PHYS 503	Problem set # 2	Due: Feb. 10, 2022, 2pm

1. (10 points) Using the technique of contour integration evaluate the Matsubara sum

$$C_{kk'}(\omega_n) = \frac{1}{\beta}\sum_m \mathcal{G}_k(\omega_m)\mathcal{G}_{k'}(\omega_n - \omega_m).$$

Here $\mathcal{G}_k(\omega_m)$ is the Matsubara phonon Green's function derived in class and $\omega_m = 2\pi m/\beta$. This type of sum is associated with the so-called phonon bubble Feynman diagram and plays an important role in the theory of electron-phonon coupling in metals and semiconductors.

2. (10 points) Derive the expression for the time-ordered phonon Green's function at non-zero temperature given by Eq. (2.3.16) in the textbook.

3. (20 points) According to the Lindemann criterion a crystal melts when the root-mean-square fluctuation of the ion position reaches a significant fraction of the lattice constant: $\sqrt{\langle u_j^2\rangle} = c_L a$, where $c_L = 0.10 - 0.15$ is the Lindemann number.

a) Starting from the phonon Green's function derived in class find an expression for the T-dependence of $\langle u_j^2\rangle$ in a generic crystal with dispersion $\Omega_{\mathbf{k}}$.

b) Now consider a model for the cubic crystal lattice in d-dimensions of ions with masses M connected to their nearest neighbors by springs as in problem 3 of the previous homework. For $d = 1, 2, 3$ find its melting temperature T_M in terms of M and K. What does this imply for the stability of low-dimensional crystals?

Hints: Focusing on acoustic modes similar to problem 3 in the previous hwk one finds the phonon dispersion in the cubic lattice

$$\Omega_{\mathbf{k}} = 2\sqrt{\frac{K}{M}}\left[\sum_{i=1}^{d}\sin^2(ak_i/2)\right]^{\frac{1}{2}}.$$

The k-space integrals that appear in the calculation of $\langle u_j^2\rangle$ are most conveniently treated using the standard Debye approximation discussed in PHYS 502 or in your undergraduate CM course. Consult Ashcroft & Mermin or Kittel textbook if you need a refresher on the the Debye model.