraviolet)

For $Z = 3$: $\lambda_1 = 1.01 \times 10^{-8}\,m$, $\lambda_2 = 1.35 \times 10^{-8}\,m$ (ultraviolet)

4.17 a) $V(r) = -G\frac{Mm}{r} \qquad \left(\frac{e^2}{4\pi\epsilon_0} \rightarrow GMm\right)$

b) $a_g = \frac{\hbar^2}{GMm^2} = 3.34 \times 10^{-138}\,m$

c) $n = \sqrt{\frac{r_0}{a_g}} \approx 2.53 \times 10^{74}$

d) $\Delta E \simeq 2.09 \times 10^{-41}\,J, \qquad \lambda = 9.52 \times 10^{15}\,m$

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